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Topological and Geometrical Properties of Braided Streams

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Abstract. Strong relationships among dimensionless properties of braided streams indicate that similarity is preserved in streams with the same average number of channels but of different sizes. The degree of braiding in streams, conveniently measured by the average number of channels bisected by lines crossing the channel, increases with the product of discharge and gradient, but decreases with higher variance in discharge. A random walk simulation model of braiding duplicates many of the numerical relationships observed in natural streams, suggesting that most of the downstream variation in the number of channels in braided streams is explainable by local fluctuations in discharge and sediment in transport, as opposed to large-scale factors such as valley constriction.

INTRODUCTION

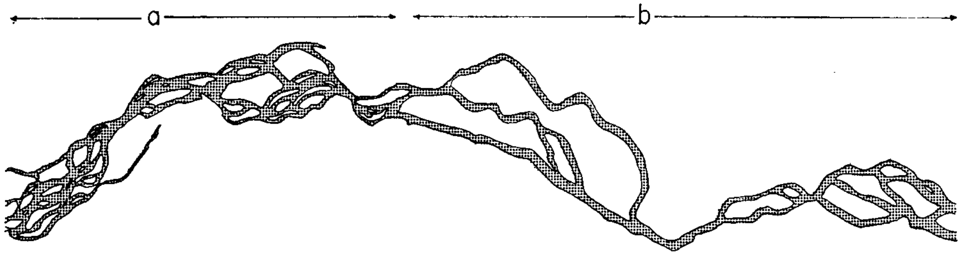
The planimetric properties of single thread streams have received extensive quantitative study, especially of their hydraulic geometry and meandering behavior. Some quantitative studies of stream meandering have been descriptive [*Leopold and Wolman*, 1957, 1960; *Dury*, 1965; *Speight*, 1967; *Chang and Toebes*, 1970], some have related meandering behavior to hydrologic factors [*Schumm*, 1968, 1969; *Bagnold*, 1960; *Chang and Toebes*, 1970], and some have proposed theoretical or simulation models of stream meandering [*Langbein and Leopold*, 1966; *Thakur and Scheidegger*, 1968; *Scheidegger*, 1967; *Surkan and Van Kan*, 1969]. Numerical properties of braided streams have received much less attention because of the comparative rarity of highly braided streams and because of the lack of obvious quantifiable properties. The hydrological factors controlling braiding in streams have been discussed by *Leopold and Wolman* [1957], *Fahnestock* [1963], and *Brice* [1964].

Leopold and Wolman [1957] proposed the division of stream patterns into single thread channels (straight or meandering) and braided streams with multiple anastomosing channels (anabranches). This distinction is arbitrary because most meandering or straight streams have a few islands along their length, and braided streams differ among themselves in the average number of channels at a point. The continuum of channel patterns from unbranched to highly braided has been quantified by the braiding index introduced by *Brice* [1964, p. D27], which is the ratio of twice the length of the islands in the reach of stream divided by the length of the reach. Although braiding in some streams appears to be best described as islands in a single channel (Figure 1*a*), others are better considered as anastomosing channels (Figure 1*b*). The second approach is used in this paper, which uses the numerical properties of the channel segments rather than those of the islands.

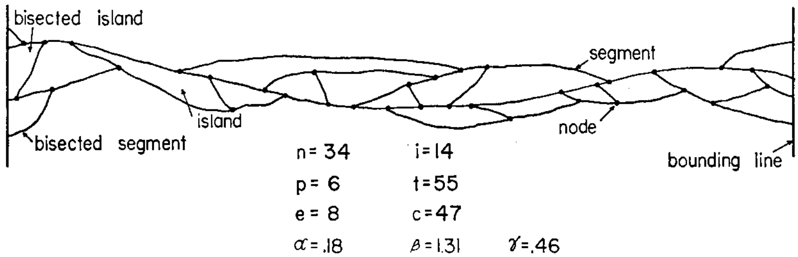
This paper reports four interrelated studies: (1) the topology of braided streams, (2) quantitative relationships between several properties of braided streams measurable from maps, (3) relationships between the degree of braiding and regime factors of discharge, sediment transport, and gradient, and (4) random walk simulation studies of stream braiding.

TOPOLOGY OF BRAIDED STREAMS

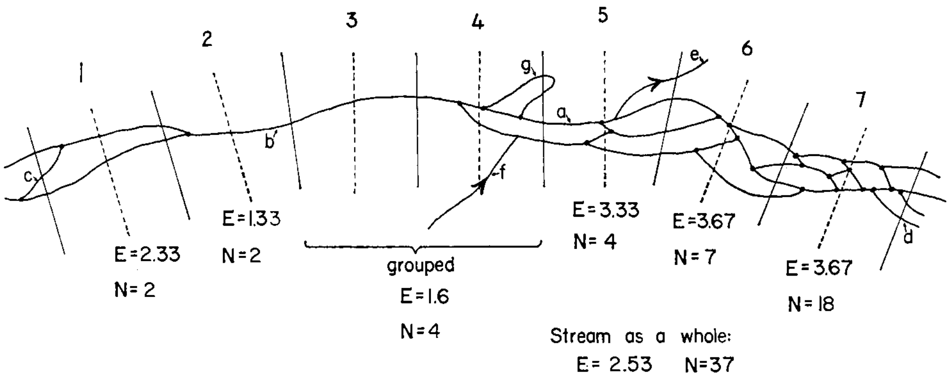
A braided stream is characterized by channel segments (anabranches), nodes where segments branch or join, and islands enclosed by segments (Figure 1). If a braided stream is sectioned by two arbitrary, crosscutting lines subject to the restrictions that the lines are far enough apart that no segment is crossed by both lines and that neither line passes directly through any node, then nodes, islands, and segments are numerically related. The following definitions are made (Figure 1):



(c)



(d)



(e)

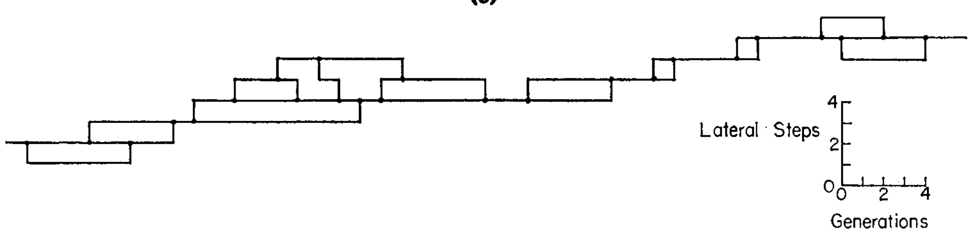


Fig. 1. Features of braided streams. (*a* and *b*) Channel with numerous small islands is contrasted with anastomosing channels on the Kuskokwim River, Alaska. Small channels are omitted. (*c*) Topologic definitions on braided streams. (*d*) Conventions in measurement on braided streams. E and N are defined in notation. If one or more sections in sequence possess only a single segment (section 3, case b), the one-channeled section(s) are grouped with the first higher numbered section with multiple segments (section 4). Braids crossing the end of a section are included in the data for the data for the section of next higher number for counting and length measurements (cases a and c), or ignored if they leave the measured length of stream (case d). Channels e and f, which do not reunite with the stream, were excluded, and were not considered to have created a node at their junction. Segments that are twice intersected by a crossline are counted only once (case g). (*e*) Simulated braided stream (model 1, step width 3, bias 0.75).

t, total number of segments, including segments lying entirely within the section (entire segments) and those bisected by the lines bounding the section (bisected segments);
e, total number of bisected segments;
c, total number of entire segments;
i, total number of unbisected islands within the section;
p, total number of islands bisected by the bounding lines;
n, total number of nodes within the section.

The topological theory of networks shows that the following relationship is always true in a planar network [*Berge,* 1962, p. 27; *Kansky,* 1963, p. 10]:

$$i = t - (n + e) + 1 \quad (1)$$

The number of islands i is termed the cyclomatic number in graph theory. In addition, the following two relationships are always true:

$$e = p + 2 \quad (2)$$

and

$$c = t - e \quad (3)$$

The joining of more than three segments at a node is rare in natural braided streams. If no more than three join at a node, then two additional relationships hold

$$t = 1 + 3i + 2p \quad (4)$$

and

$$n = 2i + p \quad (5)$$

If the sample section is long so that $i \gg p$ then

$$n \cong \tfrac{2}{3}t \quad (6)$$

Three topologic indices have been proposed to measure the degree of connection between nodes on a planar graph [*Garrison and Marble,* 1962; *Kansky,* 1963]

$$\alpha = \frac{t - (n + e) + 1}{2(n + e) - 5} \quad (7)$$

and

$$\beta = \frac{t}{n + e} \quad (8)$$

and

$$\gamma = \frac{t}{3(n + e - 2)} \quad (9)$$

The alpha index is the ratio of the observed number of islands to the greatest possible number of islands for a given number of nodes. The gamma index is the ratio of the observed number of channel segments to the greatest possible number of segments for the given number of nodes. In a large section of braided stream where $i \gg p$ and $n \gg e$, substitution of equation 6 into equations 7, 8, and 9 shows that α, β, and γ approach the values $\frac{1}{4}$, $1\frac{1}{2}$, and $\frac{1}{2}$, respectively.

INTERRELATIONSHIPS BETWEEN PROPERTIES OF BRAIDED STREAMS

The quantitative variables include only those that can be determined from topographic maps; these include measures of the number of channel segments, their length and width, and such properties as channel gradient, and wavelength and sinuosity of channel meandering (notation).

Twenty-six streams were selected for measurement subject to the following criteria:

1. A wide range in the degree of braiding among the streams selected, as estimated by visual appearance.
2. Availability of recent topographic mapping at a scale of 1:24000 or 1:62500, or detailed maps at a larger scale.
3. At least one channel traceable along the stream with measurable width on the map.
4. Absence of large incoming tributaries or man-made levees and channels.

With the exception of these restrictions, the streams measured were selected to be as representative as possible of streams in the United States including Alaska, ranging in size from the Mississippi River to an ephemeral arroyo with 4 square miles of drainage area. (Information on the streams measured and copies of the data are available from the senior author.)

Systematic or random effects upon measured data have probably been introduced in the process of map compilation because of the following factors:

1. Lack of universal criteria for definition of channels or islands. *Brice* [1964, p. D31] notes

that two streams which show a similar degree of braiding on aerial photographs appear markedly different in degree of braiding on their respective topographic maps.

2. Differences in conditions under which areal photography was flown. In some photography, differentiation between water and land may be difficult, leading to confusion of islands and submerged bars.

3. Differences in river stage. A rise in stage may submerge low bars or, conversely, reactivate ephemeral channels. However, the photography is likely to have been taken near the median discharge.

These factors have probably introduced random effects and may have systematically affected average values of stream properties, especially measures of the number of channels (E and N, notation). The process of map making is less likely to be responsible for the observed correlations between stream properties.

Measurements. A typical length of channel without large incoming tributaries is selected; this is divided into sections of equal length by crosslines perpendicular to the main direction of flow. The length of each section was approximately two times the average width of the stream. Each section is then bisected as shown in Figure 1*d*. On a tracing paper overlay the center line for each channel segment is drawn and the measurements indicated in the notation list were made on each section and for the stream as a whole. Several conventions adopted in measurement are illustrated in Figure 1*d*.

Analysis of Data. Two types of relationships between variables measured from the map were investigated:

1. Relationships between dimensionless measures and ratios (notation). The correlations between these scale free properties involve those properties of braided streams that are independent of the size of the stream. Several of the scale free variables have a factor of unity subtracted in the formula; this permits these variables to equal zero in a single thread stream (or a nonmeandering stream in the case of the sinuosity index).

2. Relationships between dimensioned variables (notation) that reveal the degree to which the size of a braided stream affects its properties. Included with these variables are the gradient, which is inversely related to the width and meander wavelength of single thread streams, and the excess segment index, to investigate relationships between scale free and dimensioned variables.

Analysis of both scale free and dimensioned variables was further broken into:

1. Average correlations of variables between sections within each stream. The correlation coefficients between variations in quantitative variables among the sections within each stream is computed. The correlation coefficients for each variable are averaged over the total number of streams in the sample (26 here), with each correlation coefficient being weighted by the square root of the number of sections measured along each stream.

2. Correlations of variations in the average properties of the measured streams (between stream correlations). The measured variables for each stream were computed by considering the measured portion of the stream to be a single long section, except that the variables W and C were computed by averaging the average values for each section.

The measured and derived variables were first plotted versus each other on arithmetic axes. In many cases these produced curvilinear relationships that became nearly linear when the logarithms of the variables were plotted. Therefore the logarithmic transforms of all variables were taken before correlation, implying a relationship between two variables (e.g., X and Y) of the form

$$Y = KX^m \qquad (10)$$

where K and m are constants. This type of relationship is common in hydraulic geometry [*Leopold and Maddock*, 1953].

Regression parameters for the four types of relationships (scale free or dimensioned and within or between streams) are given in Tables 1 and 2. Note that some quantities were measured only once for each of the sampled streams, so that these, and their derivative scale free parameters, do not appear in the within stream correlations.

Interpretation. Many scale free parameters in geomorphology are largely independent of the size of the phenomenon involved, for example, number, length, and area ratios by order in

TABLE 1. Scale Free Correlations

Independent Variable	Dependent Variable												
	E_i	G	U_i	R_{cm}	R_{sm}	R_{sc}	R_{wm}	R_{wc}	R_{ws}	T_m	T_c	T_s	T_w
	Correlation Coefficients												
E_i		0.11	...	0.46	−0.25	−0.52	...	...	...	0.59	0.71	0.55	...
G	0.07		...	0.07	0.02	−0.01	...	...	...	0.08	0.10	0.14	...
U_i	−0.44	0.09		...	...	...	...	...	...	...	...	...	...
R_{cm}	0.80	0.16	−0.30		0.27	−0.64	...	...	...	0.04	0.53	0.35	...
R_{sm}	−0.41	0.25	0.22	0.13		0.64	...	...	...	−0.67	−0.36	0.05	...
R_{sc}	−0.89	0.07	0.37	−0.59	0.71		...	...	...	−0.61	−0.74	−0.21	...
R_{wm}	0.22	−0.11	−0.26	0.43	0.50	0.09		...	...	...	...	...	...
R_{wc}	−0.49	−0.28	−0.02	−0.48	0.32	0.61	0.56		...	...	...	...	...
R_{ws}	0.63	−0.36	−0.47	0.27	−0.57	−0.65	0.43	0.20		...	...	...	...
T_m	0.80	−0.12	−0.32	0.35	−0.85	−0.93	−0.22	−0.49	0.68		0.80	0.63	...
T_c	0.96	0.00	−0.36	0.71	−0.57	−0.97	0.04	−0.60	0.63	0.90		0.76	...
T_s	0.96	0.10	−0.31	0.81	−0.28	−0.80	0.23	−0.50	0.52	0.74	0.92		...
T_w	0.89	−0.18	−0.46	0.59	−0.50	−0.82	0.39	−0.13	0.90	0.81	0.87	0.84	
	*Fitted Exponents**												
E_i		...	−0.31	0.89	−0.32	−0.85	...	−0.36	0.49	0.92	1.45	0.60	1.09
G	...		...	...	...	...	...	...	−0.25	...	...	...	...
U_i	−0.63	...		...	...	0.50	...	...	−0.52	−0.52	−0.78	...	−0.80
R_{cm}	0.72	...	...		...	−0.51	0.28	−0.32	...	0.36	0.96	0.46	0.65
R_{sm}	−0.51	...	...	...		0.85	0.45	0.30	−0.55	−1.22	−1.06	...	−0.77
R_{sc}	−0.93	...	0.27	−0.69	0.59		...	0.48	−0.52	−1.11	−1.52	−0.52	−1.05
R_{wm}	...	...	...	0.65	0.54	...		0.57	0.46	...	...	...	0.66
R_{wc}	−0.66	...	...	−0.72	0.35	0.79	0.55		...	−0.76	−1.21	−0.42	...
R_{ws}	0.82	−0.52	−0.43	...	−0.59	−0.81	0.41	...		1.01	1.23	0.42	1.42
T_m	0.70	...	−0.20	0.34	−0.60	−0.77	...	−0.32	0.46		1.18	0.40	0.86
T_c	0.64	...	−0.17	0.53	−0.30	−0.62	...	−0.29	0.32	0.69		0.38	0.71
T_s	1.53	...	...	1.44	...	−1.23	...	−0.59	0.64	1.36	2.23		1.64
T_w	0.73	...	−0.27	0.54	−0.33	−0.65	0.24	...	0.57	0.76	1.08	0.43	
	*Common Logarithm of Multiplicative Constant**												
E_i		...	−0.92	−0.01	0.56	0.19	...	1.02	0.84	−0.12	0.24	0.43	1.27
G	...		...	...	...	...	...	...	0.23	...	...	...	...
U_i	−0.43	...		...	...	0.52	...	...	0.42	−0.46	−0.24	...	0.70
R_{cm}	0.08	...	...		0.48	0.11	1.38	1.01	...	0.00	0.37	0.48	1.38
R_{sm}	0.45	...	...	0.08		−0.39	1.20	0.81	1.20	0.66	1.05	...	1.86
R_{sc}	0.22	...	−0.99	0.18	0.48		1.42	0.94	0.94	0.08	0.56	0.56	1.51

R_{vm}	...	...	...	−0.76	−0.28	−0.14		0.14	0.28	...	...	...	0.55
R_{wc}	0.82	...	...	0.85	0.16	−0.73	0.90		...	0.79	1.68	0.95	...
R_{ws}	−0.57	−2.36	−0.58	...	1.04	0.77	1.04	...		−0.88	−0.61	0.16	0.16
T_m	0.16	...	−0.97	0.15	0.53	0.07	...	0.97	0.91		0.46	0.53	1.43
T_c	−0.14	...	−0.89	−0.11	0.65	0.35	...	1.10	0.76	−0.31		0.35	1.11
T_s	−0.65	...	...	−0.63	...	0.70	...	1.28	0.58	−0.69	−0.70		0.58
T_v	−0.89	...	−0.59	−0.63	0.99	0.98	1.07	...	0.09	−1.07	−1.07	−0.09	

Values in upper right section are within stream correlations. The centered dots indicate parameters not measured. Values in lower left section are between-stream correlations. Correlation coefficients of ±0.33 and ±0.39 are critical values for 90% and 95% levels of significance, respectively, in the between-stream correlations.

* Regression parameters show between-stream relationships. The fitted equation has the form $Y = KX^m$, where X and Y are the independent and dependent variables, respectively, K is the multiplicative constant, and m is the exponent. Entries are given only for correlations significant at the 90% level of significance.

See notation list for definition of symbols.

stream networks, and the ratio of the meander wavelength to the channel width in meandering streams [*Leopold and Wolman,* 1960, table 1]. Several observations suggest that the same may be true of braiding in streams:

1. The between stream, scale free parameters of braiding strongly correlate among themselves, despite large variations in the size of the stream (Table 1).

2. The within stream scale free correlations (Table 1) are close to those between streams, as would be expected if the scale free properties are independent of scale.

3. Most of the strong correlations between dimensional properties (Table 2) are due to their having the same units of measurement (miles); regression slopes between these properties are generally near ±1.0.

However, moderate correlations occur between the excess segment index E_t, a scale free variable, and several dimensional variables (Table 2), e.g., the number of channels per mile N_t. This would not be expected if dimensionless characteristics of braiding are independent of the size of the stream. However, these correlations are probably due to the particular selection of streams in this sample. An independent sample of 74 streams resulted in almost no correlation between E_t and N_t (Table 3).

Two generalizations about braided streams are confirmed by this study. *Leopold and Wolman* [1957, p. 48] note that an increase in the number of channels along a stream is associated with an increase in gradient. A moderate positive correlation is indicated within streams between the excess segment index E_t and the gradient G (Table 1). *Leopold et al.* [1964, p. 292] find that braided streams are less sinuous than single thread streams. The excess segment index E_t and the sinuosity index U_t are negatively correlated (Table 1).

BRAIDING RELATED TO HYDRAULIC REGIME

The correlations reported above allow the prediction of several properties of braided streams once one parameter of that stream is measured, for example, the excess segment index E_t or the number of channels per mile N_t. However, these correlations cannot be used for a priori prediction of the degree of braiding of the stream. Braiding evidently occurs under certain condi-

TABLE 2. Dimensional Correlations

Independent Variable	Dependent Variable						
	N_l	E_i	G	S	M	C	W
				Correlation Coefficients			
N_l		0.64	0.08	−0.49	−0.07	0.39	...
E_i	0.53		0.11	−0.26	−0.01	0.51	...
G	0.65	0.07		0.06	−0.09	0.09	...
S	−0.70	0.20	−0.64		−0.02	0.01	...
M	−0.33	0.43	−0.61	0.68		0.10	...
C	−0.04	0.78	−0.42	0.67	0.84		...
W	−0.34	0.53	−0.63	0.78	0.80	0.82	
				Fitted Exponents			
N_l		0.64	0.88	−0.63	−0.43	...	−0.49
E_i	0.44		...	...	0.47	1.00	0.64
G	0.48	...		−0.43	−0.61	−0.49	−0.68
S	−0.77	...	−0.94		1.00	1.14	1.25
M	−0.25	0.39	−0.62	0.47		0.98	0.88
C	...	0.61	−0.37	0.39	0.72		0.77
W	−0.23	0.44	−0.58	0.49	0.73	0.88	
				Common Logarithm of Multiplicative Constant			
N_l		−0.58	−3.90	0.11	−0.62	...	0.87
E_i	1.13		...	...	−1.24	−0.87	0.15
G	2.59	...		−1.87	−2.87	−2.06	−1.64
S	0.71	...	−3.46		−0.49	0.07	1.10
M	0.93	0.65	−3.55	−0.12		0.45	1.28
C	...	0.61	−3.09	−0.39	−0.66		0.80
W	1.28	0.07	−2.68	−0.79	−1.35	−0.92	

For explanation see Table 1.
See notation list for definition of symbols.

tions of water flow, sediment transport, and bank erodibility [*Leopold and Wolman*, 1957; *Fahnestock*, 1963; *Brice*, 1964]. Therefore indices of these regime factors might allow the prediction of the degree of braiding.

Several hydraulic characteristics of 74 streams throughout the United States were collected from published reports to determine their relationship to the degree of braiding measured by the parameters E_i and N_l measured from topographic maps. The hydraulic variables include gradient, bed material size, mean annual flood, and a measure of the variability of discharge (notation).

Both simple and multiple regressions were used to determine the relationships between braiding and hydraulic regime (Table 3). These correlations may be compared with the conclusions of other studies:

1. *Leopold and Wolman* [1957, p. 60] note that braiding is favored when both gradient and discharge are high, and single thread meandering for lower values. This is supported by the present study because the excess segment index E_i increases as gradient and discharge increase (Table 3).

2. *Fahnestock* [1963, p. A58] suggests that high variability in discharge may be associated with a high degree of braiding, whereas *Brice* [1964] finds little association. However, the multiple regressions indicate an inverse relationship between E_i and the variability of discharge.

Although significant correlations occur between indices of braiding and the regime factors, the degree of explanation is low, perhaps owing to the following criteria:

Differences in mapping procedures. Although the process of map compilation probably has little effect upon the correlations between properties of braided streams (see previous discussion), systematic inclusion or deletion of channels affects both of the measures of the degree of braiding, E_i and N_l. Use of aerial photographs and standardized procedures would help to reduce operator bias (see discussion in *Brice* [1964, p. D27–D32]).

The influence of bank erodibility. Stability of channel banks, due to cohesive sediment [*Schumm*, 1960; *Brice*, 1964, pp. D32–D35; *Fahnestock*, 1963, pp. A57–A58] or to a protective cover of vegetation [*Brice*, 1964, pp. D32–D35; *Mackin*, 1956] reduces channel width and inhibits development of islands. Bank erodibility is difficult to assess. The type and density of bank vegetation differs greatly among the sampled streams and is probably an important factor not accounted for. *Schumm* [1960] has used the percentage of silt and clay in the banks to measure channel stability. However, a separate regression for 22 streams for which *Schumm* [1960, Table 1] had measured the silt-clay content of the channel banks produced little additional explanation of the degree of braiding.

TABLE 3. Correlations of Braiding and Hydraulic Parameters

	Dependent Variable					
Independent Variable	E_i	N_l	Q_f	R_q	D	G
			*Correlation Coefficients**			
N_l	0.00					
Q_f	0.04	−0.42				
R_q	−0.23	0.14	−0.20			
D	−0.02	0.29	−0.36	−0.11		
G	0.21	0.27	−0.65	0.13	0.48	
			Fitted Exponents†			
E_i		...	...	−0.28	...	0.21
N_l	...		−0.52	...	0.60	0.32
Q_f	...	−0.34		−0.23	−0.62	−0.61
R_q	−0.19	...	−0.17		...	...
D	...	0.14	−0.21	...		0.27
G	0.21	0.23	−0.68	...	0.87	
			Common Logarithm of Multiplicative Constant†			
E_i		...	...	1.01	...	−2.70
N_l	...		4.33	...	−0.55	−3.14
Q_f	...	2.23		2.07	2.38	−0.49
R_q	−0.45	...	4.05		...	...
D	...	0.93	3.85	...		−2.85
G	−0.07	1.59	1.90	...	2.48	

Equation	Multiple Correlation Coefficient
Multiple Regressions‡	
$E_i = 0.24\ G^{.41} Q_f^{.29}$	0.31
$E_i = 1.9\ R_q^{-.21} G^{.24}$	0.33
$E_i = 0.58\ R_q^{-.19} G^{.41} (Q_f^{.25})$	0.39
$G = 0.14\ D^{.16} Q_f^{-.51}$	0.70
$G = 0.21\ D^{.16} Q_f^{-.52} E_i^{.24}$	0.74
$D = 200\ (R_q^{-.28})(N_l^{.40}) G^{.82}$	0.54
$Q_f = 260\ E_i^{.18} N_l^{-.31} G^{-.65}$	0.71

* Correlation coefficients of ±0.20 and ±0.23 are critical values for 90% and 95% levels of significance, respectively.

† For explanation see Table 1.

‡ Variables in parentheses are significantly correlated with the dependent variable at a 90% level of significance. Others are significant at a 95% level of significance.

See notation list for definition of symbols.

Past hydrologic history. Major floods tend to widen channels both immediately by lateral erosion and over longer periods of time by stripping the vegetation from the banks, whereas channels tend to narrow by growth of vegetation and deposition of alluvium during periods with high discharges but only minor flooding [*Schumm and Lichty*, 1963]. Thus the elapsed time since major floods is probably a major uncontrolled factor. Recent changes in climate may likewise have affected erosion, deposition, and vegetative growth on stream banks.

SIMULATION MODELS OF BRAIDING

The downstream fluctuation in the number of channels and other properties of braided streams might arise either through specific local influences, such as valley narrowing by exposure of bedrock or changes in bank vegetation, or on the other hand it might be due to the nature of the flow and sediment transport within the channel. The following three observations suggest that the second of these is most important in most braided streams:

1. The within stream fluctuations in dimensionless properties are similar to between stream variations (Table 1). Such regularity would not necessarily occur if local factors determined the degree of braiding.

2. Fluctuations through time in number and areal position of channels correlate with variations in flow conditions [*Fahnestock*, 1963, pp. 43–57].

3. Many, and perhaps most, braided streams flow on alluvium in wide, flat valleys so that local constrictions and similar large-scale local factors are probably unimportant.

A braiding pattern appears to develop in streams through buildup of bars by local deposition within the channel, probably due to areal and temporal variance in flow conditions and sediment in transport [*Leopold and Wolman*, 1957, p. 53; *Fahnestock*, 1963, p. 56]. Local diversions of flow during high discharges may also be important [*Brice*, 1964, p. 27]. The factors producing the local variations that induce braiding are probably so numerous and so rapidly changing that a satisfactory deterministic modeling of braiding would be difficult to construct. However, the braided pattern produced by these innumerable, small-scale factors, past and present, might be simulated by a random process of branching and reuniting. Several such models are reported below.

Model 1. Stream braiding is simulated by a random walk through successive generations during which the stream grows in a longitudinal direction (i.e., upstream or downstream). During each generation existing segments extend longitudinally one step without lateral (cross-stream) shifting. At the close of each generation, channels shift laterally by a number of steps determined by a probability function described below. During these lateral steps channels may coalesce to form a single channel that grows longitudinally during the following generation. Segments may also branch at the close of a generation.

Both the number of lateral steps and the occurrence of branching are randomly determined by the probability function. The probability of a stream head taking N lateral steps from its present position X is assumed to be proportional to the area beneath a section of a normal curve. The boundaries of the section are determined by the average of the positions of the stream heads $\langle X \rangle$, the step width (W in units of standard deviations), and the bias of the movement toward the average position (B also in units of standard deviations). A random normal deviate R from a normal distribution with mean zero and standard deviation of unity determines the number of lateral steps. Using $I[Z]$ to represent the

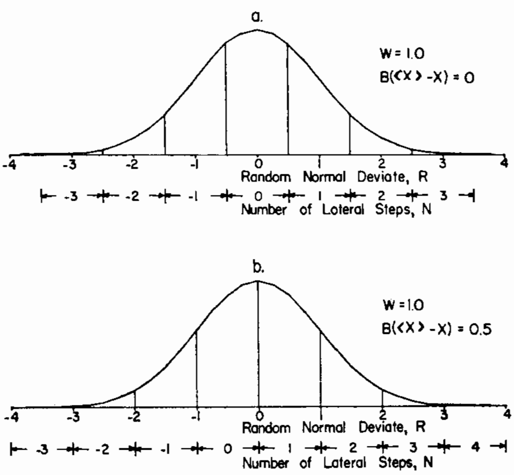


Fig. 2. Examples of the probability function for lateral steps in the simulation of braided streams.

integer part of the expression Z, the probability function is (also see Figure 2)

$$N = \begin{cases} I[(D/W) + 0.5] & D \geq 0 \\ -I[(-D/W) + 0.5] & D < 0 \end{cases} \quad (11)$$

where

$$D = R + B(\langle X \rangle - X) \quad (12)$$

The simulation model thus has two adjustable parameters, B and W, that are specified at the beginning of the simulation. The bias B must always be greater than zero, otherwise branching of channels would lead to a continual increase in the number of channels.

The lateral movement of channels is conducted according to the following process, with the restriction that splitting and coalescing of channels produces not more than three channels joining laterally at any generation:

1. Channel segments existing at the close of the generation are examined in a random serial order. Each segment is initially active. Each active channel is subjected to the following action:

a. The number of lateral steps, positive (Figure 3, model 1, case b), negative, or zero (case a), is randomly determined according to the probability function. However, not all

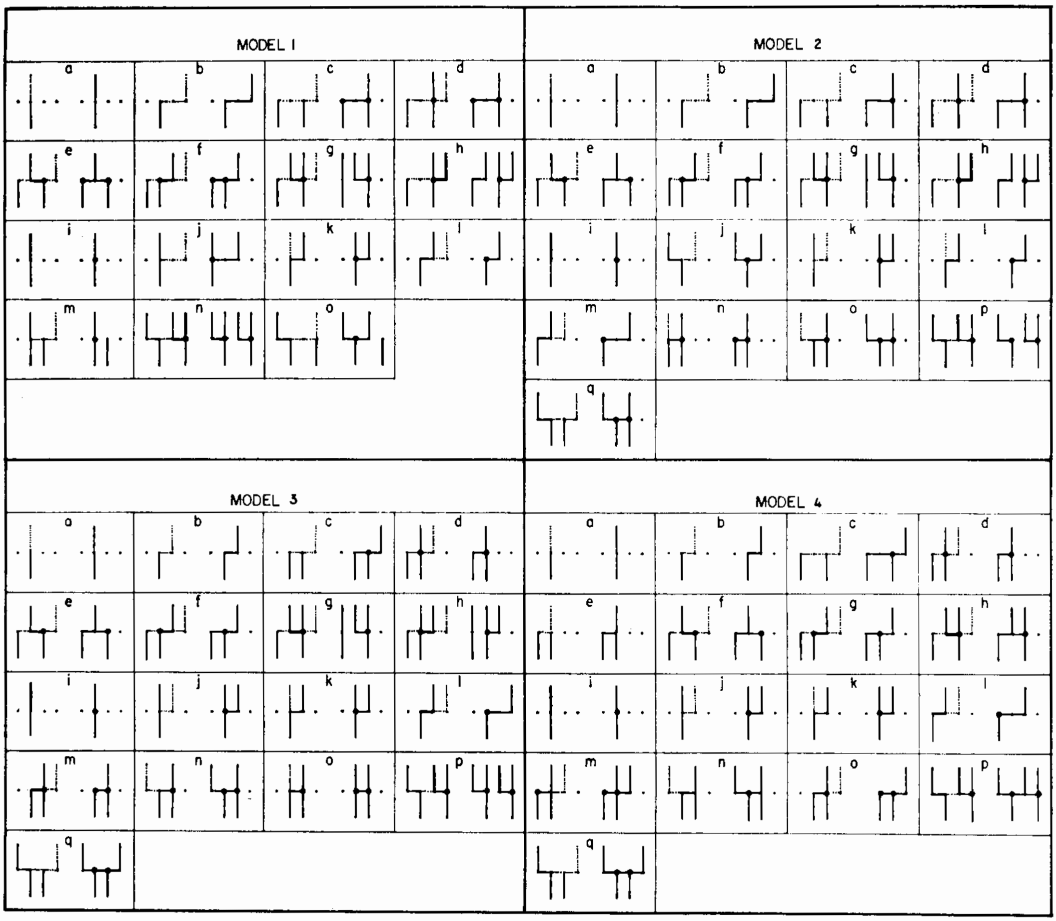


Fig. 3. Rules for branching and coalescence of braids. In each box the left-hand diagram shows the action indicated by the randomly generated number of lateral steps (dotted lines) and the right-hand diagram shows the action taken. Solid lines show existing channels. Lines below the row of dots show channels existing during the nth generation, and those above the $(n + 1)$th generation. Circles indicate inactive stream heads.

steps are taken if the channel encounters an existing segment, in which case the streams may join (cases c, d, e, and f) or not (cases g and h) depending on whether this would violate the three-junction rule. If two segments join, then both are deactivated (cases c, d, e, and f).

b. If the channel is still active, the possibility of splitting is examined by a second random number of lateral steps; if this second lateral movement would be in the same direction as the first (case l), or if both are zero (case i), no branching occurs. A splitting can only occur if no more than three segments meet at a point (cases j and k), and the second channel cannot unite with another (cases m, n, and o).

c. The channel is deactivated.

2. The new disposition of channels produced by this process is carried through to the end of the succeeding generation.

All simulations started with a single stream and were terminated after a specified number of generations (100 to 200). A typical simulation is illustrated in Figure 1*e*.

Models 2, 3, and 4. The simulation rules for these models are identical to those of model 1 except that the rules for branching and coalescence were varied. Model 2 is more restrictive than model 1 in branching of single channels (Figure 3, model 2, case l), but is more liberal in branching of connected channels in that it allows more than three segments to join at a point in some cases (cases o and q). Models 3 and 4 have the same rules for branching of single channels as model 1, but they allow greater freedom of coalescence and branching of connected channels (Figure 3).

Analysis of simulated networks. Simulated and natural streams may be compared only through use of dimensionless parameters. The excess segment index E_i is directly comparable in natural and simulated streams, and variations in the average value of this index, because of different combinations of step width and bias parameters, are used to compare the dimensionless properties of natural and simulated streams. Dimensionless combinations of average stream width, segment length, and number of segments per length of channel may be measured on simulated streams as well as on natural ones. The following conventions were adopted in measuring these:

1. In the simulation model the choice of longitudinal and lateral scale is arbitrary. Because most segments of natural streams are elongated in the longitudinal direction (Figure 1), the length of a segment in the simulation model is defined as the number of generations during which the segment exists. In such a case the dimensionless ratio $T_s = N \cdot S/L$ equals the average number of streams bisected in a lateral direction (i.e., E); braiding in most natural streams closely approximates this equality (Figure 4).

2. The total width of simulated streams was taken to be the total number of lateral units within the two outside segments, including the outside segments; a single channel has a width of unity. In natural streams the width of the main channel M does not vary greatly during within stream variations in the total number of channels (Table 2). Therefore the width as measured above is assumed to be proportional to the width ratio R_{cm} in natural streams.

3. Average stream width C and the average segment length S were decreased by unity in order that these parameters would have minimum values of zero rather than unity.

Because of the independence between the lateral and longitudinal scales in the simulation

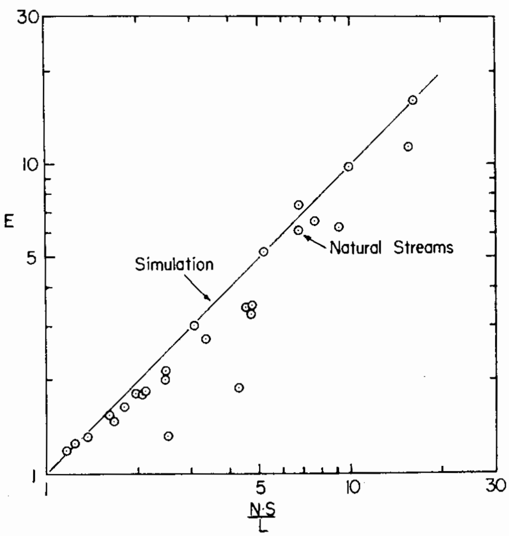


Fig. 4. Comparison of the ratio $N \cdot S/L$ for natural and simulated streams, plotted versus E. Symbols are explained in the notation.

models, dimensionless ratios involving both channel width and segment length vary greatly with changes in either the step width or the bias. The effects of variations in both parameters was investigated only for model 1 (Figure 5). For a given step width but different biases the dimensionless parameters vary systematically with the excess segment index, and the curves for different step widths are generally parallel (Figure 5). The slopes of these relationships for model 1 are generally close to those of natural stream networks except at low values of E_i (the intercepts of these curves are unimportant, for they depend upon the entirely arbitrary choice of longitudinal and lateral scales). The break in the relationships plotted in Figure 5 for model 1 is probably due to the limiting of segment lengths to values of one generation or greater, because a smaller break occurs for higher values of the step width (which correspond to longer average segment lengths).

The remaining models (2 and 4) were investigated only at a step width of 3.0. Model 2 appears to simulate most closely braiding in natural streams, for the relationships between the excess segment index and other dimensionless parameters are approximately linear (on logarithmic paper) and the slopes of the relationships are very close to the regressions obtained between natural streams (Figure 5). The more restricted rules for channel splitting, which lead to longer average segment lengths, probably account for this better fit to natural data.

Although the choice of step width for the simulation models is arbitrary, the use of large step widths not only produces better simulations of natural braiding, but it also allows the simulation of streams with lower values of the excess segment index. In the limiting case of very high biases, simulated streams consist of long single segments alternating with two-channel sections that extend only one generation longitudinally. In this case, the minimum obtainable excess segment index can be predicted (Figure 6). These predicted values compare closely with the minimum values actually obtained in the simulations (Figures 5 and 6).

The variance in the number of segments in lateral cross sections (Var E) is nearly identical for natural and simulated networks having the same excess segment index (Figure 5). This supports the supposition that the variance in the number of segments in a downstream direction may be accounted for by small-scale fluctuations in flow conditions in the channel rather than by systematic changes of local factors. The few natural streams with abnormally large variance may, however, be affected by large-scale factors within the measured reach.

The length of links in digital simulations of dendritic stream networks approximately follows a geometrical distribution [*Smart et al.*, 1967; *Howard*, 1971]. The length of links in natural streams is close to an exponential distribution [*Smart*, 1968], which is the continuous equivalent of the geometrical distribution, although the gamma distribution appears to give a slightly better fit [*Shreve*, 1969; *James and Krumbein*, 1969]. The distribution of braid lengths in natural and simulated networks might also follow these simple distributions. One test for the closeness of approximation of the geometrical and exponential distribution to simulated and natural braid lengths, respectively, is to see whether the mean and variance are related as with these distributions.

Under the geometric distribution the variance divided by the mean times the mean minus one should be equal to unity; similarly for the exponential distribution the variance divided by the square of the mean should be unity. The geometrical distribution seems to fit the simulated braid lengths very well except that the variance is too great when braiding is weak (low E_i, Figure 5). This is due to a contrast between long segments in the single-channel sections and short segments where the channel splits. A similar pattern is found in natural streams with respect to the exponential distribution, except that the variance may be too low for highly braided streams (Figure 5), perhaps because of operator bias in definition or measurement of segment lengths.

Although the numerical relationships for the simulation model are similar to those of natural braided streams, the process of generation is obviously not comparable. In natural streams the braided pattern develops and changes through time simultaneously along the length of the channel, whereas in the simulated streams the braiding is developed sequentially in the longitudinal direction. Once a pattern is formed in the simulation, it is no longer modified. Probably a large number of random models might satis-

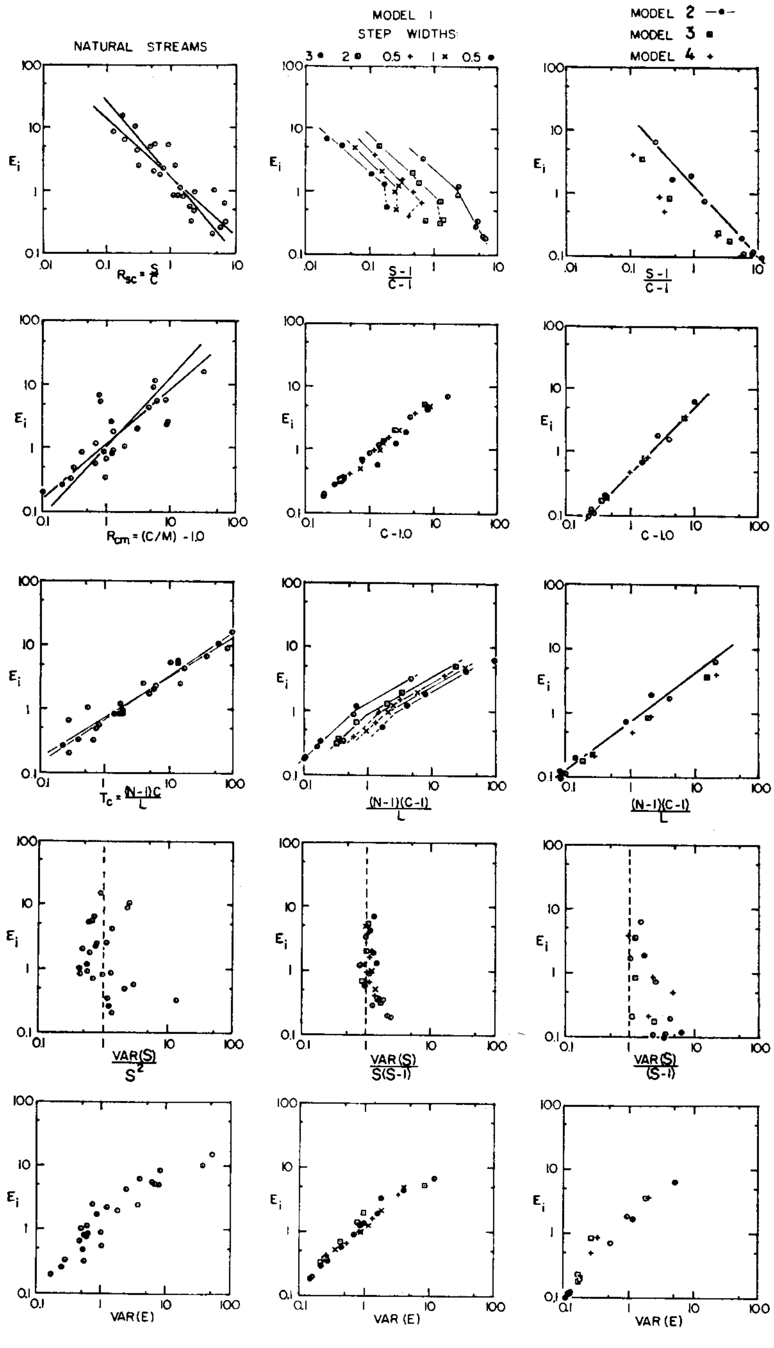


Fig. 5. Numerical properties of natural braided streams compared with simulated ones, plotted versus the excess segment index E_i. Mean values for individual natural streams are indicated by circles with the regression lines for both axes considered as independent variables in turn. Like symbols for simulation models indicate separate simulations with the same step width but different biases.

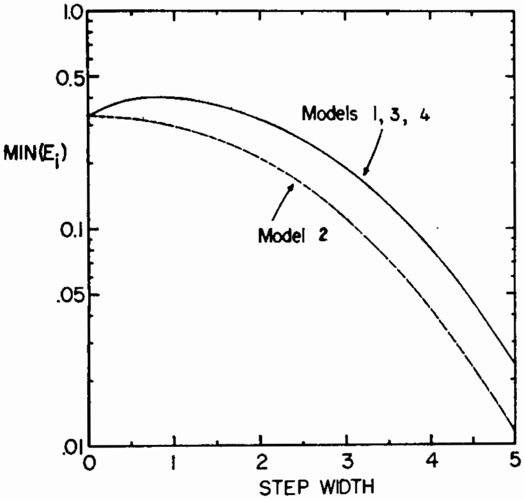


Fig. 6. Predicted minimum values of the excess segment index E_i, as a function of the step width. The minimum values obtain for large biases. If S is the probability of zero lateral steps in the simulation model, then the probability P of splitting of a single stream at the end of any generation is $2S(1 - S) + \frac{1}{2}(1 - S)^2$ in models 1, 3, and 4, and $S(1 - S) + \frac{1}{2}(1 - S)^2$ in model 2. Since each generation of a single stream is independent, the distribution of single stream lengths is geometric with mean $1/P$. The excess segment index produced by single channels alternating with single generation islands will therefore be $[(1/P) + 2]/[(1/P) + 1] - 1$ (plotted).

factorily simulate the numerical properties of braided streams without necessarily simulating the actual process of generation. For example, the rules for splitting and coalescence of channels (Figure 3) could be extensively modified. A similar situation exists for simulation models of dendritic stream networks [*Howard*, 1971].

NOTATION

Parameters measured for each stream section:

- E, average number of segments bisected by the crosslines at the ends and interior of the section;
- S, average length of all segments entirely within the section and those entering the section from a lower numbered section in miles;
- N, total number of segments entirely within the section and entering the section from a lower numbered section;
- M, average width of the widest channel through the section in miles;
- C, average width of the stream between the outermost segments within the section in miles;
- G, stream gradient in feet per feet;
- L, length of the section in miles.

Parameters measured only once for each stream:

- U, sinuosity of the widest channel that can be followed along the stream, i.e., the ratio of the length along the channel to the straight line distance between the ends of the measured length of stream;
- W, average wavelength, in miles, of the widest channel that can be followed along the stream, measured as the average crest to crest length.

Derivative scale free parameters:

- $E_i = E - 1$, excess segment index;
- $U_i = U - 1$, sinuosity index;
- $R_{cm} = (C/M) - 1$, width ratio index;
- $R_{sm} = S/M$, segment length–width ratio;
- $R_{sc} = S/C$, segment length–channel width ratio;
- $R_{wm} = W/M$, wavelength–segment width ratio;
- $R_{wc} = W/C$, wavelength–channel width ratio;
- $R_{ws} = W/S$, wavelength–segment length ratio;
- $T_m = [(N - 1)\cdot M]/L$, segments per segment width unit;
- $T_c = [(N - 1)\cdot C]/L$, segments per channel width unit;
- $T_s = [(N - 1)\cdot S]/L$, segments per segment length unit;
- $T_w = [(N - 1)\cdot W]/L$, segments per wavelength unit.

Derivative dimensioned parameters:

- $N_l = (N - 1)/L$, segments per mile of channel.

Hydraulic parameters:

- Q_f, mean annual flood in cubic feet per second;
- R_q, ratio of the mean annual flood to the mean annual discharge;
- D, median grain size of the channel bed in millimeters.

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