CAVES AND KARST

Research in Speleology

Volume 13, No. 1 January/February 1971

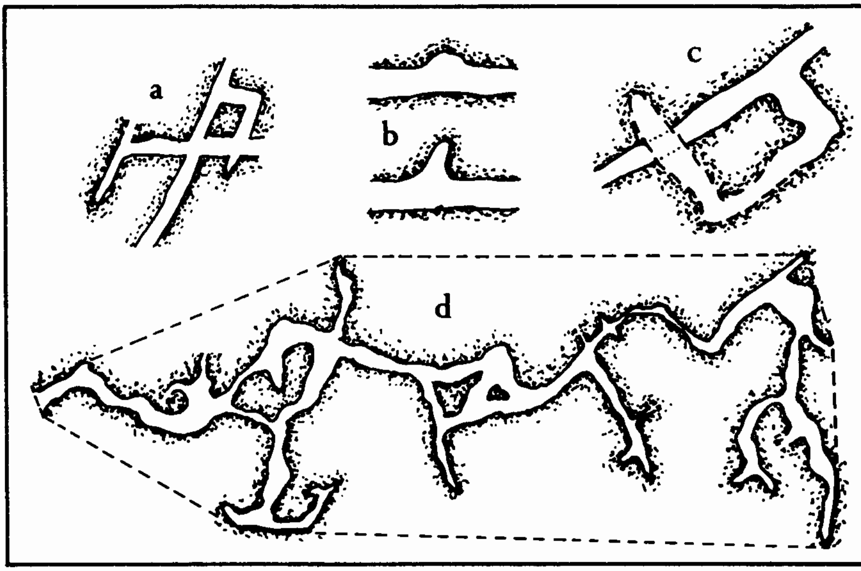

Figure 1. Hypothetical cave patterns showing measurement conventions: **(a)** Joint controlled passages having 2 junctions of 4 passages and 2 junctions of three; **(b)** An external node on a dead end passage is defined to occur only where that passage is at least 1½ to 2 times as long as it is wide. Thus the alcove in the top passage does not define an external node, whereas a passage link and external node are defined for the alcove in the lower passage; **(c)** Two passages that cross but do not connect cannot close a loop and form an island. The shaded interior between the two passages is therefore not an island; **(d)** The area enclosed by the periphery of the cave (A_p) is defined as the minimum-area convex polygon that will enclose the mapped passages (dashed line), whereas the cave area (A_c) is defined as the passage area within the periphery (not including islands). For the cave shown, the number of islands, i, is 3 and the number of external nodes, e, is about 21.

QUANTITATIVE MEASURES OF CAVE PATTERNS

By Alan D. Howard*

Abstract: Several measures of the topologic complexity of cave networks and of the intensity of solution in caves have been measured on 2 groups of caverns; i.e., 25 caves in Indiana formed by subterranean free-surface streams or beneath integrated water-levels, and 3 large caves apparently formed by artesian ground-water flow. Of the 2 groups, the artesian caves are topologically more complex and have had a greater percentage of the original limestone dissolved. Quantitative measurement of a cave pattern may help to distinguish the type of ground-water flow that formed the cave.

Introduction

The analysis of socio-economic and physical networks has been a fruitful area of geographic research in recent years (Haggett & Chorley, 1969). The properties of stream networks have been extensively quantified (Strahler, 1968), and considerable

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CAVES AND KARST is a publication of Cave Research Associates. Subscriptions are available for $3.25 per year (six issues) or $7.50 for Volumes 13 through 15. Mid-year subscriptions receive the earlier numbers of the volume.

Correspondence, contributions, and subscriptions should be addressed to: **CAVE RESEARCH ASSOCIATES,** 3842 Brookdale Blvd., Castro Valley, Calif. 94546

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progress has been made towards theoretical explanation for, and simulation of, stream-network features (For bibliographies of recent work see SMART *(in press, a)* and HOWARD, ET. AL., 1970). Recently, these quantitative studies have extended from consideration of dendritic channel networks to the highly interconnected networks of braided streams and delta distributaries (HOWARD, ET AL., 1970; SMART, *in press, b).* Many of these quantitative descriptions of stream networks are applicable to the closely related networks of cave passages. Apparently the only such study to date has been of the sinuosity of cave passages (DEIKE & WHITE, 1969).

The pattern of passages in a cave is determined by the nature of ground-water flow through the cave at the time of its formation (as well as by the lithology and fracture pattern of the rock). For example, a maze pattern (highly interconnected) is thought to originate from artesian flow conditions (HOWARD, 1964: 8; DEIKE, 1960), whereas a dendritic network presumably results from solution by underground free-surface streams (HOWARD, 1964: 17; WHITE, 1969). In addition to the pattern of passages, other cave features have been used for interpreting the mode of cave origin; such as, 1) the number of levels and the degree of their interconnection, 2) the horizontality or dip of passages; 3) the nature of cave sediments; 4) present water flow through the passages, if any; and 5) the overall pattern of cave passages in relation to topography, lithology, structure, and geomorphic history.

Often, however, many of these criteria are not utilizable for a particular cave because of a lack of vertical control during mapping, because of an inaccurate knowledge of the cave network at the time of its formation (inaccessible, undiscovered, or destroyed portions of the cave, or fortuitous connections between sections of cave not formed simultaneously through collapse or formation of secondary solutional features such as domepits), or because of a lack of first-hand knowledge of cave features. The most common type of detailed information about a cave is the planimetric map of its passages. Two types of quantitative measurements that can be read from cave maps; i.e., measures of topologic complexity and measures of solution intensity, are introduced below and tested for their usefulness in distinguishing among caves of different origins.

Cave topology

A cave pattern is most easily quantified by defining the topological relationships between cave passages. For this purpose a terminology is needed. The point where 3 or more passages join defines an *internal node* (or *junction).* A change of direction of a single passage, even though abrupt, does not constitute a node. An *external node* is defined at the dead end of a passage and at cave entrances. The passage directly connecting any two nodes is termed a *link.* A bedrock pillar enclosed by a loop of passages is called an *island* (Figure 1).

The symbol n is defined as the total number of internal nodes on a cave map, e as the total number of external nodes, t as the total number of links, and i as the total number of islands. A number of quantitative relationships may be stated relating these quantities if the following simplifying assumptions are made about cave patterns:

1) Cave passages are assumed to lie in a plane or to be topologically projectable onto a plane without overlap of passages on different levels. Many caves are essentially on one level, so that they satisfy this assumption very well. Some caves have different levels, but no, or few, loops can be made in the cave using different levels. In such a case the quantitative relationships stated below are not seriously inaccurate. Caves with a well developed three-dimensional pattern (intimately interconnected both within and between levels) obey only the first relationship given below. However, such patterns are infrequent, and the three-dimensional pattern appears to be good evidence for an artesian origin (HOWARD, 1964: 8), so that the criteria developed below are superfluous for such patterns.

2) Passage nodes are assumed to involve the junction of no more nor less than 3 passage links. This is true to an excellent approximation in surface drainage networks, but is more often violated in cave networks, where two joints enlarged by solution commonly intersect (Figure 1a). Even in caves, however, the number of such instances are generally small compared to three-link junctions.

An important quantitative relationship between the topologic parameters defined above comes from graph theory (BERGE, 1962: 27):

$$i = t - (n + e) + 1 \quad . \tag{1}$$

In addition, if the above assumptions are met, an additional relationship may be stated (after HOWARD, ET AL., 1970: 1676):

$$t = 1 - i + 2n \quad . \tag{2}$$

Because there are 2 equations and 4 variables *(t, e, n,* and *i)*, determination of any two for a particular cave pattern indirectly specifies the other two. The number of external nodes *e* and the number of islands *i* are easy to measure on cave maps and are subject to little error.

The degree of connectivity of passages in a planar network can be measured by parameters such as the indices α, β, and γ defined in Table 1. For a large cave network with a very high degree of interconnection and few dead end passages or exits $(i >> e)$, the indices α, β, and γ approach the values ¼, 1½, and ½, respectively. On the other hand, for caves with few loops and a dendritic channel pattern $(e >> i)$, the above indices approach the values 0, 1, and ⅓, respectively.

A. Measures of Connectivity		
Parameter	*Definition**	*Alternate Definition*†
α	$\frac{i}{2(n+e) - 5}$	$\frac{i}{4i + 4e - 9}$
β	$\frac{t}{n+e}$	$\frac{3i + 2e - 3}{2i + 2e - 2}$
γ	$\frac{t}{3(n + e - 2)}$	$\frac{3i + 2e - 3}{6i + 6e - 12}$

B. Measures of Solution Intensity	
Parameter	*Definition*
A_c/A_p	A_c = Area of cave passages A_p = Area within cave periphery
L/A_p	L = Total length of cave passages
t/A_p	t = Total number of passage links in cave (see text)

Table 1. Quantitative measures of cave patterns.

* See HOWARD, ET AL. (1970: p. 1676).

† The alternate expression assumes the validity of Equation 2.

Measures of solution intensity

The degree of removal of bedrock by solution can be directly measured by the ratio of cavern volume to the original volume of soluble limestone within the periphery of the cave. Because of the difficulty in measuring cave volume, several less accurate measures of solution intensity measurable on cave maps can be used, such as the ratio of the area of cave passages to the original area of the cave (Table 1). The use of passage area on the map as a measure of volume of cave passage requires the assumption that the cave passages have a regular cross-section. Two systematic factors may bias estimates of passage volume from its area. Firstly, the cross-sectional shape of a passage may vary with both the lithologic sequence and with the mode of origin of the passage (WHITE, 1959). Secondly, mapping practices differ among surveyors (some may measure the maximum width in a given cross-section, whereas others may use the average width), and the passage width is less accurately mapped than passage location, length, and orientation (passage width is exaggerated for clarity on some maps).

The two ratios L/A_p, passage density, and t/A_p, link density, (see Table 1) are also rough indicators of solution intensity. Caves with the same passage density have the same total length of passage in equivalent planimetric areas. Caves with the same link density have an equivalent degree of branching and interconnecting of passages. Passage density and link density are defined similarly to drainage density and link frequency in stream networks (SHREVE, 1967). In most cavernous limestones the proportion of the original fractures that have been enlarged into passages is small, so that variations in the passage and link densities between caves may be attributed to differences in the selectivity and intensity of solution.

Parameters as indicators of cave origin

The utility of the above parameters of connectivity and solution intensity as indices of the mode of origin of cave systems was investigated by measuring them for two sets of caves that are thought, on the basis of the cave pattern and their overall features and relationships, to have been formed under different ground-water flow regimes. The first set of 3 caves are considered to have been formed by large-scale artesian ground-water flow; namely, Jewel Cave, South Dakota (HOWARD, 1964: 12; DEAL, 1962: 130-141, Plate 3), Breathing Cave, Virginia (DEIKE, 1960), and Anvil Cave, Alabama (WHITE, 1969; TARKINGTON, ET AL., 1965: 1-11, cave map).

The second group of 25 caves is composed of the larger ($n + e \geq 10$) Indiana caves for which maps appear in POWELL (1961). Most or all of these originated by free-surface stream flow or integrated ground-water circulation (most have streams along major passages that connect to disappearing streams and sinkholes). Conventions adopted in measurement of the quantities listed in Table 1 are illustrated in Figure 1.

Solution by artesian ground-water flow results in enlargement of a large percentage of the original fractures in the limestone, resulting in a dense, maze-like cave pattern as well as a large volume of solution due to the stability of the ground-water flow (HOWARD, 1964: 8). Therefore, all of the indices of connectivity and solution intensity listed in Table 1 would be relatively large. On the other hand, free-surface stream or integrated ground-water circulation is more selective in joint enlargement, forming simple cave patterns (HOWARD 1964: 7). Such ground-water flow is often shortlived in any single passage due to changing ground-water levels, underground capture, and surface erosion. Therefore all the indices in Table 1 would be relatively small.

Measurements of the two groups of cave patterns generally bear out the above expectations (Figure 2). Although 6 different parameters were measured (Table 1), several are strongly interrelated. As would be expected from the similarity of their definition, the parameters α, β and γ are very strongly correlated (Figure 2b), so that measurement of any one closely determines the others. Similarly the passage density and the link frequency are closely fitted by the following empirical equations for caves of the two groups under

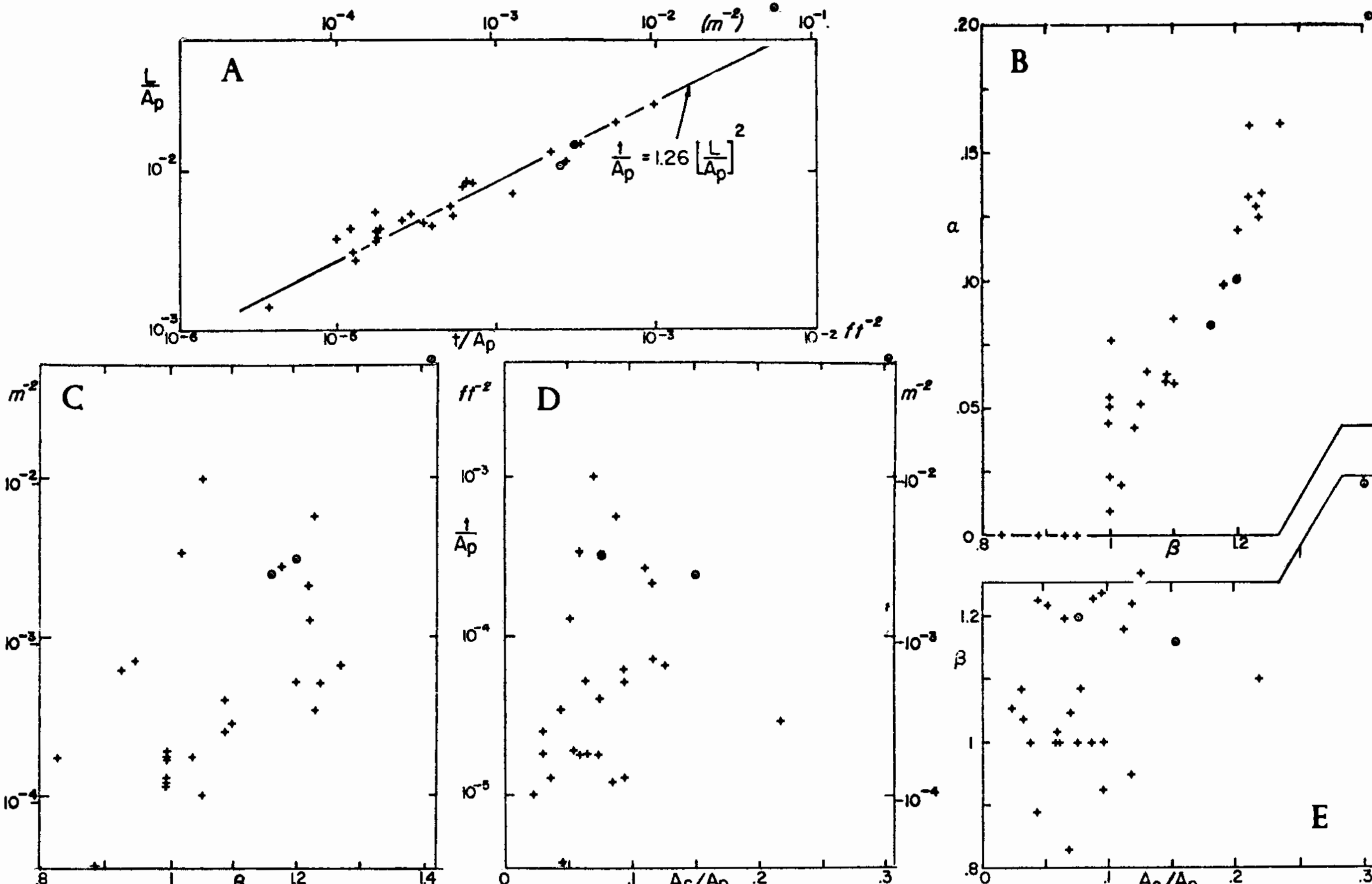

Figure 2. Relationships between cave-pattern parameters. Crosses show values for caves presumed to have originated by free-surface stream or integrated ground-water flow, while circles show caves of presumed artesian origin. The parameters A_c/A_p, α and β are shown on a linear scale, whereas t/A_p and L/A_p are shown on a $\log_{10}$ scale.

study (also see Figure 2a):

$$\frac{t}{A_p} = 1.26\left(\frac{L}{A_p}\right)^2 \quad (3)$$

A similar equation is found to hold in surface-stream networks, where the square of the drainage density is proportional to the link density (SHREVE, 1967), but the constant of proportionality is smaller, about 0.8 (HOWARD, 1971). Shreve (1967) shows that the relationship given in Equation 3 results if both the average passage density and the average link length are uniform over the area of a single cave (although both vary between different caves). The value of the constant in Equation 3 is apparently not related to the origin of the cave, because points for caves of presumed similar origin are scattered both above and below the general trend (Figure 2a).

However, when any of the connectivity indices (α, β or γ) is plotted versus either A_c/A_p or one of the quantities in Equation 3 (t/A_p or L/A_p) considerable scatter results (Figure 2c, 2e). Similar scatter results if A_c/A_p is plotted versus t/A_p or L/A_p (Figure 2a). Therefore, 3 (but not any 3) parameters of the 6 in Tables 1 and 2 are necessary to accurately characterize a cave pattern.

Of these three measures of cave pattern, a density-frequency statistic (L/A_p or t/A_p) is most successful in discriminating between the presumed artesian caves and the Indiana caves (Figure 2a). Linear discriminant analysis (KING, 1969, p. 204-212) allows combination of all three measures of cave patterns into a function that gives a single parameter for classification of cave patterns. The least-squares discriminant function for the caves measured in this study is (using t/A_p, A_c/A_p, and β):

$$z = 0.083 \log_{10}(t/A_p) + 1.02\, A_c/A_p + 0.11\beta \ . \quad (4)$$

Caves with values of Z above 0.444 would be suspected to be artesian in origin, whereas lesser values of Z would indicate free-surface streams or an integrated ground-water reservior. For the caves measured, only Boone's Cave, Ellers' Cave, and Marengo Caverns of the Indiana caves have values of Z greater than 0.444, while Breathing Cave—of presumed artesian origin—has a value less than 0.444. A visual comparison of Breathing Cave with those Indiana caves with the most complex patterns reveals little difference. Indeed, the most impressive evidence for an artesian origin for Breathing Cave is not its network pattern but the dip of its passages, its submerged-solution features, and the regional pattern of karst drainage (DEIKE, 1960).

Limitations of the method

The numerical measures of cave pattern introduced above assume either that the original fracture pattern is comparable for all caves considered, or that differences in fracture pattern are less important in determining cave patterns than are differences in ground-water regime. These conditions should be satisfied in areas where several open, cross-cutting fracture sets are present. However, several structural situations can occur that would affect cave patterns:

1) If the fracture density is low compared to the size of the cave, the cave pattern is necessarily topologically simple in plan, and the quantitative parameters introduced above are very sensitive to small variations in cave pattern. This difficulty can be avoided by limiting the analysis to larger caverns.

2) If the orientations of open fractures are limited to a small range of azimuth, then the cave pattern will be topologically simple for all ground-water regimes. In cases of highly anisotropic fracture azimuths, the resultant cave pattern could vary with the predominant direction of ground-water flow.

3) Caves formed along bedding planes may have a pattern that is considerably different in scale or complexity from caves formed by the same ground-water regime in fractured rocks.

4) The areal extent and pattern of cave passages can be affected by lateral boundaries of the soluble rock; for example, by facies change, folding, or faulting.

In addition to the influence of structure, the numerical properties measured from maps are affected by the completeness of the map. Most cave maps are imperfect records of the cave pattern at the time of its formation, both due to incomplete exploration or mapping and due to present inaccessability of passages formerly connected. However, if the cave pattern was originally areally uniform and if the portion of the cave lying within the perimeter of mapped passages (Figure 1) is completely represented, then the quantitative measures introduced above should be fairly accurate measures of the properties of the original cave pattern. However, the completeness of mapping is difficult to assess. Some cave mapping emphasizes the wide or long passages while neglecting the short dead ends or minor loops; such practices obviously bias the quantitative measures of cave pattern.

In view of these limitations, the interpretation of cave origin should not rest solely on the pattern of passages. However, the quantitative measures of cave pattern should provide a standardized means of comparison for one line of evidence for cave origin.

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