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Modelling Fluvial Systems: Rock-, Gravel- and Sand-bed Channels

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CHANNEL BED TYPES: ROCK, GRAVEL AND SAND

The beds of stream channels can generally be classified as bedrock, sandy alluvium, or gravelly alluvium. Bedrock channels are distinguished by the absence of alluvial sediment, except in isolated scour holes. The sandy, or fine-bed alluvial channels have beds dominated by sand with small percentages of gravel, and the reverse is true for the gravel, or coarse-bed channels. Downstream transitions between these channel types are usually abrupt. Howard and Kerby (1983) document the sudden transitions in badlands between shale-floored rills and gullies and sand-bed channels or pediments downstream. Yatsu (1955) and Shaw and Kellerhals (1977) cite rapid downstream transitions from gravel to sand beds in streams in Japan and Canada, respectively. Empirical data on the grain sizes of bed sediment collected from widespread localities generally show bimodal distributions, with peaks in the sand and gravel sizes and a paucity of granule sizes (2 to 10 mm) (Williams, 1978b). Channel armouring downstream from dams is an example of a temporal change from fine- to coarse-bed channels. Howard (1980) discussed the reasons for such thresholds in natural channels; a summary is provided in this chapter. The implications of these thresholds for modelling and prediction of fluvial landforms are also examined, as are the limitations and uncertainties of the model proposed by Howard (1980). Finally, suggestions are made for future directions of theoretical modelling, field research, and flume experimentation that would help improve our understanding of these thresholds.

Bedrock channels

Bedrock channels are defined as channel segments which lack a coherent bed of active alluvium. In addition to consolidated rocks, 'bedrock' in this context can also include cohesive fluvial deposits now undergoing dissection. Bedrock sections are common along many rivers in mountainous regions, often occurring as short riffles or waterfalls, but most commonly forming the headwater tributaries in otherwise alluvial channel networks.

Bedrock channels remain free of bed sediment because of steep gradients. They generally transport sediment concentrations comparable to alluvial channel segments occurring upstream or downstream, but sediment is not deposited on the bed, even during waning flows. Thus bedrock channels generally carry bed sediment in less than capacity quantities, in contrast to alluvial channels, which have gentler gradients than bedrock channels for a given discharge-sediment load regime and which generally transport a capacity bed material load (figure 4.1). In a bedrock channel segment lying downstream from an alluvial channel, the bed sediment during waning flows is preferentially deposited on the low-gradient alluvial section. For bedrock channels in headwater areas, the lack of low-flow deposition occurs because of high tractive forces and because sediment supply primarily occurs from runoff during rising stages, whereas transport continues during waning stages.

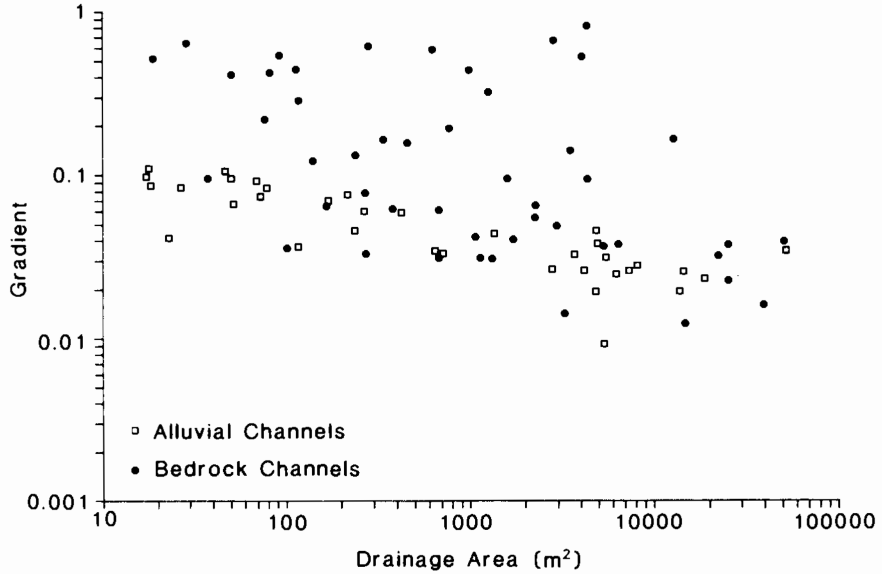

Figure 4.1 Plot of gradient versus drainage area for alluvial and bedrock channels in a badlands in Virginia, USA
(after Howard and Kerby, 1983)

One factor that may contribute to the sharpness of the transition between bedrock and alluvial channels is enhanced rebound of sediment grains on hard bedrock relative to impacts of grains upon a bed composed of grains of similar size, where much of the momentum of the impacting grains is lost to frictional losses and small displacements of bed grains. This phenomenon is best documented in aeolian environments (Bagnold, 1941; Ellwood et al., 1975). Bagnold suggests that the transport capacity of grains over hard surfaces is larger than over loose grains of the same size as those in motion; as a result, downstream transitions from bedrock to alluvial beds should be abrupt due to the sudden loss of mobility.

The gradient of bedrock channels exhibits no necessary correlation with discharge and sediment characteristics, which contrasts with strong correlations in alluvial channels (figure 4.1). Rather, the gradients are determined by the distribution of bedrock resistance to erosion along the channel and the erosional history of the channel network. However, the rate of erosion of bedrock channels is determined jointly by the bedrock resistance and the hydraulic regime (discharge, sediment load, and size distribution of supplied sediment).

Erosion of bedrock channels requires both weathering and detachment, which often go hand in hand. Resistant rocks may be eroded primarily by sediment abrasion. By contrast, limestone beds are primarily attacked by solution. In the case of gully erosion in shales, weathering may be of secondary importance to detachment, so that erosion rates are either related to the erosive capability of the sediment load, or are simply due to hydraulic plucking when the shear stress exceeds a critical value. Therefore no universal model of bed erosion can be formulated for bedrock channels. However, Howard (1980) and Howard and Kerby (1983) suggest a model for detachment-limited rills and gullies in badlands in which erosion rates are proportional to high-flow bed shear stress. If the hydraulic geometry of badland bedrock channels shows consistent downstream relationships, then the rate of channel bed erosion, E, depends upon drainage area, A, local channel gradient, s, and bed erodibility, K. In particular, if the width-depth ratio and hydraulic roughness remain constant downstream, then erosion proportional to shear stress implies:

$$E = K\,A^{0.38}\,s^{0.81} \tag{4.1}$$

Data from rapidly eroding badland channels (Howard and Kerby, 1983) indicate drainage area and gradient exponents of 0.44 and 0.68, respectively, which are reasonably close to the predicted values.

Bedrock channels and alluvial channels with either fine or coarse beds commonly coexist in many drainage basins. Short sections of bedrock channel commonly occur where especially resistant rocks occur; erosion rates lag when resistant rocks are exposed, resulting in steepened gradients and a transition to a bedrock channel. However, resistant bedrock also acts as a local base-level control to channel sections lying upstream, and a zone with a low rate of erosion, and correspondingly low gradients, commonly occurs above resistant outcrops (Howard, 1980); these low gradient sections are generally alluvial, even if the majority of the channel system is bedrock. Alluvial channel segments also commonly occur above other base-level controls, such as oceans, lakes, or reservoirs. Rejuvenation of drainage by relative change of land-ocean levels can also create bedrock channel sections in otherwise alluvial rivers, such as the nearly ubiquitous bedrock rapids at the fall line along Atlantic Coast rivers in the United States. These rivers are generally alluvial above the fall line. Howard (1980) also suggests that downstream transitions from bedrock to alluvial channels may occur even in basins undergoing steady-state erosion, although exclusively bedrock channel networks would occur for very rapid erosion rates (very high relief) and exclusively alluvial channels for very low erosion rates.

Change in the sediment supply and hydraulic regime can convert bedrock channels to alluvial ones, or vice versa. High rates of sediment supply and low discharges during winter in badlands in the eastern United States result in steep channel gradients, and many of the washes are aggraded with sandy alluvium (Howard and Kerby, 1983). However, during the summer smaller sediment yields due to reduced mass wasting and slope surface sealing, coupled with high intensity rainstorms, result in bed degradation and gradient reduction. Some channels with relatively thin winter alluvial mantles are completely stripped of alluvial cover during the summer; thus they are seasonally converted to bedrock channels.

Fine- and coarse-bed alluvial channels

Bed material transport rates increase non-linearly with increases in flow primarily because of the existence of a threshold of motion. Howard (1980) showed that this non-linearity implies the existence of threshold transitions between coarse and fine alluvial beds. Channels with coarse, usually gravel beds experience bed sediment transport only during high-flow stages, with generally slow net rates of gravel transport. On the other hand, channels with fine, primarily sandy alluvial beds exhibit motion at moderate stages and are characterized by high rates of sand transport. Abrupt spatial (downstream) or temporal thresholds can occur between these bed types even where the hydraulic regime varies gradually. Alluvial channels tend towards an equilibrium gradient which is just sufficient, over a period of years, to transport the supplied sediment with the available discharges (Mackin, 1948; Howard, 1982). However, in some channels it is the sparse load of coarse gravel that determines the equilibrium gradient, whereas in others it is the requirement to transport the more abundant sediment near the median size that controls the gradient. Transport continuity is maintained in alluvial channels by mutual adjustment of gradient and cross-section geometry, but the latter also depends on bank material characteristics in a complex way. The adjustment of gradient alone can, however, be assessed by considering the transport of bed material for a particular specific discharge (discharge per unit width).

The reasons for these thresholds can be illustrated by consideration of the channel gradient that would be required to transport each size fraction of bed sediment in a given hydraulic regime under the assumption that transport of each size fraction occurs independently. This is a variation on the normal engineering usage of transport formulae in which gradient is specified and transport rate is calculated. The critical assumption in Howard's (1980) analysis is that the required gradient for a given specific discharge can be calculated independently for each size fraction of supplied sediment, with each size fraction having a characteristic long-term rate of supply. Although interactions obviously occur between different grain sizes in transport (for example, Einstein's (1950) transport relationship uses a 'hiding factor' to account for such interactions for naturally graded alluvial beds), the size composition of bed sediment in natural channels is generally a small portion of the total size range of supplied sediment. The size fractions represented on the bed are presumably those that at that section of the channel are the most difficult to transport, that is, which require the steepest gradient. Supplied sediment which is finer than the bed is transported as wash load in below-capacity concentrations, whereas grains coarser than the dominant bed size are carried in such small quantities that they are dispersed over the bed, or they may be areally sorted into riffles or cobble bars. Thus, of the total range of supplied sediment, a relatively restricted portion dominates the bed and determines the required gradient and is carried in capacity quantity. Howard (1980) therefore assumes that the required gradient for the channel can be determined by finding that size range which, when its characteristic supply rate and size is input into an appropriate sediment transport formula for the dominant discharge, predicts the largest gradient.

Figure 4.2 shows a typical plot of required gradient as a function of grain size. In making the calculations the Einstein-Brown sediment-transport relationship (Henderson, 1966) was used and the quantity of supplied sediment was assumed to follow a lognormal distribution (characterized by the log-mean grain size, D, and the variance, V). For the fine grain sizes an arbitrary value of the Manning resistant coefficient, n, was assumed, but for the coarse sediment grain roughness was assumed to dominate so that n varied as the $1/6$th power of grain size. The particular plot shown in figure 4.2 assumes a specific water discharge, q, of $10\,m^2\,s^{-1}$, a sediment concentration, c, of 0.01, $D = 0.3\,mm$,

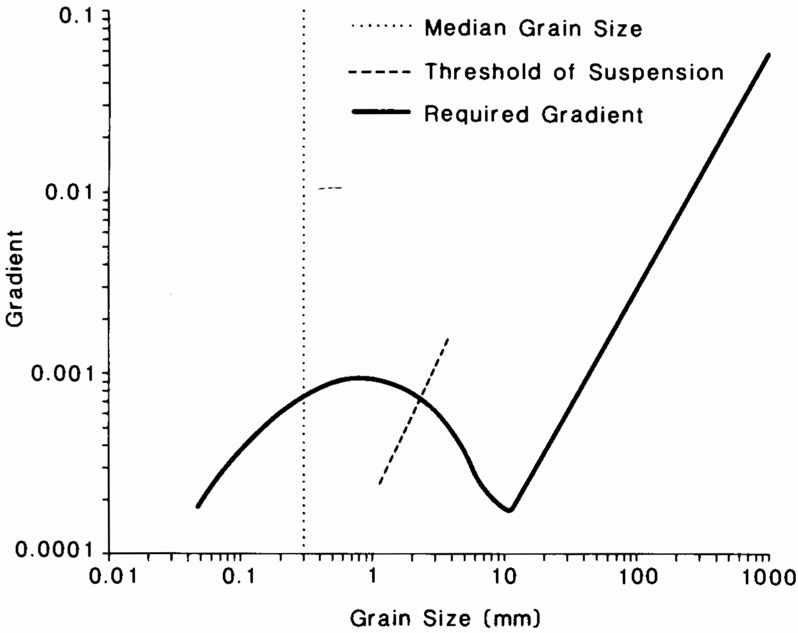

Figure 4.2 Curve of required gradient versus grain size. Median grain size is 0.3 mm. Gradients calculated using Einstein-Brown transport formula
(after Howard, 1980)

V = 0.87, and n = 0.02. Calculations of required gradient using other total-load sediment formulae exhibit the same general form of relationship to that shown in figure 4.2. In particular, for the input conditions noted above, the required gradient has a local maximum for a grain size somewhat larger than the median grain size, a local minimum near 10 mm, and an indefinitely increasing required gradient as grain size increases beyond 10 mm. In the coarse size ranges the required gradient is determined by the threshold of motion and is nearly independent of the quantity of sediment, whereas in the fine size ranges the sediment load is important in addition to the grain size.

Naive interpretation of the indefinitely increasing required gradient in the coarse size range would suggest that alluvial channel gradients are invariably determined by the coarsest grains in transport and the bed is correspondingly dominated by coarse grains. However, as is evident from the occurrence of numerous sand-bed channels, coarse grain sizes do not always dominate the bed. This is because above some *critical* grain size, weathering and slope erosion supply so little sediment of that size and coarser, that they are incapable of forming a coherent alluvial bed. If this critical grain size is coarse enough such that the corresponding required gradient is larger than the peak required gradient in the fine size range, then the bed of the channel can be expected to be composed primarily of grain sizes close to the critical grain size that is, it will be a coarse-bed channel. Because of the steep gradient, fine grain sizes will seldom accumulate on the bed, except possibly as an infilling between the coarse grains. On the other hand, if the supply of coarse grain sizes is so restricted that the critical grain size is small enough that its required gradient is less than that for the maximum in the fine sediment sizes, then the bed will be dominated by fine grain sizes in the range near the peak required gradient. In this case, coarser grain sizes will be present in the alluvial bed, but their concentration will be diluted by the abundant fine bed material. Sediment that is much finer than the size corresponding to the largest required gradient will primarily be carried as wash load and is seldom deposited on the bed.

Implications of the fine- to coarse-bed threshold

Conditions favouring gravel or sand channels. Figure 4.3 shows the effect on the required gradient curves of varying sediment concentration (a), specific discharge (b), and variance in grain size (c and d). Figure 4.4 shows the effect of variations in the mean grain size. In each of these figures the remaining parameters were held constant. As sediment concentration increases (figure 4.3a) the valley between the fine-grain peak and the threshold curve deepens; for example, at a concentration of 0.0001 a coarse grain-size about six times the size corresponding to the peak in the sub-millimeter sizes gives the same critical gradient; this size ratio increases to about 60 for a concentration of 0.1. This suggests that alluvial streams with high imposed loads are more likely to be fine bed. Following similar reasoning, figure 4.3b shows that narrower streams, when compared to wider streams with higher specific discharges but the same sediment concentrations and total discharges, are more likely to be fine bed. Increase in the mean grain size in transport (figure 4.4) increases the grain size of the peak required gradient in the fine size range as well as the local minimum required gradient. For the Einstein-Brown sediment transport relationship streams with coarser average sediment size are somewhat more likely to be coarse-bed channels. However, other sediment transport formulae are either indifferent to average sediment size or predict the opposite of the Einstein-Brown relationship. Streams with a narrow range of supplied sediment sizes show a strong peak in the required gradient in the fine size range, and thus are more likely to be fine-bed channels. Channels receiving a wide range in sediment sizes have a less pronounced fine-grained peak gradient, or for a very wide range of sizes, essentially no fine-grained peak, so that the channel will probably be coarse-bed (figures 4.3c and 4.3d). In the case of very large variance in supplied sizes, the required gradient is not a strong function of grain

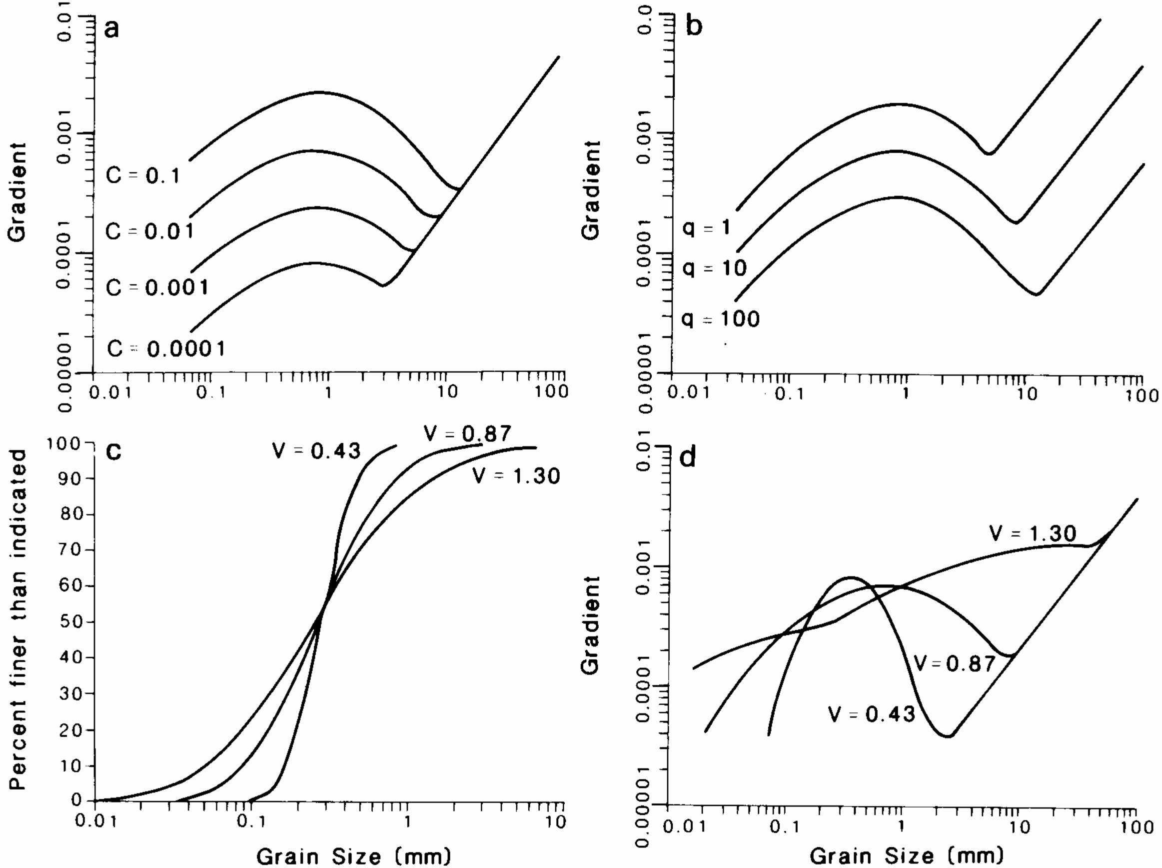

Figure 4.3 Curves of required gradient versus grain size, showing effects of variations in sediment concentration (a); specific discharge in $m^2 s^{-1}$ (b); and variance (V) in grain size distribution (c and d). Other parameters held constant in each case (after Howard, 1980)

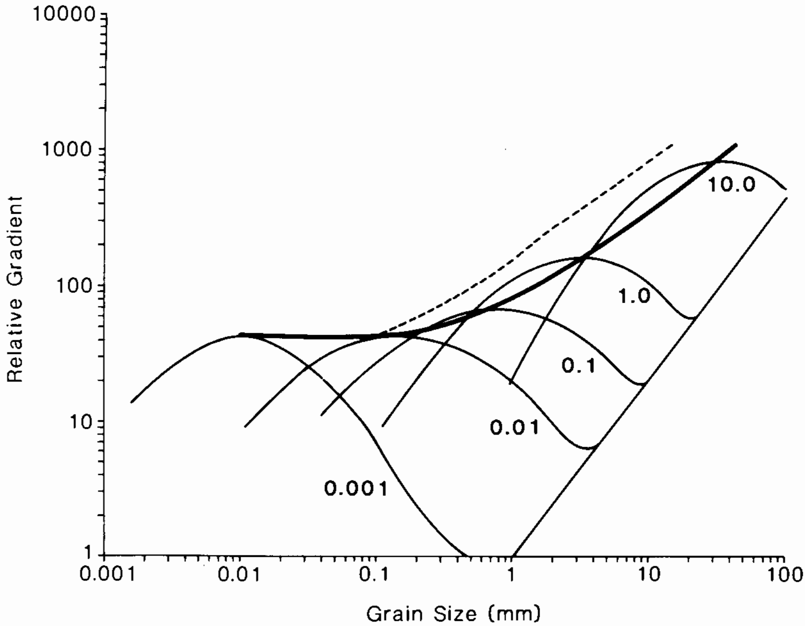

Figure 4.4 Variation of maximum required gradient for fine-bed alluvial channels with average particle size. Dashed line shows relationship between median particle size of supplied sediment and channel gradient, whereas solid line shows relationship between bed material size and gradient
(after Howard, 1980)

size, so that the bed may be characterized by a large range of grain sizes, possibly areally sorted into riffles and pools.

In summary, fine-bed channels would be expected in streams where concentrations are high but the grain size range is narrow, as in many badlands and in the semi-arid Great Plains, where erosion of poorly consolidated sedimentary rocks yields high sediment yields but little coarse detritus. On the other hand, coarse-bed channels are favoured by low sediment loads and relatively large proportions of coarse detritus, such as in mountain areas with resistant bedrock, steep slopes, and a predominant role of physical weathering.

Spatial transitions between bed types Transitions between coarse and fine beds in a channel system may occur spatially (downstream) or temporally at a given location. In natural channel systems, the most common spatial transition is the threshold change from headwater coarse-bed channels to downstream sand-bed channels (Yatsu, 1955; Shaw and Kellerhals, 1977). This threshold occurs because grain size in gravel channels generally decreases more rapidly downstream than it does in sand channels, due to more effective comminution and sorting of gravels (although Hack (1957) and Brush (1961) show that local geologic or physiographic settings may counteract or even reverse the normal downstream fining of gravel channels). The typical downstream decrease in the grain size, which is critical for maintenance of a coherent gravel bed, will trigger a change to a sand-bed channel if the required gradient drops below that for the sand size range.

Local increase of channel width increases the required threshold gradient more than the fine-bed peak gradient. Thus in streams with nearly equal required gradients for the fine and coarse portions of the load, a local width increase might trigger conversion to a coarse-bed channel. Some riffle-pool sequences might be due to this mechanism, since riffles commonly occur in areas of divergent flow and increasing channel width (Keller and Melhorn, 1973). Similarly, Howard and Dolan (1981) show that the Colorado River in the Grand Canyon is a sand-bed channel where the width is constricted by talus or resistant bedrock, but cobble bars are common where less resistant rocks allow channel widening.

In the same river transitions from fine- to coarse-bed channels, followed shortly downstream by a return to sand-bed conditions, occur at nearly every rapids (Howard and Dolan, 1981). The cause of the local changes to coarse-bed channels is the local injection of boulder-rich sediment where steep sidewall tributaries debouch into the main river. The steep tributaries have high competence during intense thunderstorms, permitting transport of boulders up to several metres in size. Because of the abundant gravel, the critical grain size for forming a coherent gravel bed is increased, resulting in the transition to a threshold-of-motion gravel-bed channel with a gradient several times that of the between-rapids sandy reaches. However, the long-term rate of supply of coarse boulders from these tributaries is insufficient to force throughput of these coarse boulders along the canyon; rather, the boulders are comminuted in place by weathering, abrasion, and fracturing until they are transportable with available discharges at the between-rapids gradients. Because of the high sand loads and the narrow river, the Colorado River remains a sand-bed channel except at the rapids despite the high rate of local supply of coarse debris from the canyon walls (except, as noted above, at local wide channel sections where transitions to a gravel bed occur).

Temporal transitions between bed types A change of hydraulic regime can trigger conversion of sand-bed channels to gravel, or the reverse. The armouring that occurs below dams in otherwise sand-bed channels is a classic example (Williams and Wolman, 1984). The interception of nearly all the sediment load of the stream by the dam reduces the sediment size/required gradient curve to essentially a threshold relationship, so that the stream would generally convert to a coarse-bed channel. This conversion is aided and accelerated by the bed degradation that accompanies the reduction in sediment load, which rapidly concentrates at the bed surface coarse-grained sediment that was formerly dispersed in small quantities through the alluvium. The general reduction in peak discharges also aids this conversion, because it may render the coarsest bed material no longer transportable.

The reverse case, of conversion of a gravel channel to a sand bed, can occur as a result of increased sediment supply. Timber harvesting, forest fires, and poor agricultural practices can increase sediment yields by up to an order of magnitude relative to geologic norms. In addition, such increases commonly increase the relative proportion of sand to gravel. If the channel is initially gravelly, aggradation and the conversion to a sand bed will occur if the additional load requires a gradient for transport in the sand size range that is greater than the existing gradient. For very coarse gravel channels this is unlikely to occur.

Transport rates of bed sediment in gravel channels are generally very low when compared with rates of sand transport in sand-bed channels (although gravel channels may transport large quantities of sand, largely in suspension). As a result, the time-scale for adjustment of gradients in gravel streams to altered hydrologic conditions will generally be much longer than for sand-bed channels of equivalent size. In fact, some of the coarse cobble pavements of alpine streams may have originated during periods of accentuated physical weathering and mass movement during the late Pleistocene (Miller, 1958; Brush, 1961). Sand-bed channels respond much more rapidly to change in regime, with response times on the order of months to hundreds of years, depending upon the size of the alluvial channel (Howard, 1982).

The difference in characteristic response-times for gravel and sand rivers can result in short-term adjustments to altered hydraulic regime that are distinct from the ultimate response; for example, channelization of a river by elimination of meandering can increase channel gradients by up to twice the natural value. The response of sand channels will be to degrade their beds (Daniels, 1960; Daniels and Jordan, 1966; Emerson, 1971; Piest et al., 1977); if there is a sufficient gravel component in the alluvium, an armoured bed can result due to the slow transport rates of the gravel (Schumm et al., 1984), even though the ultimate response would be a sand-bed channel with approximately the same gradient as existed before channelization (since the hydraulic regime has not been directly altered). Conversely, aggradation due to change in hydraulic regime or a rising base-level may result in temporary conversion of a gravel river to sand; for example, if upstream deforestation or urbanization result in a greater influx of sediment to a gravel channel without change in the grain size distribution, the ultimate equilibrium channel, given the changed regime, may be a steeper gravel channel. However, if the influx of sediment is sufficient to raise the required gradient for the fine sediment above the existing gradient, the immediate response will be a temporary conversion of the channel to a sand bed. In fact, if the increase in sediment supply is short-lived, the steeper gravel channel will never form, and the sand bed will be scoured away.

A rise in local base-level (such as by construction of a dam) requires upstream alluviation, with the ultimate equilibrium channel having approximately the same gradient and bed type as the original channel (since the hydraulic regime is unchanged). However, if the channel is originally gravel, the initial alluviation will occur by deposition of the sand load at a gradient smaller than the original channel gradient, because of the slow transport rates of gravel sizes. For practical purposes, the long-term equilibrium may not be established during the lifetime of the dam.

Downstream hydraulic geometry of coarse- and fine-bed channels. As cited previously, confirmation of these concepts and the existence of the threshold relationship between coarse- and fine-bed channels come partly from observations of sudden downstream transitions from coarse- to fine-bed channels, as well as of the paucity of channels with beds dominated by intermediate grain sizes. In addition, sediment transport relationships indicate that the downstream hydraulic geometry of fine- and coarse-bed channels should differ, particularly in the relationships between gradient and the variables of discharge and size of bed sediment. Howard (1980) shows that coarse-bed channels with beds near the threshold of motion at peak flows will exhibit a dependency of gradient on grain size (D) and specific discharge (q) of the form

$$s = K_1 q^a D^b, \tag{4.2}$$

where, for the Shields threshold-of-motion criterion, the exponent a, has a value of -0.86 and b a value of 1.29. Data from eastern USA coarse-bed streams (Hack, 1957; Brush, 1961) show estimated values of a near -1.0, close to the predicted value, but a value of b near 0.5, lower than predicted.

Transport formulae predict rather different exponents for fine-bed channels for which dominant shear stresses are well above threshold of motion:

$$s = K_2 q_s^e q^f F(D), \tag{4.3}$$

where F(D) can be approximated by D^g for narrow ranges of grain size. Values for e, f and g vary considerably for different transport formulae but cluster near 0.6, -1.0, and 1.1, respectively, for channels of medium sand. Data on downstream hydraulic geometry from fine-bed channels in Virginia badlands (Howard and Kerby, 1983), Utah badlands (Howard, 1980), sandy Great Plains streams (Schumm, 1960), and ephemeral streams in New Mexico (Leopold and Miller, 1956; Renard and Laursen, 1975) are generally consistent with the e and f exponents in equation (4.3) when downstream changes in width and in the relative values of q and q_s are accounted for (Howard, 1980). However, these channels have insufficient ranges in grain size to evaluate g. Furthermore, multicolinearity amongst the independent variables in both equations (4.2) and (4.3) results in bias in the least-squares exponents which obscures the empirical testing of theoretical predictions.

The difference in downstream gradient changes in gravel- and sand-bed rivers underscores the necessity for stratifying hydraulic geometry data on the basis of bed type (see chapter 5). Hydraulic engineers generally recognize this necessity by proposing different relationships for sand and gravel bed rivers (e.g. Parker, 1978a; 1978b; Chang, 1979a; 1980). In a related vein, Carson (1984) suggests that the meandering-to-braiding transition in alluvial streams occurs at higher slopes (for a given discharge) in gravel streams than in sand-bed channels because of the higher threshold of motion (see chapter 6).

Deficiencies of intermediate bed particle sizes. The curves of required gradient versus grain size (figures 4.2–4.4) show a local minimum between the peak in gradient in the fine size range and the monotonically increasing gradients in coarse grain sizes. This minimum is in the range of 1 to 10 mm for reasonable ranges in the median size of sediment supplied from upstream. This size range corresponds to the commonly observed deficiency of these grain sizes in fluvial sediments (Yatsu, 1955; Slatt and Hoskins, 1968; Church and Gilbert, 1975; Emmett, 1976; Williams, 1978b). The usual explanations for this deficiency are either that weathering produces little detritus in this size range or that transport comminution processes are particularly effective in this size range relative to sand, which has lower momentum and is generally monomineralic. However, the deficiency may be due in part to sorting processes related to the minimum in the required gradient curves.

The relative proportion, P_D, of different grain sizes on the bed is proportional to the ratio

$$q_{sD}/(Dv_D), \tag{4.4}$$

where q_{sD} is the specific sediment discharge of grains of size D measured in volume of solids per unit time and channel width, and v_D is the average velocity of grain movement. Grains in the sand size range are strongly represented in the bed due to large q_{sD} and small grain size despite large v_D. Coarse gravel is strongly represented due to low v_D despite low q_{sD} and large D. However, because of the high actual gradient relative to the required gradient for grain sizes near the minimum, P_D is low due to low q_{sD} and high v_D; that is to say, the small quantity of granule-sized bed material moves relatively rapidly along the channel, so that it is poorly represented in fluvial sediments. If the bimodal distribution of fluvial sediments is largely due to this sorting process, then the missing granule sizes should be found in their original proportions in still-water sediment sinks, whereas they will also be missing in such deposits if original scarcity or differential comminution is the main cause of the deficiency. If the sorting process is responsible for the deficiency, then the grain size range of the deficiency will vary among fluvial environments in relation to differences in the mean and variance of sediment sizes supplied to the channel system. This may help to account for the lack of bimodal grain size distributions in aggregate averages of grain size distributions from diverse fluvial environments (Shea, 1974).

Thresholds in sand-bed channels

Chang (1979a, 1979b, 1984, 1985, 1986) has proposed the existence of a threshold in channel morphology and bed type distinct from those discussed above. Flume experiments and some field data have demonstrated the existence of multiple-valued average depths as a function of flow velocity in sand-bed channels of constant gradient (figure 4.5) due to the successive occurrence of different bedform types (dunes, upper-stage flat, bed, and antidunes). As a result, bed material transport rates also exhibit non-linear behaviour. By accounting for variations in flow resistance with bed type, Chang has shown that for a given sediment and water discharge, the channel gradient required for equilibrium generally exhibits a minimum value for a particular channel width (figure 4.6). In some cases two local minima occur in equilibrium gradient associated with upper- and lower-stage flows, respectively. The narrower width is associated with the upper-stage flow. Chang calculates the required gradient for a given channel width, water discharge, and sediment discharge by assuming a flow depth and calculating velocity and gradient from sediment transport relationships, and then calculating discharge using predicted flow resistance. If the calculated and assumed discharge do not agree, the flow depth is adjusted in additional iterations until agreement occurs.

As a second and independent assumption, Chang hypothesizes that channels adjust their width to minimize gradient, thereby also minimizing the rate of energy dissipation. Under conditions where two local minima occur in required gradient (for relatively high sediment transport rates), the one of lower gradient (subcritical flow) would be the more stable. Chang finds, however, that the two minima can become equal if the bank steepness of the narrower upper regime channel is steeper. Chang suggests that these two minima may thus coexist as riffles (wide, shallow, lower-stage flow) and pools (narrow, deep, upper-stage

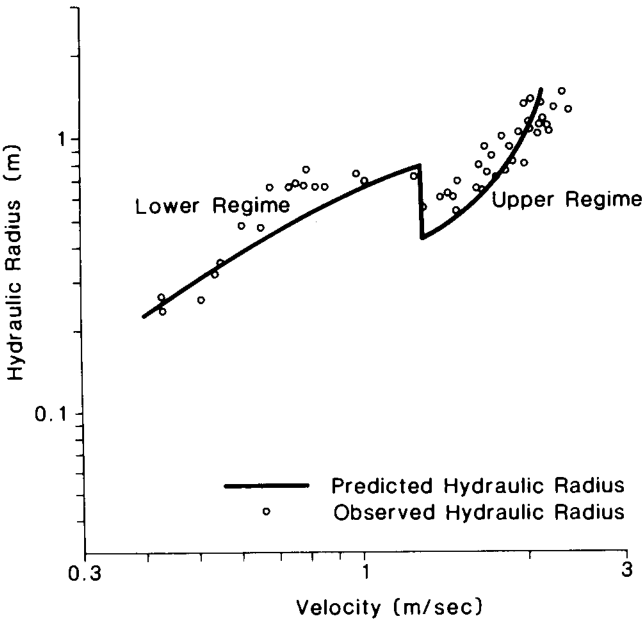

Figure 4.5 Observations of channel depth as a function of flow velocity for the Rio Grande River, with curve showing values predicted by Engelund's (1966) flow resistance model (after Raudkivi, 1976)

flow) in the same channel reach. He therefore implies that riffles and pools in steep gradient sand-bed streams represent threshold transitions between these two minima.

Chang's analysis seems to provide fairly accurate predictions of channel width and depth for a given discharge and observed gradient and bed material size. It also suggests that for channels with low sediment loads, channel width increases with discharge but is relatively independent of sediment load and channel gradient. However, for high sediment loads channel width is an increasing function of both discharge and sediment load (or gradient). Channels of the latter type are likely to be braided due to the high width-depth ratio, whereas the narrower channels associated with lower sediment transport rates are likely to be meandering or straight. Chang's analysis is actually somewhat more involved than suggested here, with greater attention to channel planform pattern and a distinction between four possible regions of regime behaviour.

Although Chang's analysis is innovative and worthy of further empirical testing, the hypothesis that channel width is optimally adjusted to provide minimum channel gradient has less basis in established hydraulic principles than the co-assumption that channel gradients vary with channel width due to changes in flow resistance. Several potential difficulties with the model

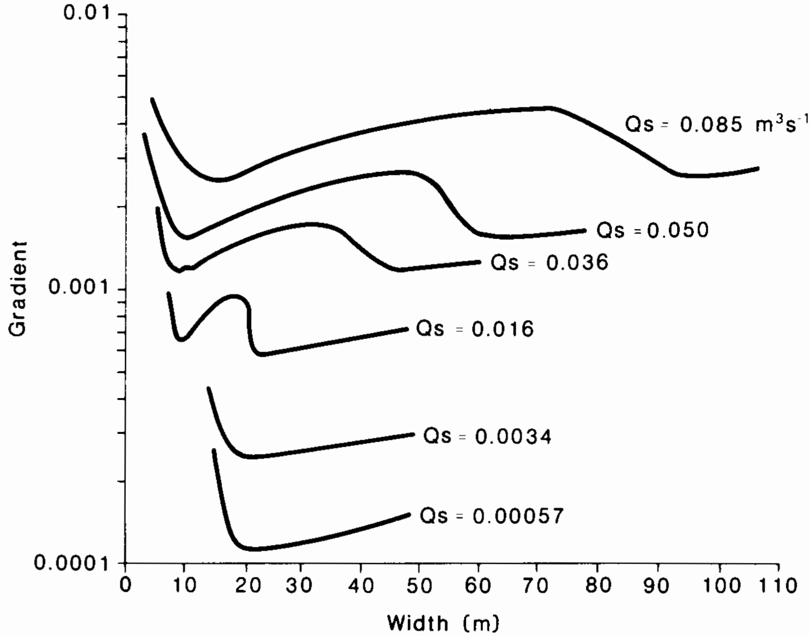

Figure 4.6 Calculation of equilibrium gradient as a function of channel width for fixed sediment grain size, discharge, and bank slope and for various values of sediment discharge (Qs). Curves illustrate minima in equilibrium gradient due to variations in flow resistance (after Chang, 1985)

assumptions can be mentioned. Firstly, the assumption of optimal width adjustment has little direct basis in fluid mechanics and sediment transport. Like other optimal models of channel geometry, such as minimum variance models of hydraulic exponents (Langein, 1964; Williams, 1978a), minimum rate of work models of channel junctions (Howard, 1971; Roy, 1985), and other minimum energy expenditure models (Yang, 1971; 1976; Cherkauer, 1973), the optimal geometry has not been directly linked with causal mechanisms for its attainment (however, see the discussion of extremal hypotheses by Davies, chapter 9). A case can be made for a stabilizing feedback mechanism for a channel that is narrower than the optimum, since the increased gradient and greater flow depth relative to the optimum would increase fluid stresses on the banks, and help to restore the optimum. However, in a channel wider than the optimum it is unclear that decreased flow depth would offset the effects of steeper gradient in decreasing bank shear stresses and encouraging deposition.

Secondly, channel width is subject to multivariate controls, as discussed below, so that width is likely to depart strongly from the hypothesized optimum in many channel networks.

Thirdly, natural streams are subject to widely varying flows. Each discharge is associated with a different optimal width. Although a dominant discharge and associated optimal width may occur, the effect of the varying discharges will probably be to broaden and weaken the gradient minimum associated with the optimal width, correspondingly diminishing the efficiency of any mechanisms tending to adjust the width to an optimal value. In addition, if the stream is subject to rapidly varying discharges, equilibrium bedforms may not fully develop at each flow stage, further blurring the optimum conditions (Allen, 1976a; 1976b).

DISCUSSION

This discussion focuses upon several related issues, including: the limits of applicability of the scenario predicting channel bed types outlined in the preceding sections; the relationship of the bed-type model to other components of fluvial morphology such as cross-sectional shape and river pattern; and the prospects for quantitative modelling of channel evolution.

Model limitations

Several assumptions underlying the calculations of the required gradient versus grain size relationship (figures 4.2–4.4) deserve closer scrutiny. The most crucial assumption is that the required gradient can be calculated independently for different grain-size categories - that is, that these size fractions are hydraulically independent. One justification for this has been suggested that is, the narrow range of bed sediment sizes compared to the total range of supplied sediment. Sediment finer than the range of grain sizes represented on the bed is generally carried as wash load, in below-capacity quantities, so that the assumption seems valid for finer grain sizes. However, all grain sizes in transport that are coarser than the modal bed sediment size are also carried as bedload, so that interaction between size ranges is probable (see p. 8 and p. 73). The basic argument remains valid, however, as long as the method gives reasonable estimates of required gradient for both coarse and fine components. For a sparse population of coarse grains on a fine bed, the effect of the coarse grains on the motion of the fine bedload is probably less important than the enhancing effect of the fine bed on the mobility of the coarse grains (due to greater exposure). The size range of bed sediment in sand channels is generally small, so that the assumption of independence should not be grossly in error.

A related issue is the choice of range of grain sizes included within each grain size category for calculating the required gradient. For the fine grain sizes, it is apparent from equation (4.3) that the required gradient is dependent upon the sediment discharge of the selected grain size range, and that this will tend towards zero as the grain size range narrows, giving a seemingly paradoxical result. On the other hand, inclusion of too wide a grain size range will include grains carried as wash load in calculation of the peak required-gradient and will reduce the resolution of the resulting relationship. Clearly there must be an optimal grain size range dictated by the degree of interaction of related grain sizes. One indicator of this optimum is the range of grain sizes dominating the bed; the range used in the required gradient calculation should probably be neither much larger nor much smaller than this range. Another indication may be the range of grain sizes near the peak in the required-gradient curves, which suggests a dependency of the range width on the variance of supplied grain sizes (figure 4.3); however, the peak varies in width among different assumed transport relationships for the same hydraulic regime.

For the coarse grain sizes near the threshold of motion the model assumes that the Shields relationship governs the threshold of motion, which requires that grains act independently of one another. However, recent research has shown that bed particles ranging from about one-third to four times the median size of the *subsurface* bed material are entrained at nearly the same discharge (Parker, Klingeman and McLean, 1982; Andrews, 1983). Furthermore, this research shows that a surface pavement of grains coarser than both the mean bedload transported and the median subpavement grain size (which are nearly equal) is a stable feature of gravel-bed streams for transport conditions near the threshold of motion (Parker, Dhamotharan and Stefan, 1982; Parker and Klingeman, 1982). At first this near-equality of mobility over a grain size range of about 12 times seems to negate the use of Shields-type threshold equations to calculate required gradients. However, as Carson and Griffiths (1985) point out, one must be careful not to confuse threshold criteria formulated and used for different purposes. Firstly, the median size of the sub-pavement bed material *can* be used with the Shields (entrainment) dimensionless shear stress to predict the onset of motion (although a somewhat smaller constant, 0.033, is suggested than the generally accepted value of about 0.06). Secondly, the equality of mobility does not apply to grains either coarser or finer than the range quoted above. Finer grains move in suspension or wash load, and are only represented in the bed to the degree that they filter down between the coarser bed particles. Particles coarser than 4.2 times the sub-pavement median grain size are not as mobile as smaller grains (otherwise gravel channels would be overwhelmed with house-sized boulders), but have a constant dimensionless shear stress of about 0.02 (Andrews, 1983). Thus, with the exception that a certain range of sizes about the median has nearly constant mobility, these recent findings do not contradict the general pattern proposed in the model. In fact, the range of grains with nearly constant mobility can be viewed as an indication of the range of strongly interacting grain sizes (discussed in the previous paragraph) characteristic of the gravel component.

The assumption of independence of transport for different size ranges clearly will be inapplicable for channels characterized by very high sediment concentrations or mudflows. The sorting processes responsible for the threshold changes in bed type cannot operate in such viscous flows.

The calculations also assume that a dominant discharge can be defined which is equivalent in transporting capability to the wide range of natural discharges. For a narrowly-graded supply of sediment this poses few problems. However, with a well-graded sediment input, the dominant discharge will vary with grain size, becoming larger for coarser sizes. Although the calculations do not account for this effect, its inclusion would not change the basic character of channel behaviour and the threshold behaviour. More problematic is the effect of varying discharges and varying sediment supply in restricting availability of certain size ranges of bed sediment, as a result of either temporary bed armouring during waning and low flows hiding underlying fine sediments, or influx of fine sediment during runoff events burying coarse sediment (summer runoff from desert floods has this effect on the Colorado River in the Grand Canyon; Howard and Dolan, 1981). However, these are probably second-order effects that do not change the essential nature of the thresholds.

The calculations also assume a flow resistance for fine sediment that is independent of grain size and required gradient. In fact, as the discussion of the Chang papers has suggested, the occurrence of bedforms implies that flow resistance, and hence transport capacity and required gradient, will vary with the assumed size of bed sediment and the channel width. More elaborate, iterative calculations of required gradient would be required to account for this effect, or a nomograph approach could be used (Parker and Anderson, 1977). This refinement would be valuable in predictive use of the required gradient for fine-bed channels, but the effect is not in conflict with the occurrence of the coarse to fine threshold transition.

Although the required gradient calculations offer an explanation of the threshold between channel types, the model is not particularly useful in making *a priori* predictions of whether a coarse- or fine-bed alluvial channel or a bedrock channel would occur for those natural streams operating close to a threshold transition. The required gradients depend crucially upon the size-distribution and rate of supply of sediment supplied by slope erosion to the channel system. Since supply and grain size range cannot be easily predicted from morphological characteristics of the drainage basin, intra-stream measurements are required. The fine component of the load, and therefore the peak required gradient in the fine grain size range, can be reasonably estimated from long-term measurements of bed and suspended load. However, the critical grain size for coarse-bed streams and the corresponding required gradient depend upon the relatively small rate of supply of coarse grain sizes and upon the frequency of flood peaks. Sampling of coarse bedload is not routinely undertaken, especially not for long time-periods. The estimation problem is further compounded by the variable fabric of gravel beds; for example, Reid et al. (1985) show that recently reworked gravel beds are more readily entrained, so that there is an ordering effect (Wolman and Gerson, 1978; Brunsden and Thornes, 1979) in entrainment. Also complicating prediction are the actions of sorting (Brierly and Hickin, 1985) and comminution (Schumm and Stevens, 1973), as well as the possibility of disequilibrium between channel bed type and hydraulic regime due to the longer time-scale for adjustment of coarse-bed than fine-bed channels. The occurrence of bedrock channels depends upon past erosional history and resistance of the bedrock to erosion, factors which are difficult to quantify. However, the measurement of channel gradient will indicate whether it is steep enough such that an alluvial bed will not form, although because of the difficulty in estimating the critical grain size, this may not be possible for the bedrock, coarse-bed threshold except for gradients well above the threshold.

Channel cross section and planform

The present model of bed types and channel gradients in alluvial channels is formulated in terms of specific discharges of water and sediment, so that predictions of hydraulic geometry require specification of bank material control of channel width as an additional independent influence. This approach has been taken for several reasons. Firstly, transport relationships are defined in terms of specific discharges, allowing development of the threshold model with fewest additional hypotheses and constraints.

Secondly, channel width is subject to multivariate control, so that in some cases knowledge of the hydraulic regime may not be sufficient to predict width; for example, along most of the Colorado River in the Grand Canyon the channel walls are bedrock or talus although the bed is alluvial (Howard and Dolan, 1981); the channel width is therefore narrower than would be the case for alluvial banks.

Thirdly, even in the more common case of channels with self-formed alluvial banks, the width is determined by different components of the sediment load from the bed (with a greater role exercised by the suspended and wash load), and width responds to different aspects of the flow regime. In particular, channel width is sensitive to the effects of large floods (Schumm and Lichty, 1963; Burkham, 1972). Deterministic predictive models of cross-sectional geometry have so far been limited to cases where the bed and bank sediment are the same. Li et al. (1976) have modelled stable gravel channels where bed and banks are at the threshold of motion. The predicted width-to-depth ratio of about 7 to 8 is close to the average value of about 10 observed by Brush (1961) in eastern United States gravel-bed streams. Parker (1978b; 1979) has extended this analysis to the more frequent case of gravel channels with stable banks but which experience finite sediment transport rates at high discharges; he shows that equilibrium channel widths are greater than for the stable channel case, which in turn requires steeper gradients. Parker (1978a) has also provided a model for sand-bed channels with sand banks in which the bank profile is determined by the balance between the downslope transport of bank sediment and the deposition from suspension. Pizzuto (1984) has applied this model to sand-bed channels. However, only empirical models are available for the more typical case of banks finer than the bed material, particularly when the banks are cohesive and/or vegetated.

'Closure' of predictive models for channel gradients requires determination of channel width; channel width may be estimated by locally-determined width-discharge hydraulic geometry, by multivariate hydraulic geometry relationships that incorporate information on bank sediment characteristics and, possibly, vegetation (Schumm 1960; 1969; Charlton et al., 1978; Osterkamp et al., 1983; Andrews, 1984; Hickin, 1984; Gurnell and Gregory, 1984), or by theoretical models where appropriate.

Comprehensive discussion of channel pattern and its effects upon gradient and bed type is beyond the scope of this chapter. In heavily engineered streams with protected banks the channel pattern is determined. But for natural streams knowledge of the equilibrium channel pattern is important not only to be able to predict valley gradients in addition to channel gradients, but also because the channel pattern has mutual interactions with cross-sectional shape and sediment transport mechanisms: for example, transport efficiency in braided channels should be lower than for a single-thread channel of equivalent hydraulic regime because of increased bank resistance (although differences in bedforms in single-thread and braided channels may complicate the issue). Also, several authors have suggested that a meandering pattern is more efficient in sediment transport than a straight channel (e.g. Langbein and Leopold, 1966; Yang, 1971; Chang, 1979b).

Furthermore, valley slope may be an independent variable during short-term adjustments of channels to change in hydraulic regime. Schumm (1968) for example suggests that the Murrumbidgee River in Australia responded to changes in hydraulic regime by combining changes in channel width and sinuosity which absorbed the change in regime without aggradation or entrenchment. More likely is that the valley gradient constrained the possible short-term responses of channel width and sinuosity to changes in regime. Thus, were the regime to be indefinitely held constant following a change in regime, systematic aggradation or entrenchment would probably eventually change the valley gradient. The short-term decreases in sinuosity in response to widening of channels during floods and the increase in sinuosity as the channel gradually narrows (Schumm and Lichty, 1963; Burkham, 1972) confirm this interpretation.

More complete discussions of the extensive literature on channel patterns are provided in texts by Richards (1982) and Knighton (1984), in recent papers by Carson (1984) and Ferguson (1984), and in the chapter by Ferguson in this volume.

Prospects for simulation modelling

The analysis presented in this chapter is limited in quantitative treatment to equilibrium bed morphology and gradient, although some qualitative conclusions have been made concerning transient response. In view of the dynamic nature of geomorphic processes and landforms, equilibrium models are informative about the structure of the system, although of restricted predictive applicability (Howard, 1982). In recent years more general temporal simulation models have been developed for particular fluvial systems. These include aggradation and degradation models based upon routing of fine-grained sediment e.g. Howard, 1982; Park and Jain, 1986; and references in these papers), models of channel armouring below dams (e.g. Garde et al., 1977, and Lu and Shen, 1986), and simulations of meander pattern development (Howard and Knutson, 1984). Such models oversimplify the natural interactions between processes and morphology, restricting the range of their applicability or rendering them of heuristic value only. However, as knowledge of these interactions improve, simulation models will become more general and useful in examining both short- and long-term fluvial responses to disturbances.

CONCLUSION

The existence of thresholds in bed type between bedrock, gravel, and sand-bed channels seems well-established, and the general factors leading to these thresholds have been examined in this chapter. Thresholds may also occur in bed type within fine-bed channels as a result of transitions between bedform regimes. However, several questions remain to be answered, such as the limits of applicability of the fine- to coarse-bed threshold for high sediment concentrations or for large variance in sizes of supplied sediment. Furthermore, the role of the thresholds, if any, in the formation of riffles and pools is uncertain.

Therefore, further examination of the threshold concept as applied to channel bed types is desirable, either by more refined theoretical modelling, through application of field observations, or by flume experiments. Flume experimentation involving input of sediments with a large variance in grain size and with variable discharges would be particularly valuable.

REFERENCES

Allen, J. R. L. 1976a: Time-lag of dunes in unsteady flows: an analysis of Nasner's data from the R. Weser, Germany. *Sedimentary Geology*, 15, 309-21.

Allen, J. R. L. 1976b: Computational models for dune time lag: population structures and the effects of discharge pattern and coefficient of change. *Sedimentary Geology*, 16, 99-130.

Andrews, E. D. 1983: Entrainment of gravel from naturally sorted riverbed material. *Geological Society of America Bulletin*, 94, 1225-31.

Andrews, E. D. 1984: Bed-material entrainment and hydraulic geometry of gravel bed rivers in Colorado, USA. *Geological Society of America Bulletin*, 95, 371-8.

Bagnold, R. A. 1941: *The Physics of Blown Sand and Desert Dunes*. London: Methuen.

Brierly, G. J. and Hickin, E. J. 1985: The downstream gradation of particle sizes in the Squamish River, British Columbia. *Earth Surface Processes and Landforms*, 10, 597-606.

Brunsden, D. and Thornes, J. B. 1979: Landscape sensitivity and change. *Transactions of the Institute of British Geographers, New Ser.*, 4, 463-84.

Brush, Jr. L. M. 1961: Drainage basins, channels, and flow characteristics of selected streams in central Pennsylvania. *US Geological Survey Professional Paper*, 282-F, 145-81.

Burkham, D. E. 1972: Channel changes of the Gila River in Safford Valley, Arizona, 1846-1970. *US Geological Survey Professional Paper*, 655-G.

Carson, M. A. 1984: The meandering-braided river threshold: a reappraisal. *Journal of Hydrology*, 73, 315-34.

Carson, M. A. and Griffiths, G. A. 1985: Tractive stress and the onset of bed particle movement in gravel stream channels: different equations for different purposes. *Journal of Hydrology*, 79, 375-88.

Chang, H. H. 1979a: Geometry of rivers in regime. *Journal of the Hydraulics Division American Society of Civil Engineers*, 105, 691-706.

Chang, H. H. 1979b: Minimum stream power and river channel patterns. *Journal of Hydrology*, 41, 303-27.

Chang, H. H. 1980: Geometry of gravel streams. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 106, 1443-56.

Chang, H. H. 1984: Modeling of river channel changes. *Journal of Hydraulic Engineering*, 110, 157-72.

Chang, H. H. 1985: River morphology and thresholds. *Journal of Hydraulic Engineering*, 111, 503-19.

Chang, H. H. 1986: River channel changes: adjustments of equilibrium. *Journal of Hydraulic Engineering*, 112, 43-55.

Charlton, F. G., Brown, P. M. and Benson, R. W. 1978: The hydraulic geometry of some gravel rivers in Britain. *Hydraulics Research Station Report*, IT 180.

Cherkauer, D. S. 1973: Minimization of power expenditure in a riffle-pool alluvial channel. *Water Resources Research*, 9, 1613-28.

Church, M. and Gilbert, R. 1975: Postglacial fluvial and lacustrine environments. In *Glaciofluvial and Glaciolacustrine Sedimentation*, Society of Economic Paleontology and Mineralogy, Special Publication, 23, 22–100.

Daniels, R. B. 1960. Entrenchment of the Willow Drainage Ditch, Harrison County, Iowa. *American Journal of Science*, 258, 161–76.

Daniels, R. B. and Jordan, R. H. 1966: Physiographic history and the soils, entrenched stream systems and gullies, Harrison County, Iowa. *US Department of Agriculture, Technical Bulletin*, 1348.

Einstein, H. A. 1950: The bed-load function for sediment transportation in open channel flow. *US Department of Agriculture, Soil Conservation Service, Technical Bulletin*, 1026.

Ellwood, J. M., Evans, P. D. and Wilson, I. G. 1975: Small scale eolian bedforms. *Journal of Sedimentary Petrology*, 45, 554–61.

Emerson, J. W. 1971: Channelization: a case study. *Science*, 172, 325–6.

Emmett, W. W. 1976: Bedload transport in two large, gravel-bed rivers, Idaho and Washington. In *Proceedings of the Third Federal Inter-Agency Sedimentation Conference*, 5-1–5-12.

Engelund, F. 1966: Hydraulic resistance of alluvial streams. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 92, 315–26.

Ferguson, R. I. 1984: The threshold between meandering and braiding. In K. V. H. Smith (ed.), *Channels and Channel Control Structures*, Berlin: Springer Verlag, 6-15–6-29.

Garde, R. J., Ali, K. A. and Diette, S. 1977: Armoring process in degrading streams. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 103, 1091–5.

Gurnell, A. M. and Gregory, K. J. 1984: The influence of vegetation on stream channel processes. In T. P. Burt and D. E. Walling (eds), *Catchment Experiments in Fluvial Geomorphology*, Norwich: Geo Books, 515–35.

Hack, J. T. 1957: Studies of longitudinal stream profiles in Virginia and Maryland. *US Geological Survey Professional Paper*, 294-B, 45–97.

Henderson, F. M. 1966: *Open Channel Flow*. New York: Macmillan.

Hickin, E. J. 1984: Vegetation and river channel dynamics. *Canadian Geographer*, 28, 111–26.

Howard, A. D. 1971: Optimal angles of stream junctions: geometric, stability to capture, and minimum power criteria. *Water Resources Research*, 7, 863–73.

Howard, A. D. 1980: Thresholds in river regimes. In D. R. Coates and J. D. Vitek (eds), *Thresholds in Geomorphology*, London: Allen and Unwin, 227–58.

Howard, A. D. 1982: Equilibrium and time scales in geomorphology: application to sand-bed alluvial streams. *Earth Surface Processes and Landforms*, 7, 303–25.

Howard, A. D. and Dolan, R. 1981: Geomorphology of the Colorado River in the Grand Canyon. *Journal of Geology*, 89, 269–98.

Howard, A. D. and Kerby, G. 1983: Channel changes in badlands. *Geological Society of America Bulletin*, 94, 739–52.

Howard, A. D. and Knutson, T. R. 1984: Sufficient conditions for river meandering: a simulation approach. *Water Resources Research*, 20, 1659–67.

Keller, E. A. and Melhorn, W. N. 1973: Bedforms and fluvial processes in alluvial stream channels: selected observations. In M. Morisawa, (ed.), *Fluvial Geomorphology*, New York: Allen and Unwin, 253-83.

Knighton, D. 1984: *Fluvial Forms and Processes*. London: Edward Arnold.

Langbein, W. B. 1964: Geometry of river channels. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 90, 301-12.

Langbein, W. B. and Leopold, L. B. 1966: River meanders - theory of minimum variance. *US Geological Survey Professional Paper*, 422H.

Leopold, L. B. and Miller, J. P. 1956: Ephemeral streams - hydraulic factors and their relation to the drainage net. *US Geological Survey Professional Paper*, 282-A.

Li, R., Simons, D. B. and Stevens, M. A. 1976: Morphology of cobble streams in small watersheds. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 102, 1101-17.

Lu, J-Y. and Shen, H. W. 1986: Analysis and comparisons of degradation models. *Journal of Hydraulic Engineering*, 112, 281-99.

Mackin, J. H. 1948: Concept of the graded river. *Geological Society of America Bulletin*, 59, 463-512.

Miller, J. P. 1958: High mountain streams - effects of geology on channel characteristics and bed material. *New Mexico Bureau of Mines and Mineral Resources, Memoir*, 4.

Osterkamp, W. R., Lane, L. J. and Foster, G. R. 1983: An analytical treatment of channel-morphology relations. *US Geological Survey Professional Paper*, 1288.

Park, I. and Jain, S. C. 1986: River-bed profiles with imposed sediment load. *Journal of Hydraulic Engineering*, 112, 267-80.

Parker, G. 1978a: Self-formed straight rivers with equilibrium banks and mobile bed. Part 1. The sand-silt river. *Journal of Fluid Mechanics*, 89, 109-25.

Parker, G. 1978b: Self-formed straight rivers with equilibrium banks and mobile bed. Part 2. The gravel river. *Journal of Fluid Mechanics*, 89, 127-46.

Parker, G. 1979: Hydraulic geometry of active gravel rivers. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 105, 1185-1201.

Parker, G. and Anderson, A. G. 1977: Basic principles of river hydraulics. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 103, 1077-87.

Parker, G. and Klingeman, P. C. 1982: On why gravel bed streams are paved. *Water Resources Research*, 18, 1409-23.

Parker, G., Dhamotharan, S. and Stefan, H. 1982: Model experiments on mobile, paved gravel bed streams. *Water Resources Research*, 18, 1395-1408.

Parker, G., Klingeman, P. C. and McLean, D. G. 1982: Bedload and size distribution in paved gravel-bed streams. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 108, 544-71.

Piest, R. F., Elliot, L. S. and Spomer, R. G. 1977: Erosion of the Tarkio Drainage System: 1845-1976. *American Society of Agricultural Engineers Transactions*, 20, 485-8.

Pizzuto, J. E. 1984: Equilibrium bank geometry and the width of shallow sand bed streams. *Earth Surface Processes and Landforms*, 9, 199-207.

Raudkivi, A. J. 1976: *Loose Boundary Hydraulics*. Oxford: Pergamon.

Reid, I., Frostick, L. E. and Layman, J. T. 1985: The incidence and nature of bedload transport during flood flows in coarse-grained alluvial channels. *Earth Surface Processes and Landforms*, 10, 33-44.

Renard, K. G. and Laursen, E. M. 1975: Dynamic behavior model of ephemeral stream. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 101, 511-28.

Richards, K. 1982: *Rivers: Form and Process in Alluvial Channels*. London: Methuen.

Roy, A. G. 1985: Optimal models of river branching angles. In M. J. Woldenberg (ed.), *Models in Geomorphology*, Boston: Allen and Unwin, 269-85.

Schumm, S. A. 1960: The shape of alluvial channels in relation to sediment type. *US Geological Survey Professional Paper*, 352-B.

Schumm, S. A. 1968: River adjustment to altered hydrologic regimen - Murrumbidgee River and paleochannels, Australia. *US Geological Survey Professional Paper*, 598.

Schumm, S. A. 1969: River metamorphosis. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 95, 255-72.

Schumm, S. A. and Lichty, R. W. 1963: Channel widening and flood-plain construction along Cimarron River in southwestern Kansas. *US Geological Survey Professional Paper*, 352-D, 71-88.

Schumm, S. A. and Stevens, M. A. 1973: Abrasion in place: a mechanism for rounding and size reduction of coarse sediments in rivers. *Geology*, 1, 37-40.

Schumm, S. A., Harvey, M. D. and Watson, C. C. 1984: *Incised Channels - Morphology, Dynamics, and Control*. Littleton, Colorado: Water Resources Publications.

Shaw, J. and Kellerhals, R. 1977: Downstream grain size changes in Albertan Rivers (Abstract). In *First International Symposium on Fluvial Sedimentology*, Calgary.

Shea, J. H. 1974: Deficiencies of clastic particles of certain sizes. *Journal of Sedimentary Petrology*, 44, 985-1003.

Slatt, R. M. and Hoskins, C. M. 1968: Water and sediment transport in the Norris Glacier outwash area, upper Taku Inlet, southeastern Alaska. *Journal of Sedimentary Petrology*, 38, 434-56.

Williams, G. P. 1978a: Hydraulic geometry of river cross sections - theory of minimum variance. *US Geological Survey Professional Paper*, 1029.

Williams, G. P. 1978b, Bankfull discharge of rivers. *Water Resources Research*, 14, 1141-54.

Williams, G. P. and Wolman, M. G. 1984: Downstream effects of dams on alluvial rivers. *US Geological Survey Professional Paper*, 1286.

Wolman, M. G. and Gerson, R. 1978: Relative scales of time and effectiveness of climate in watershed geomorphology. *Earth Surface Processes*, 3, 189-208.

Yang, C. T. 1971: Potential energy and stream morphology. *Water Resources Research*, 7, 311-22.

Yang, C. T. 1976: Minimum unit stream power and fluvial hydraulics. *Journal of the Hydraulics Division, American Society of Civil Engineers*, 102, 919-34.

Yatsu, E. 1955: On the longitudinal profile of the graded river. *Transactions of the American Geophysical Union*, 36, 655-63.