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EQUILIBRIUM AND TIME SCALES IN GEOMORPHOLOGY: APPLICATION TO SAND-BED ALLUVIAL STREAMS

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ABSTRACT

Equilibrium is defined as a single-valued, temporally invariant functional relationship between the values of an output variable and the values of the input variable(s) in a geomorphic system. Disequilibrium occurs if the output deviates from the functional relationship by more than a consensual degree. Natural geomorphic variables are characterized by a relaxation time. Output variables are insensitive to cyclical inputs with frequencies much greater than the relaxation time, but can respond completely for sufficiently low frequencies. Rapid trends, recent step changes or pulse inputs, and intermediate frequency inputs can cause disequilibrium.

The gradient of sand-bed alluvial channels (the output variable) is determined by sediment and water delivery from slopes (the input variables), and changes in this hydraulic regime require regrading by erosion and deposition. Initial stages of adjustment to changed regime in a long, unbranched channel with sediment and water delivery only at the upstream end propagate downstream, but later stages of adjustment occur simultaneously throughout the reach. In a dendritic channel network the gradient responds rather uniformly throughout the network to changes in regime during all stages of adjustment. The time scale of adjustment to changed regime depends upon the size of the channel network (or stream length), the sediment and water discharges, and to a lesser degree upon the magnitude of the change.

Grade as defined by Mackin (1948) is synonymous with equilibrium as used in this paper if 'a period of years' is replaced by 'a time period commensurate with the relaxation time of the gradient'. The use of the term grade is best restricted to a single-valued relationship between channel gradient and the hydraulic regime.

KEY WORDS Equilibrium Grade Alluvial Time scale Relaxation time

INTRODUCTION

Science is an art of pragmatic idealization wherein the complex interactions of mass and energy in nature are represented by simplified models employing verbal, mathematical, mechanical or electrical analogues. Such idealization is applied not only to the modelling of structures and flows of the natural system, but on occasion also to its temporal behaviour. The concept of equilibrium is an important model of the behaviour of natural systems, implying some type of dynamic balance or constancy between controlling and responding elements. Although equilibrium has long been a central concern in geomorphology, it has been used with little uniformity in definition and usage, so that a number of different types of temporal or spatial relationships have been proposed as characterizing equilibrium. The major problems seem to be imprecise, overlapping, and conflicting definitions, and a plethora of similar terms (see the discussions by Chorley and Kennedy, 1971. Ch. 6, and Allen, 1974). If the equilibrium concept is to be of continued utility within geomorphology, it must be within the context of precise definition (preferably quantitative) and adequate operational rules for its application and recognition in the behaviour of natural systems. In addition, the proposed definition should as much as possible conform to the essential features of prior usages and connotations of equilibrium. It is the purpose of this paper to extend my earlier consideration of geomorphic equilibria (Howard, 1965) by presenting a quantitative definition of equilibrium, and testing its applicability to the concept of regime and grade in alluvial streams.

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The properties of equilibrium will be introduced and illustrated with the behaviour of a simple linear system and then extended to the more complicated case of sand-bed alluvial streams, using dimensional analysis, idealized numerical simulations, and limited field evidence to explore the applicability and utility of the defined usages. Alluvial streams were chosen to illustrate the equilibrium concepts because they are in many respects the simplest and best understood geomorphic systems and they share with other landforms the property of being spatially distributed, so that delays in completion of response to altered fluxes of mass or energy (lag or relaxation times) are occasioned by finite transport rates.

Equilibrium in alluvial streams has generally been discussed within the conceptual framework of grade. The final discussion of sand-bed channels explores the correspondences between the concepts of grade and equilibrium, as well as a brief summary of some of the limitations and complications of these concepts as applied to natural streams.

Related to the concept of equilibrium is the time scale required for its reestablishment following change or disruptions in the controlling variable. The determination of the time scale of responses in alluvial streams remains a central concern even when the nature of inputs is such that equilibrium is never attained.

The concluding discussion returns to the question of the untility of the equilibrium concept: does the utility outweigh the complications and uncertainties in applications to geomorphic systems?

THE CONCEPT OF EQUILIBRIUM

Within this paper equilibrium refers to a specific type of temporal relationship between external factors (the inputs) and a responding internal parameter (the output) measured within a (geomorphic) system and having the following characteristics:

1. Measurable changes in the inputs must cause measurable changes in the output either immediately, or after a finite time. This eliminates the trivial case of input variables which have no effect upon the output. Likewise eliminated from consideration are those components of input signals which cause no output response (e.g., high frequency components, as discussed below).
2. The value of the output at a given time is related by a single-valued, temporally invariant functional relationship to the value(s) of the input(s) at the same time, within a consensual degree of accuracy. Although the equilibrium value of the output does not specifically depend upon past values of the inputs, only certain types of input histories will result in equilibrium.
3. The input and output must satisfy the above requirements for a length of time that is an appreciable fraction of the relaxation or lag time of the system output. This restriction eliminates from inclusion within equilibrium the momentary (or 'accidental') satisfaction of the functional relationship, such as periodically occurs with a cyclical input. This requirement could be alternatively phrased as a requirement for an asymptotic approach to equilibrium.

This characterization of equilibrium is in some respects more liberal than certain previous usages, and in some respects more restrictive. It is more liberal in allowing within the equilibrium behaviour a degree of departure from the defined functional relationship between input and output. It is more restrictive by limiting the equilibrium to pairwise relationships between one or more input variables and the output; thus, equilibrium is not considered to be a property of the geomorphic system as a whole.

Natural systems require a finite length of time for the system to respond fully to a change in input. This lag, or relaxation time, is a consequence of the size and mass of geomorphic systems. Because of the limited energy available, processes like weathering and erosion work slowly within the landscape to produce changes resulting from alterations of process or base level. Similarly, the spatial extension of geomorphic systems and the potentiality for mass and energy storage within the landscape imply finite time between distributed inputs of mass or energy and their transport from the system within, for example, streams. The effect of relaxation times on system behaviour is discussed below within the context of a simple linear system. This heuristic example is used to introduce the requisite conditions for the occurrence of equilibrium.

Types and time scales of equilibrium

The occurrence of equilibrium between the relevant system property (output) and the forcing external variables (inputs) depends upon the balance between the relaxation time of the system property and the rate, type, and timing of input changes. This balance is illustrated by considering a simple linear system whose output, or response, $y(t)$, is a weighted average of past values of a single external variable, $x(t)$. The weighting function will be assumed to be a negative exponential, so that:

$$y(t) = \lambda \int_0^{\infty} x(t-\tau) \exp\{-\lambda\tau\} \, d\tau, \quad (1)$$

where $T = 1/\lambda$ is the characteristic relaxation time of the system (the relaxation time is sometimes alternatively defined as the time, T, such that $\exp\{-\lambda T\} = 0{\cdot}5$(Waide and Webster, 1976)).

The conditions for equilibrium and the characteristics of system dynamics are best illustrated by considering the response of the system to one or more types of simple inputs, such as step changes, trends, sinusoids, and impulses (Schwarz and Friedland, 1965). This response function approach is common in biological (Milsum, 1966), ecological (Waide and Webster, 1976), and hydrological (Sharma, 1980) systems analysis, with the unit hydrograph and unit sediment graph being the most common hydrological usage. Brunsden and Thornes (1979) and Pickup and Rieger (1979) have anticipated the use of response functions and the convolution integral (equation (1)) in geomorphology. A step change in input is used to introduce the requirements for equilibrium.

Equilibrium occurs when the value of $y(t)$ is sufficiently close to the value that it would have if the input variable(s) were constant over an infinite time ($x(t) = C$). This ultimate equilibrium value of the output, for our system, is:

$$y(t) = \lambda \int_0^{\infty} C \exp\{-\lambda\tau\} \, d\tau = C \quad (2)$$

Equilibrium can be defined without reference to the initial state as the requirement that the response be within a given fraction of its ultimate value, so that, if $x(t)$ undergoes a step change at time T_0 from C_1 to C_2, then $y(t)$ will not be in equilibrium until a time T_1 when:

$$|C_2 - y(T_1)| = \varepsilon \, . \, C_2, \quad (3)$$

where ε is the consensual limit of error. This definition seems appropriate for those geomorphic variables which are always positive (e.g., channel width, depth, and gradient) and whose variance is assumed to be proportional to the magnitude of the variable (as in log-linear regression). For $t \geqslant T_0$ the value of $y(t)$ is:

$$y(t) = C_2 + (C_1 - C_2) \exp\{-\lambda(t - T_0)\}. \quad (4)$$

Thus the duration to the new equilibrium depends upon the magnitude of the step change:

$$t \geqslant T_1 = T_0 - \ln\{(\varepsilon \, . \, |C_2|)/|C_1 - C_2|\}/\lambda. \quad (5)$$

Note that for changes in input less than $\varepsilon \, . \, C_2$ the equilibrium is not disturbed.

The degree of approach to the ultimate value could be alternatively expressed as:

$$(C_2 - y(t))/(C_2 - C_1) \leqslant \varepsilon \quad (6)$$

As an example, for equation (6), 95 per cent adjustment to the ultimate value ($\varepsilon = 0{\cdot}05$), occurs for:

$$t \geq T_1 = T_0 + 3{\cdot}0/\lambda, \quad (6a)$$

This latter definition is used below in presentation of the results of computer simulations of alluvial channel aggradation and entrenchment due to step changes in inputs because the time scale is not dependent upon the magnitude of the change. However, the time scales so derived can be converted to those given by equation (3) by a suitable adjustment of the parameter ε.

The inputs to natural systems are not necessarily constant, or step-wise variant, but may be complex functions of time. Many geomorphic processes, such as runoff, can be considered to be quasi-periodic (with, for example, a yearly cycle). Such perioidc inputs can be represented conceptually as a sum of sinusoidal inputs with different frequencies superimposed upon an average value plus, perhaps, step changes, trends, or impulses. Looking first at a single frequency component with an average value:

$$x(t) = a \sin\{\omega t\} + C, \text{ so that}$$

$$y(t) = A \sin\{\omega t - \theta\} + C, \text{ where} \tag{7}$$

$$A = a\beta/\sqrt{\beta^2+1} \text{ and } \theta = \arctan 1/\beta, \text{ where } \beta = \lambda/\omega.$$

For this type of input the system response is a constant signal with a damped and delayed superimposed sinusoidal component. The parameter A/a is conventionally termed the magnitude ratio, and θ is the phase shift (also see Pickup and Rieger, 1979). These, in turn, are functions of the parameter β (termed here the response ratio) relating the input frequency to the system relaxation time. The magnitude ratio and phase shift are plotted as a function of the response ratio in Figure 1.

Using the proportional deviation definition of equation (3), equilibrium is considered to occur if the following inequality pertains:

$$M_x\{y(t) - Y(t)\}/A_v\{y(t)\} \leq \varepsilon, \tag{8}$$

where ε is the consensual limit of deviation, $y(t)$ is the output, $Y(t)$ is the ultimate value of the output were the present value of the inputs maintained indefinitely, and $M_x\{\}$ is the maximum value and $A_v\{\}$ is the average value of the enclosed functions occurring over one input cycle. For the input given by equation (7) $Y(t)$ equals $x(t)$, and equation (8) reduces to:

$$a \sin\{\theta\} C \leq \varepsilon, \tag{8a}$$

where θ is determined by the response ratio (equation (7)).

Alternatively, using the proportional response concept of equation (6) the degree of response of the output to just the cyclical component of the input can be assessed by replacing $A_v\{y(t)\}$ in equation (8) by the amplitude, a, so that equilibrium occurs if:

$$\sin\{\theta\} \leq \varepsilon. \tag{8b}$$

For small values of the response ratio the sinusoidal component of the input has negligible influence on the output or upon the presence or absence of equilibrium, that is, it is filtered out. This type of response is discussed by Howard, 1965; Chorley and Kennedy, 1971; Allen, 1974; and Brunsden and Thornes, 1979. For the input represented by equation (7), the contribution of the sinusoidal component to the output can be considered to be negligible if:

$$M_x\{y(t)\}/A_v\{y(t)\} = A/C = a\,.\,\beta/(C\sqrt{\beta^2+1}) \leq \varepsilon. \tag{8c}$$

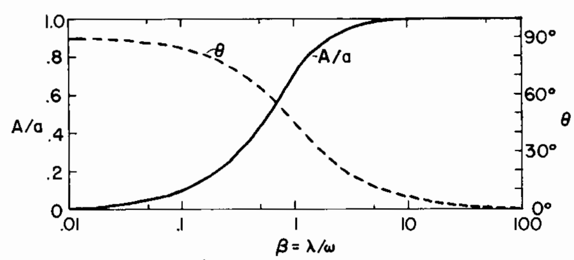

Figure 1. The magnitude ratio, A/a, and phase lag, θ, plotted as a function of the response ratio, β, for a linear system with finite memory

The degree of filtering out of sinusoidal inputs for high frequency inputs and long relaxation times can also be measured in a manner similar to equation (8b) by the ratio A/a in equation (7). For example, $A/a \leqslant 0{\cdot}05$ for $\beta \leqslant 0{\cdot}05$.

For large-amplitude cyclical inputs characterized by response ratios near unity, the output is not in equilibrium due to the damped, delayed response.

If a time trend is superimposed, such that:

$$x(t) = a \sin\{\omega t\} + C + \delta t,$$

then the response shows a bias:

$$y(t) = A \sin\{\omega t - \theta\} + C + \delta t - \delta/\lambda. \tag{9}$$

In this case the response can be considered to be in disequilibrium if the bias, δ/λ, is sufficiently large compared to other components of the response:

$$\delta/\lambda \geqslant \varepsilon(C + \delta t) \tag{10}$$

Another choice of signal decomposition is into a series of impulses of varying magnitude and frequency of occurrence, as is discussed by Pickup and Rieger (1979). Such a choice of input representation would seem appropriate for describing the response of slowly adjusting system parameters (such as channel gradient) to short-duration inputs, such as excess sediment yields resulting from fire or clear-cutting. The unit impulse is the limiting representation of a pulse of unit area as the duration of the pulse approaches zero. A single impulse of magnitude M, occurring at time T_0, and superimposed upon a constant input, C, will cause a disequilibrium whose duration depends upon the impulse amplitude and the relaxation time of the system. if equation (3) defines equilibrium, then it will occur for times $t \geqslant T_1$ such that:

$$t \geqslant T_1 = T_0 - \ln\{(\varepsilon \,.\, C)/(M\lambda)\}/\lambda. \tag{11}$$

Note that equilibrium is not disturbed if $T_1 \leqslant T_0$.

If the unit impulses occur with sufficient frequency compared to the relaxation time of the system output, then the output will remain essentially constant, with a value proportional to the frequency and magnitude (duration times amplitude) of the pulses. Thus the high frequency part of the input is filtered out, and the output may be considered to be in equilibrium with the average value of the input.

Summary: equilibrium and disequilibrium

The inputs to a linear system can be resolved into components consisting of superimposed periodic signals with different frequencies plus, possibly, an average value, trends, step changes, and pulses. Some of these input components may be irrelevant to the occurrence of equilibrium, either due to their small amplitude, due to the insensitivity of the output to the input, or because they are high frequency cyclical components characterized by a small response ratio. The remaining input components can be cojointly examined for equilibrium by the criteria presented above. Thus a system response will be in disequilibrium if there is an effective input component with one of the following characteristics: 1) The elapsed time since the input has undergone a step change is insufficient for the output to have responded (equation (5)); 2) The effects of an isolated pulse have not been recovered from to the defined level (equation (11)); or 3) The system is subjected to a rapid trend component (equation (10)). Finally, the system will be in disequilibrium to large-magnitude input components with periods of the same order of magnitude as the relaxation time.

Equilibrium as used in this paper refers to a type of temporal relationship between two 'signals' (the inputs and the output), so that, strictly speaking, equilibrium is not a property of the natural system being modelled. The import of measured equilibrium or disequilibrium depends upon the correspondences between the measured variables and the natural system, including both pragmatic and theoretical issues such as the selection of relevant variables, definition of system boundaries, cause and effect relationships, fluxes of mass and energy, determination of system structure, and measurement accuracy. These issues are largely beyond the scope of the present paper, excepting a few points discussed below which highlight differences between the present use of equilibrium and other definitions.

Equilibria in natural systems generally imply thermodynamic steady states, in that the systems are open to exchange of mass or energy with the environment. However, equilibrium as used in this paper implies nothing about the internal structure of the system or about the physical nature of the inputs or outputs. Therefore, the term 'equilibrium' has been used in lieu of 'steady state' due to the broader connotations of the former term.

The selection of variables and measurement procedures to represent a physical attribute of a system is based on pragmatic criteria, one of which is generally to maximize the chance of uncovering an equilibrium relationship. All measurements of natural systems involve temporal and areal /volumetric averaging, and the selection of an appropriate spatio-temporal measurement scale is important in determining the possibility of equilibrium. For example, if measurements of shear and velocity are made at the temporal and physical scale of turbulent eddies, then the concentration of bed sediment over an alluvial bed (measured at the same resolution) will show a wide scatter (disequilibrium) in its relationship to flow variables. However, if flow properties and sediment concentrations are measured over time scales which average (filter out) turbulence and concentration fluctuations, then an equilibrium transport relationship is likely to be discovered.

EQUILIBRIUM IN IDEALIZED SAND-BED ALLUVIAL STREAMS

The equilibrium concepts introduced above can be fruitfully applied to the response of the gradient of alluvial channels to variations in hydraulic regime (input of water and bed sediment) and to changes in base level. Gradient response in sand-bed alluvial channels differs in several respects from the simple linear system discussed above: 1) The response is non-linear; 2) Channel systems are spatially distributed; 3) Secondary responses may occur due to adjustment of other channel properties such as width and sinuosity, as well as thresholds due to the non-linearity of sediment transport mechanics.

Despite these complications, recognition of the occurrence of equilibrium retains both heuristic and practical utility for fluvial geomophology. The most important of the complicating factors is the spatial distribution of stream systems, whose partial analogue in the linear system is the relaxation time. The spatial routing of water and sediment inputs is discussed below with two examples: 1) The response of the gradient of a long stretch of stream channel with water and sediment input only at the upstream end; and 2) The effects of altered regime on gradients within a dendritic channel network with distributed inputs. The simulation models described below involve considerable idealization of natural fluvial processes. However, the models are hopefully realistic enough to portray the general patterns of channel gradient response to changes in hydraulic regime or base level and the requisite conditions and time required for establishment of equilibrium. This section concludes with a discussion of the implications for equilibrium and time scales in gradient response of the necessary oversimplifications introduced in the numerical models.

Response of a channel section

The response of a flume-like, sand-bed alluvial channel (with sediment and water input only at the upstream end) has been investigated both in flume studies and in numerical simulations, with reasonable agreement (Gessler, 1971; Bhamidipaty and Shen, 1971; Soni, Garde, and Ranga Raju, 1980). Such studies are extended here by investigating the response of a channel section to sinusoidally varying water or sediment input and by discussion of the implication of channel behaviour for equilibrium concepts.

The numerical simulations discussed here are limited to the long-term gradient response of lengthy channel sections such that the water flow can be considered to be locally uniform. For convenience, channel width is considered to be temporally constant and uniform in the downstream direction.

Bed material transport is assumed in the simulations to occur at relatively high concentrations so that transport is not near the threshold of movement of the bed material (for naturally varying flow transport by the dominant discharge is assumed to be at high concentrations). Under these circumstances, and additionally assuming a flow resistance equation (Manning's equation is used here), most bed-material transport equations can be summarized as follows for a given sediment density and water temperature:

$$q_s = (KS^A q^B)/\mathrm{f}(d), \tag{12}$$

where q_s and q are specific sediment and water discharges (units l^2/t), S is the channel gradient, f(d) is an increasing function of grain size, and A and B are positive exponents. Although various sediment transport formulas differ in the values of the exponents and grain size function (Howard, 1980), the exact values are not critical to the conclusions drawn here. For the present simulations, grain size was held constant and the exponents A and B were given the values 2·0 and 1·8, respectively, approximating the Einstein–Brown transport formula (Henderson, 1966, p. 440; Howard, 1980).

In the numerical simulations the stream was broken into 20 or more segments. The most downstream segment was fixed in elevation at its lower end in most simulations, corresponding in nature to a control point such as a bedrock outcrop. At the beginning of most simulations constant initial values of q_s and q were assumed, and the channel profile was assumed to have the uniform gradient in equilibrium with the hydraulic regime as indicated by solving equation (12) for the gradient. During subsequent simulation increments the hydraulic regime (or base level) was altered as a function of time, causing readjustment of the channel gradient by aggradation or entrenchment. For changes requiring entrenchment, a thick alluvial bed of grain size equal to the sediment input was assumed.

The simulation proceeded by downstream iterations. At each channel segment the sediment input was the output from the next segment upstream (or the assumed sediment input to the channel for the first upstream segment), whereas the sediment outflow is given by equation (12) for the gradient prevailing in the channel segment for the given iteration. In general inflow and outflow were unequal, requiring erosion or deposition. This sediment was eroded/deposited as two triangular wedges thinning from the end of the segment into both the given segment and the one upstream, so that the change in bed elevation was equal to the product of the temporal increment times the net volumetric change in sediment storage, divided by the channel width. Since this erosion/deposition in general changed the upstream and downstream ends of the segment unequally, the gradients were recalculated prior to the next iteration. This simulation method is very similar to that presented in greater detail by Gessler (1971).

Step change in hydraulic regime

At the beginning of the simulation the gradient was specified so as to be in equilibrium with given values of water and sediment discharge. Then either q_s or q was instantaneously changed to a new constant value (a step change) and the progress of readjustment to the new hydraulic regime was numerically simulated, while the elevation of the downstream control point was held constant. Results for one case are presented non-dimensionally in Figure 2A, where the fractional readjustment of gradient to the new regime is plotted versus the relative position within the reach for several successive stages of adjustment. The pattern of response indicated that the upstream end of the channel section adjusts more rapidly than the downstream end. The non-dimensional pattern of gradient response shown in Figure 2A is essentially independent of the magnitude and sense of the change of regime or the length of the reach (although, of course, the time scale for adjustment depends upon these factors, as discussed below).

Characteristic time scales of adjustment

The characteristic time scale, T, required for attainment of equilibrium in alluvial channels following a change in hydraulic regime (T corresponds to $(T_1 - T_0)$ in equations (5) and (6a)) depends upon the volume of sediment, v_s, that must be eroded or deposited, the difference between the input and outflow of sediment from the reach, Δq_s, and the channel width, W. An idealized model of aggradation or entrenchment following a step change in input of water or sediment envisions a wedge of sediment addition or removal progressing downstream through the reach, such that the gradient on the wedge is equal to the new equilibrium gradient, and that below the wedge remaining at its old value until overtaken by the wedge. Under these conditions the time scale of adjustment for the reach is given by:

$$T = v_s/(W \,.\, \Delta q_s). \tag{13}$$

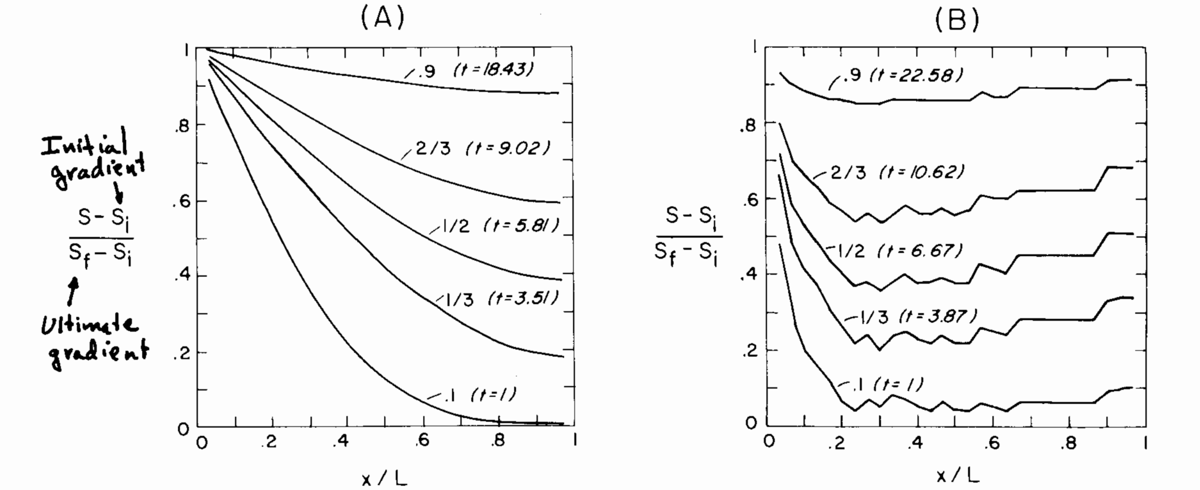

Figure 2. Dimensionless response of a long, unbranched channel (A) and channel network (B) to a step increase in sediment input

For this simplified situation of a channel reach of length L with a constant elevation of the downstream terminus and a constant, uniform width, the time scale for complete adjustment to a step change in water or sediment input will be (after Gessler, 1971):

$$T = \{L^2(S_2 - S_1)\}/(2 \,.\, \Delta q_s), \tag{14}$$

where the subscripts 1 and 2 refer to the initial and final equilibrium states (the length of the alluvial channel is assumed to be unaffected by the regrading). According to the model, the difference between influx and outflow of sediment will be (from equation (12)):

$$\Delta q_s = \{K \,.\, q_2^B(S_2^A - S_1^A)\}/\mathrm{f}(d)$$

Substituting from equations (12) and (15) into equation (14), the time scale for gradient change following a step change in sediment influx will be (discharge and grain size remaining constant):

$$T = \{L^2 \mathrm{f}(d)^{1/A}(Z)\}/\{2K^{1/A} q_2{}^{B/A} q_{s2}{}^{(1-1/A)}\}, \text{ where}$$
$$(Z) = \{(\rho^{1/A} - 1)\rho^{(1-1/A)}\}/(\rho - 1), \text{ where} \tag{16}$$
$$\rho = q_{s2}/q_{s1}.$$

The expression for the case of a step change in discharge (sediment input constant) is similar except that:

$$(Z) = (\rho^{B/A} - 1)/(\rho^B - 1), \text{ where } \rho = q_2/q_1. \tag{17}$$

Thus the characteristic time scale of the reach depends upon both the scale of the system and upon the magnitude of the change of the hydraulic regime.

The numerical simulations of aggradation and entrenchment (Figure 2A) indicate that the upstream parts of the channel approach equilibrium more rapidly than downstream parts, so that the response time increases along the channel. However, it is possible to define a reach response time, T', based upon the fractional addition or removal of the total sediment that must be deposited or eroded during regrading to the hydraulic regime (in the simulations in Figures 2, 3, and 5, the fractional regrading and the corresponding time are used to identify the gradient response curves). The fractional approach to equlibrium based upon sediment erosion/deposition corresponds closely to the average value of the fractional approach to equilibrium of the channel gradients within the reach as defined by equation (6) (Figure 2A).

The simulations indicate that the reach response time, T', for 99 per cent adjustment can be estimated using equations (16) and (17) if the indicated time scale is increased by a factor of about 3, which is independent of the simulation time and length increments. The reason for the longer time scale is that aggradation and entrenchment do not progress downstream simply as idealized above; rather, adjustments are transmitted through the reach. This diffusion of sediment reduces the contrast between the upstream and downstream gradients, and hence, also reduces the rate of accumulation or removal of sediment from the reach. For example, in the idealized model used to formulate equation (16), none of the additional sediment supplied from upstream following a step increase in q_s leaves the reach until the aggradational wedge reaches the downstream terminus. But in the more realistic simulations much of the additional sediment is transmitted through the reach prior to reestablishment of equilibrium; hence the longer time scale.

For small changes of sediment load or discharge (say, $0{\cdot}5 \leqslant \rho \leqslant 2$) the fractions involving ρ in equations (16) and 17) are approximately 0·5, allowing computation of the order of magnitude of the response time. Also under these circumstances, equation (14) can be rewritten as:

$$T \approx (L^2 \,.\, S)/(4 \,.\, g_s). \tag{14a}$$

Response to a pulse input of sediment

A forest fire or clear cutting often results in short-term increases in sediment yield which are many times normal values. This was simulated within the model channel by first establishing a uniform gradient in equilibrium with a constant water and sediment input and then disturbing this by an additional, instantaneous sediment deposition in the first upstream segment, thus increasing its gradient. The gradient response as a function of location is indicated in Figure 3A by $(S-S_e)/S_e$, where S_e is the initial and final equilibrium gradient. As would be expected, the disturbance moves downstream as a wave which spreads and diminishes in amplitude.

Sinusoidally varying hydraulic regime

Gradient changes occurring during sinusoidal variation in either sediment or water discharge were simulated in a manner similar to the step change, except that prior to measurement of the gradient

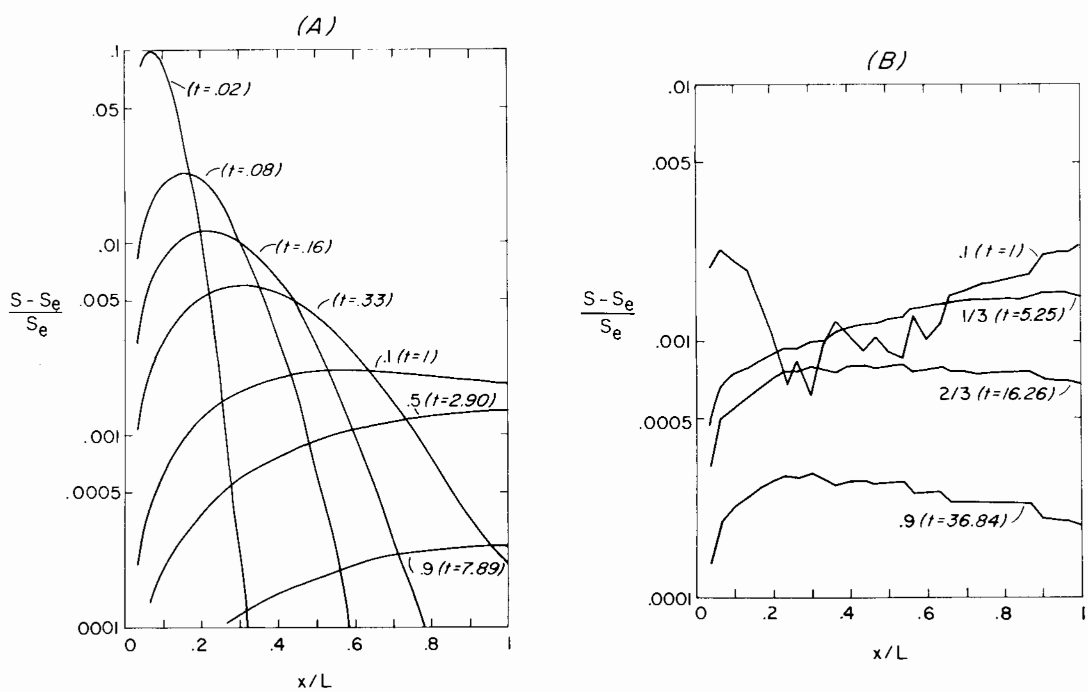

Figure 3. Dimensionless response of an unbranched channel (A) and channel network (B) to a pulse input of sediment. Values of $(S-S_e)/S_e$ are proportional to the magnitude of the pulse

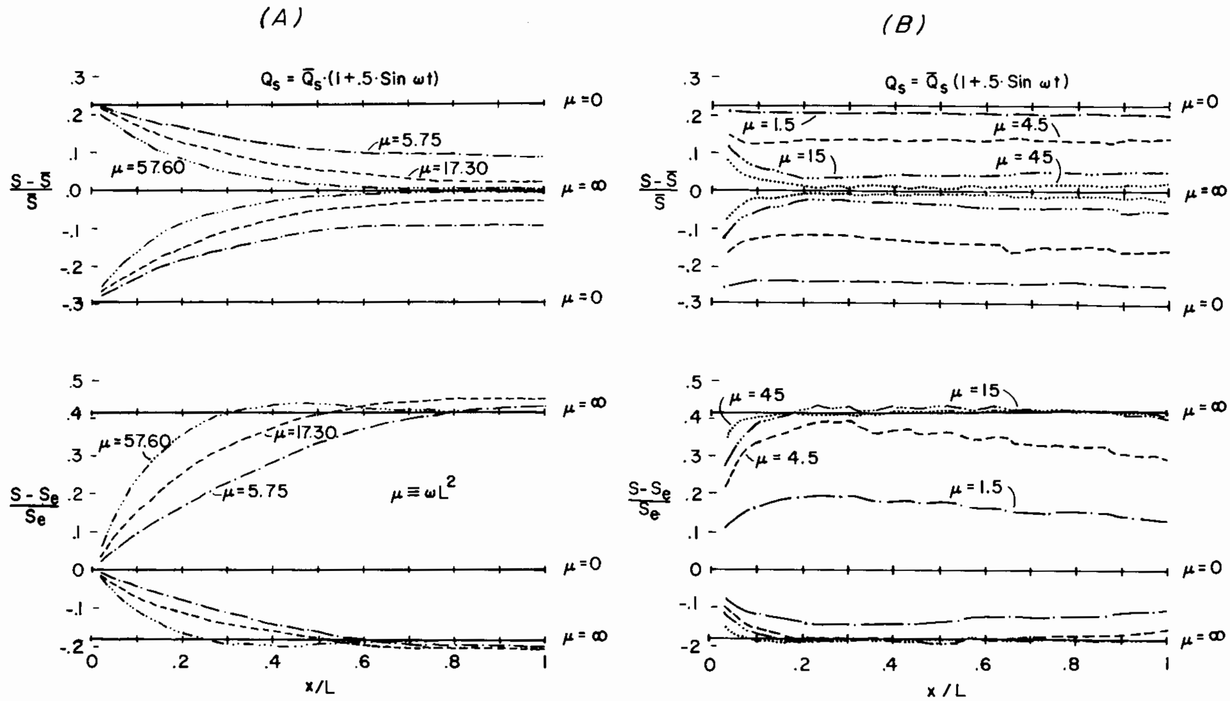

Figure 4. Response of an unbranched channel (A) and channel network (B) to a sinusoidally-varying sediment input

response occurring during a cycle of change in the hydraulic regime, the channel section was allowed to adjust to the sinusoidal input for a time period equal to that required for 95 per cent adjustment to a step change. Results of these simulations are portrayed in Figure 4A for a sinusoidally varying sediment input. The plots are scaled by the factor $\mu = \omega T$, where ω is the frequency of input variation and T is the response time from equation (18).

In the upper part of Figure 4A the instantaneous gradient, S, is compared to the gradient, $\bar{S}$, that would be in ultimate equilibrium with a constant sediment input, q_s, equal to its average value. The values of $(S - \bar{S})/\bar{S}$ that are shown are the maximum positive and negative values occurring during one cycle of variation of sediment input, plotted versus relative distance downstream. Similarly, in the lower part of the figure, the instantaneous gradient is compared to the gradient, S_e, that would be in ultimate equilibrium with the instantaneous value of the hydraulic regime; $(S - S_e)/S_e$ values are similarly extrema occurring during one input cycle.

Equilibrium occurs for very small values of μ such that $(S - S_e)/S_e$ is near zero and $(S - \bar{S})/\bar{S}$ has values near the maximum possible. For large values of μ the sinusoidal component will be filtered out so that the gradient responds to the average value of the input, such that $(S - \bar{S})/\bar{S}$ will be nearly zero and $(S - S_e)/S_e$ will be maximal. By contrast, for intermediate values of μ and for long channels the upstream portion of the channel may closely follow the cyclical input, and thus be in equilibrium, the downstream end responds to the average input (and will also be in equilibrium), whereas the central reach will be in disequilibrium due to finite lag and attenuation. Note that the positive and negative maxima of $(S - \bar{S})/\bar{S}$ and $(S - S_e)/S_e$ are asymmetrical, due to the non-linear relationship between gradient and sediment flux (equation (12)).

Response to change in base level

The profile of alluvial streams is controlled by (graded to) some base level at its downstream end; base level may be local, such as the outcrop of resistant bedrock in the channel bed or the elevation of a master river, or general, such as the level of the oceans. Changes in base level cause regrading of the

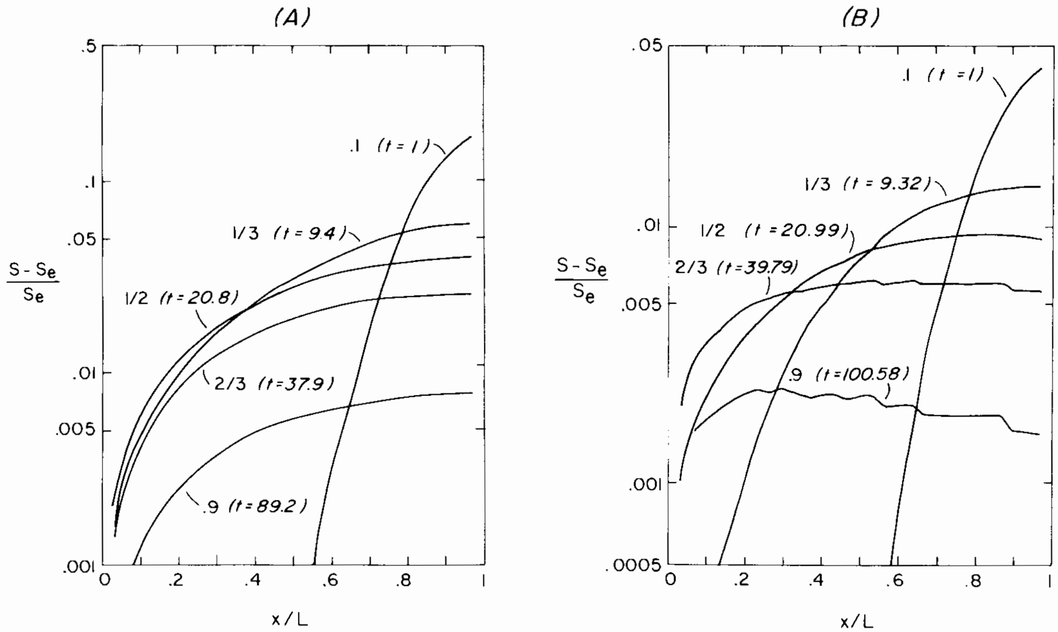

Figure 5. Response of an unbranched channel (A) and a channel network (B) to an instantaneous lowering of base level. Values of $(S-S_e)/S_e$ are proportional to the magnitude of the base level lowering

alluvial channel profile; therefore the concern with regard to equilibrium is the relationship between channel gradients and the rate of change of base level. Thus channel gradients are in equilibrium with a base level elevation which has remained constant for a period commensurate with the response time of the reach. Illustrative types of base level change are a sudden rise or fall followed by stability at the new level (an impulse when converted to rate of change) or a constant rate of rising or lowering.

The gradient response of a long alluvial channel to an instantaneous lowering of base level is illustrated in Figure 5A, under the assumption that the location of the base level remains unchanged. In this figure is plotted the fractional deviation $(S-S_e)/S_e$ of the instantaneous gradient, S, from the gradient in ultimate equilibrium with a constant base level, S_e. The curves are identified by the fractional amount of the total bed sediment removed (for a base level change of magnitude Δ this will equal $\Delta \,.\, L \,.\, W$).

Following the instantaneous gradient increase at the downstream end the gradient steepening spreads upstream as a wave of dissection which diminishes in amplitude as its effects are transmitted upstream. It is informative to compare the response to an impulse of sediment at the upstream end (Figure 3A) with the response to a sudden base level change (Figure 5A), both of which cause a sudden increase of gradient in a single channel segment. In the case of the upstream perturbation (sediment impulse) the downstream migration of the impulse causes the upstream segments to return to nearly their equilibrium value while downstream segments remain appreciably disturbed. However, in the case of the downstream perturbation (base level change) the downstream segments remain more disturbed from their equilibrium value than upstream segments throughout the simulation, due to the necessity to route the eroded sediment.

Assuming that the effective increase in gradient throughout the reach following a step change of base level of magnitude Δ averages Δ/L during the period of recovery, then the time scale for regrading of the channel would be:

$$T=\{L^2 \,.\, \mathrm{f}(d)\}/(2 \,.\, K \,.\, q^B \,.\, S), \text{ or}$$
$$T=\{L^2 \,.\, \mathrm{f}(d)^{(1-1/A)}\}/\{2 \,.\, K^{(1-1/A)} Q^{B(1-1/A)} q_s{}^{1/A}\}. \qquad (18)$$

Simulations indicate that this time scale, based upon the time, T', required for 99 per cent adjustment, should be increased by a factor of 1·8.

If the base level continuously decreases, the gradients within the channel will reach a constant value assuming that the water and sediment discharges remain constant). However, the channel gradients will be everywhere greater than those for constant base level, with the fractional increase more pronounced at the downstream end due to routing of sediment eroded from upstream. If the gradient in equilibrium with a constant base level is chosen as a standard (so that base level change is not an explicit determinant of gradient), then dropping base level introduces a bias whose magnitude increases with the rate of baselevel lowering (compare equation (9)). However, in most natural landscapes the percentage land area covered by active alluvium is small, so that for geologic rates of base level change the sediment supplied by bed erosion will be a small fraction of the sediment load. Thus the bias in channel gradients should be negligible (see Knox, 1975). Other factors influencing the effect of base level changes on gradient are discussed later.

Equilibrium in channel networks

Although the flume-like channel discussed above may be a reasonable model of sediment transporting canals and some rivers, such as the lower sections of the Nile, Colorado, and Mississippi rivers, most stream systems have a distributed input of water and sediment. This distribution of sources has a strong effect upon the pattern of responses within such an alluvial channel network. The simulation model was extended to distributed inputs by utilizing an idealized stream network generated by a random walk growth model (Figure 6). Each stream segment was assumed to receive at its upstream end an equal direct contribution of water and sediment as well as (for all but the first segment of first-order streams) the accumulated discharges from their upstream continuations and tributaries. Thus water and sediment discharges were proportional to upstream drainage areas in ultimate equilibrium. The simulations were conducted similarly to the previous ones, with water discharge assumed to respond immediately to changes in water supply, but changes in sediment discharge (discharge constant) requiring aggradation or entrench-

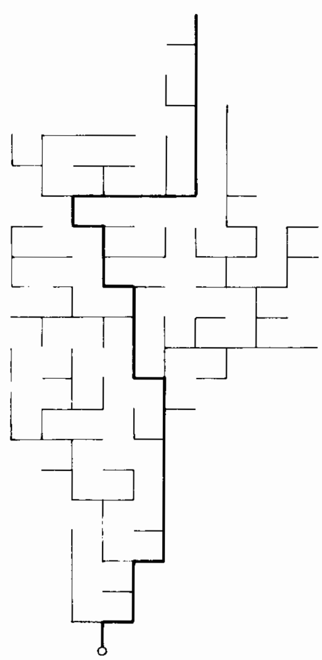

Figure 6. Idealized channel network used for response simulations. Each unit channel length is assumed to drain one unit of land surface, and the input of sediment and water is assumed to be uniform for all units of land surface. Heavy line indicates channel for which responses are portrayed in Figures 2 to 5

ment. The channel and associated valley width were assumed to be proportional to the square root of drainage area (but, as previously, width was assumed to be unaffected by temporal changes in water or sediment supply or by the resultant aggradation or entrenchment).

The response of the channel network is illustrated by considering the stream defined by the longest path length through the network (heavy line in Figure 6). Simulations of response to step, impulse, and sinusoidal changes of hydraulic regime and response to base level change were conducted as above, and the results are portrayed in Figures 2B to 5B. When the response of the stream network is compared to the unbranched channel reach (Figures 2A to 5A), it is apparent that the major difference is that, in the networks the distributed input tends cause a fairly uniform gradient response rather than the sequential downstream (or upstream in the case of base level change) propagation as is more characteristic of the unbranched reach. The specific shape of the curves of network response is somewhat sensitive to the arrangement and length of tributaries.

The simulations indicate that equations (16) and (17) can be used to estimate the response time, T', of a stream network to a step change of hydraulic regime, but the indicated response times should be increased by a factor of about 6 (for 99 per cent response) if the stream length is taken to be the length of the main stem measured from the headwater divide to the downstream terminus of the alluvial section, and the hydraulic regime is measured at the downstream end. For 90 and 50 per cent adjustments, the corresponding factors are about 3·2 and unity, respectively. The response time, T', in a network for regrading following a base level change can be estimated by multiplying the response time indicated by equation (18) by a factor of 2·4.

Another important contrast between the response of a channel network as compared to a long channel section is in the relationship of response time to channel length. In the channel section the response time increases with the square of the channel length, but for the network the rate of increase is much slower. This can be illustrated by looking at the drainage area dependence of the various terms in equations (16) or (17). The channel width increases as about $A_d^{0\cdot 4}$ and the main stream channel length, L, as about $A_d^{0\cdot 6}$. Sediment and water discharges, $(q_s \cdot W)$ and $(q\,.\,W)$, increase in proportion to $A_d^{0\cdot 8}$, so that their specific discharges increase as $A_d^{0\cdot 4}$. For the assumed values of A and B the response time, from equations (16) or (17) should therefore increase only as $L^{3/4}$.

The degree of synchroneity of aggradation or entrenchment within the channel differs markedly between distributed channel networks and long channel sections. From Figure 2A it is apparent that channels with sediment and water input only at their head undergo much aggradation or entrenchment at the upstream end prior to any marked change downstream, although later stages upstream occur together with those downstream. Thus, for a long, sand-bed channel such as the Nile River there may be partial asynchronism between alluviation/erosion events upstream and those occurring downstream. Furthermore, short-lived fluctuations in hydraulic regime may be faithfully mirrored in upstream terrace and valley fill deposits, but be missing downstream. By contrast, in a channel network with well-distributed sediment and water supply, erosion/deposition events are nearly simultaneous throughout the basin (Figure 2B), and all parts of the channel network would be expected to record these events in their deposits.

Finally, it should be noted that the gradient of a stream at the time that it completed deposition of a fill terrace was not necessarily in equilibrium with the hydraulic regime, because a change of regime can initiate incision prior to the obtaining of equilibrium with the prior regime (Bull, 1979).

Characteristic time scales of natural streams

The characteristic response times for natural alluvial channel networks vary greatly with the scale of the network and the frequency of channel flows. At one extreme are the small fans that form at the foot of bare roadcuts in soil or saprolite (L of about 1 m); the author has observed these fans to undergo epicycles of aggradation and fan-head trenching in response to different runoff stages of an individual thunderstorm. This response time measured in minutes indicates that such fans probably do not completely adjust to storm-period regime variations.

Alluvial channels in small areas of badlands are rather slower to respond. Sand-bed alluvial channel systems with drainage areas up to 5000 m^2 in an area of man-induced badlands in Virginia undergo pronounced seasonal cycles of aggradation and entrenchment, with winter/summer gradient ratios up to 1·5 (Howard, in preparation). Order-of-magnitude estimates for sediment yields, effective discharge, and flow duration for the larger badland channels suggest 99 per cent adjustment times to step change of regime of about 10 years and about 1·7 years for 50 per cent adjustment (using equation (16) with (Z) equal to 1/2). Actual response times are probably shorter, because active downcutting, occurring even during winter periods of high sediment yields and low runoff, reduces the volume of sediment deposition required for gradient increases, and alluvial channel lengths and widths decrease markedly during the summer periods of gradient reduction. Thus storm-period variations should be filtered out in the gradient response, but the channels do not completely adjust to seasonal regime changes.

Badlands in shales near the Henry Mountains, Utah (Howard, 1970) are similar in scale and morphology to the Virginia badlands, but sand-bed channels in these western badlands undergo no discernible seasonal variations in gradient. The explanation is the infrequency of runoff; the Virginia badlands are subjected to about 1·3 m of rainfall per year, whereas the Utah desert receives only about 0·1 m of yearly precipitation (the time scale in equations (16) to (18) is the cumulative duration of flow events representative of the dominant discharge, so that the actual elapsed times in ephemeral channels may be much longer). Thus these channels are insensitive to both storm-period and seasonal fluctuations in regime, because their response times are measured in decades.

Turning to larger stream networks, data presented by Andrews (1980) suggests that the headwaters of the Yampa and Little Snake rivers, with drainage areas of about 9000 km^2 would have a response time, T', for 99 per cent adjustment to a step change in hydraulic regime of about 3000–5000 years, assuming that there are no bedrock controls (which would shorten the period of adjustment) and no resultant change in channel width or sinuosity. With similar assumptions, the Mississippi River (drainage area of about 3,200,000 km^2) would have a response time of 50,000–80,000 years. Although these times seem long, one-tenth of the total adjustment occurs in about one fiftieth of the indicated times.

It is apparent from equations (16)–(18) that the response time decreases as the water and sediment discharge increases. Thus there is an asymmetry of adjustment in that increase in either water or sediment, or both, cause a more rapid adjustment than do decreases of the same magnitude. Thus, for example, the meltwater runoff and heavy sediment yields through the Mississippi River during the recessional stages of Quaternary glaciation should have caused readjustment of the river in a fraction of the response time indicated above. Similarly, the regrading following stream capture will occur more rapidly along the streams which have merged than on the beheaded stream which has lost both discharge and sediment supply.

If a section of stream is composed of alluvial sections with intervening bedrock rapids acting as control points, its response time, T_2, will be shorter than that (T_1) of an entirely alluvial stream with the same total alluvial length. The greatest difference will occur if the alluvial portions occur as N equal-sized sections, such that:

$$T_2/T_1 = 1/N \tag{19}$$

The Colorado River in the Grand Canyon is a case in point. Although the river is dominantly sand-bed, it is interrupted by many short rapids in boulders dumped into the river by tributary streams. The length of time, and the total volume of sediment deposition or erosion, required for this river to adjust to change in sand supply or discharge is much less than that of an equivalent river of uninterrupted alluvium. For example, the first 200 km below Glen Canyon Dam has rapids at intervals of approximately 4 km; this suggests a time scale, T', of about one-third year for 99 per cent adjustment to a step change in hydraulic regime (this may be an underestimate, because more scour and fill occur within the channel than would be expected by use of sediment transport relationships such as equation (12), due to stage-dependent turbulence generated in rapids (Howard and Dolan, in press)). Although the river does not completely equilibrate with the yearly runoff and sediment delivery cycle, the imposition of the upstream dam caused essentially complete reequilibration of the sand bed within a few years (Howard and Dolan, in press).

Equilibrium and the concept of grade

The concept of grade in fluvial geomorphology is closely related to that of equilibrium and is similarly burdened with imprecise and often conflicting definitions and connotations. The close relationship between the equilibria discussed above and grade is illustrated by Mackin's (1948, p. 471) classic definition, wherein a graded stream is 'one in which, over a period of years, slope is delicately adjusted to provide, with available discharge and with prevailing channel characteristics, just the velocity required for the transport of the load supplied from the drainage basin. The graded stream is a system in equilibrium . . .'. Thus Mackin seems to imply grade to require a unique functional relationship between the gradient and the hydraulic regime in the same sense as equilibrium has been used in this paper. However, if this equating of grade and equilibrium is accepted, 'a period of years' must be reinterpreted and the input and output variables must be precisely defined.

Because of the variation in the time scale among fluvial systems, it seems appropriate to replace 'a number of years' with 'a period of time commensurate with the relaxation time of the gradient'. This seems to follow the spirit of Mackin's definition, for he clearly saw that the gradient is insensitive to short-term variations in hydraulic regime, and that equilibrium requires a statistical constancy in the hydraulic regime for a time period at least as long as the response time of the stream. Mackin noted that gradients may remain in equilibrium during regrading caused by long-term variations in hydraulic regime or change in base level, calling this 'shifting equilibrium' (1948, p. 477). However, this term seems superfluous, for shifting equilibrium is no different than equilibrium between gradient and fixed values of the hydraulic regime and base level, so long as the actual gradient is within the consensual limits surrounding ultimate equilibrium.

Grade could be defined as one of at least four types of relationship between the input (hydraulic regime) and the output (channel gradient), although the first two of these clearly are not intended by Mackin's definition:

1. An equilibrium between the gradient and the hydraulic factors, measured at and applied to a given short reach. However, since from equations (16)–(18) the time scale for adjustment approaches zero for a reach of negligible length, this definition is not of geomorphic interest, for at very short time and spatial scales gradient becomes the independent variable and bed material discharge the dependent variable (Schumm and Lichty, 1965).
2. An equilibrium between the gradient and the values of the hydraulic variables at the upstream end of a reach of arbitrary length. This, when applied to reaches (generally without tributaries) where the length of the channel is determined by an engineering concern, comes close to the general usage of regime as applied to canals or altered sections of natural channels. However, the geomorphologist is generally concerned with the fluvial system as a whole, wherein the hydraulic regime is determined by runoff and sediment delivery from slopes. Even within this perspective, two additional definitions are possible.
3. An equilibrium between the gradient of each short segment in a channel network taken independently and the headwater runoff and sediment delivery from slopes (that is, some segments might be graded and others not).
4. An equilibrium between the gradients of the alluvial system taken as a whole and the runoff and sediment delivery from slopes.

If adjustment of stream systems to alteration of the hydraulic factors occurred progressively downstream, such that each channel had a characteristic interval prior to adjustment, that interval increasing downstream, then upstream reaches would reach essentially complete equilibrium before those downstream, and definition 3 would be most appropriate. Such a stream channel would have a gradient response to step changes in hydraulic regime characterized by a step function moving progressively downstream, such that, at any time those parts of the channel upstream from the step would be in complete equilibrium and those below in disequilibrium (as was idealized in deriving equation (13)). If, on the other hand, the adjustment of all channel segments within a drainage network were so interlocked that both upstream and downstream segments adjust with approximately the same time scale, then definition 4 would be most suitable. This would correspond in Figure 2 to sequentially rising horizontal lines. Unfortunately, neither situation is an entirely adequate description of natural channel adjustment.

Progressive downstream adjustment is most closely approximated in the initial stages of adjustment of long rivers with few tributaries and a concentrated source of water and sediment (Figure 2A). In the later stages of adjustment, and during all stages of adjustment in channel networks (Figure 2B) definition 4 seems most appropriate. Because of this organic response in almost all stream systems, it seems reasonable to speak of an overall time scale for adjustment to altered hydraulic regime.

Some geomorphologists, both prior to and after Mackin, have included within the concept of grade additional connotations which are unnecessary and complicating, leading to much of the criticism summarized by Kesseli (1941) and Dury (1966). One of these is W. M. Davis's suggestion that grade is associated with a particular stage of historic evolution of a drainage network, and that grade implies a particular (smooth) longitudinal stream profile (see summary in Knox, 1975). Another suggestion made by many geomorphologists including Davis (1902), Gilbert (1877), Knox (1975) and Leopold and Bull (1979) is that grade implies a balance between erosion and deposition. This suggestion is discussed in the next section.

In conclusion, grade becomes synonymous with equilibrium as applied to alluvial streams in this paper, if used according to the definition of Mackin (1948) with the recognition of variable time scales rather than 'a period of years'. Grade would then imply an equilibrium with respect to long term variations in regime and slow base level changes, and lack of response to short-term variations in hydraulic regime, and an absence of recent large-magnitude step changes, impulses, rapid trends in hydraulic regime or base level, or large-magnitude cyclical components of the hydraulic regime with periods commensurate with the gradient response time. The recognition of the insensitivity of gradient to high frequency components of the hydraulic regime and the allowance of a consensual departure from the ultimate value within equilibrium answer the criticisms of Kesseli (1941) and Dury (1975) that natural hydraulic regime is never constant, so that streams can never be graded.

Complicating factors in grade and equilibrium within natural channels

The use of equilibrium concepts in describing the behaviour of natural geomorphic systems has been introduced within the context of an admittedly oversimplified simulation model of alluvial stream dynamics. This concluding section considers the major shortcomings of the simulation model, with extended discussion only where a more realistic modelling would compromise the utility of the equilibrium concept.

The simulations, conducted under the assumption that equation (12) describes the relationship of transport rate to channel gradient, discharge, and grain size, may be inaccurate in several ways:

1. Equation (12) cannot be used for gravel channels in which transport occurs near the threshold of motion and in which sorting and comminution may be important. In such streams equilibria can occur, but with different dynamics and, in general, a much longer time scale of adjustment.
2. Certain climatic changes or base level changes, either natural or man-induced, may result in a threshold change to a different bed type (Howard, 1980), particularly during rapid erosion, which may expose bedrock or develop a lag gravel bed (Gessler, 1970; Little and Mayer, 1976; Garde, Ali, and Diette, 1977).
3. Stage-dependent roughness variations and temporary high- or low-flow bed armouring (Emmett, 1976) may require modification of equation (12).

Channel width and sinuosity have been treated here as temporally constant and independent of change in hydraulic regime, gradient, or the history of aggradation and entrenchment. This assumption is clearly an oversimplification and can alter, perhaps fundamentally, the pattern of gradient response shown in Figures 2 to 5. Variations in channel width affect the specific discharges of water and sediment in equation (12), and therefore affect the channel gradient. Changes in sinuosity directly affect the channel gradient. Studies by Schumm (1960, 1968, 1971) indicate that width (particularly as expressed by the width/depth ratio) and sinuosity are functionally related to the hydraulic regime, including the suspended and wash load characteristics. In addition, sinuosity and channel width are related to the channel gradient, with threshold transitions between channel types (Schumm and Khann, 1972).

Width and sinuosity are also affected by the history of aggradation and entrenchment, in that channel width narrows during rapid incision and widens during aggradation (Brush and Wolman, 1960; Pickup, 1977; Shepherd and Schumm, 1974; Howard, 1970; Daniels, 1960), the former reducing the time scale for adjustment, because both q_s and q increase (Bull, 1979). Conversely, an increase in channel width during rapid aggradation increases the time scale. Rapid incision may imprison meanders, perhaps exaggerating them in the case of upstream migration of base level (Schumm, 1977, p. 200). Similarly, rapid aggradation may decrease sinuosity through reduction in bank stability, thus increasing bed gradient, mitigating aggradation over the short run, and lengthening the time scale for full adjustment. Simulations by Pickup (1977) suggest that the width changes accompanying rapid incision may encourage upstream migration of knickpoints even in sandy alluvium; however, flume experiments by Brush and Wolman (1960) which allowed width to vary, showed rapid erasure of knickpoints in a manner similar to the simulations in Figure 3A.

Changes in hydraulic regime, and possibly also the occurrence of aggradation and entrenchment, may affect the bed roughness (Leopold and Maddock, 1953; Leopold and Bull, 1979), affecting transport rates and thus the channel gradient.

Because the present model assumed uniform, steady flow, it is obviously incapable of resolving small-scale effects due to bedforms, pool and riffle sequence, effects due to bedforms, pool and riffle sequence, backwater effects, and flow-regime transitions, and may result in inaccurate modelling of certain large-scale effects due to migration of transients through the system, such as knickpoints or large-scale bedforms. Several numerical models have been developed which more accurately model local flow and flow-transport interactions, for example, Gessler (1971), Pickup (1977), Chen (1979) Drew (1979), Ponce, Garcia, and Simons (1979), Thomas and Prasuhn (1977), and Soni, Garde, and Ranga Raju (1980).

In the simulations the length of the stream network was assumed to remain constant. However, hydraulic regime changes requiring aggradation will extend the headwater portions of the channel network, increasing the time scale for adjustment due to the greater amount of sediment to be deposited (Howard, in preparation). Just the reverse occurs when gradients decrease. A rise in sea level extends the control point of base level landward due to the drowning of the river valley. A falling base level extends the stream seaward by an amount dependent upon the offshore gradient. The effect of this migration is to mitigate the required aggradation or degradation. Even if the ultimate base level remains constant in elevation, the long-term delivery of sediment may push the location of the base level forward (as at the mouth of a delta or the foot of an alluvial fan), causing aggradation upstream due to the lengthening.

Slopes and channels interact in several ways not accounted for in the simulations:

1. Regrading of a stream channel will in turn affect the water, and particularly, the sediment delivery from slopes, generally in a manner that will decrease the effect of the initial change (for example, channel erosion leads to slope undercutting and gullying (Daniels, 1960), which increases the sediment yield).
2. A change in climatic regime may rapidly affect the hydraulic regime through change in runoff and slope vegetation, but there may also be long-term secondary effects (Howard, 1965) due to changes in soil properties and drainage basin morphology.
3. Release of sediment from slopes and headwater hollows may be inherently episodic under certain combinations of climate, relief, and surficial geology, causing a complex response in the channel network (Schumm, 1977).

None of the above considerations alters the utility of the equilibrium concept as applied to natural streams. Rather, they indicate that the development of more realistic models must incorporate more of these complex interactions. However, there are suggestions by modern geomorphologists that call into question several of the basic premises of gradient equilibrium in sand-bed channels. These counter-premises can be summarized as follows:

1. Channel gradients at equilibrium for a constant hydraulic regime are not determined solely by the hydraulic regime, but also by the past history of channel changes. That is, equilibrium is not equifinal. A related proposal suggests inherent indeterminacy in channel adjustments.
2. Base level (local or ultimate) has little or no effect upon upstream reaches of a channel.

3. Grade or equilibrium should refer not to a constant relationship between gradient and hydraulic regime, but to an absence of either net aggradation or entrenchment.
4. Grade refers not only to an equilibrium of gradient and hydraulic regime, but also simultaneously to an equilibrium with channel width, bed roughness, channel pattern, sinuosity, and other channel properties.

In one formulation of the first alternative hypothesis, Langbein (1964) and Maddock (1969) feel that the hydraulic relationships that govern the gradient of streams are inherently indeterminate (see also Leopold and Langbein, 1963). One source of this indeterminacy is attributed to the variation in transport rates that accompany change in bedforms, leading Maddock (1969, p. A1) to state that 'any function containing slope as a parameter will be indeterminate within wide limits'. However, these authors propose regime formulas by supposing that natural constraints exist, expressed in functions minimizing the cojoint variance of certain hydraulic variables. They retain, apparently, the notion of some level of irreducible uncertainty in these relationships due to the statistical nature of the constraints.

Several authors have criticized the hypothesis of inherent randomness in geomorphic systems, both on philosophical and methodological grounds (Watson, 1969; Howard, 1972; Smart, 1979). The degree of inherent impredictability of natural stream channels, if any, can probably be determined only after every effort is made to improve the realism of physical and mathematical models. In this regard it is appropriate to note that the most important elements of the minimum variance, minimum power, and maximum efficiency models of geomorphic processes (for example, Langbein, 1964; Leopold and Langbein, 1962; Langbein and Leopold, 1966; Maddock, 1969; Yang, 1971a, 1971b, 1971c, 1976; Williams, 1978; Chang, 1979a, 1979b) are the constraints imposed upon the model, including those of maximization or minimization. Since these constraints are used in a predictive sense, they in fact imply underlying causal mechanisms, even though they may be phrased in non-dynamic 'thermodynamic' models that do not explicitly incorporate time.

A related counterproposal is the premise that the equilibrium that results in a fluvial system as a result of change in base level or hydraulic regime depends in part upon the initial value of the system. The usual formulation suggests that a stream has several ways that it can respond to an external change (through the response of gradient, sinuosity, channel width, channel pattern, or hydraulic roughness) and that the resulting change of these properties has 'a tendency to counteract the effects of external change in such a way that the amount of internal adjustment required is minimized' (Knighton, 1977, p. 113). If two channel segments start with different initial morphology due to distinct hydraulic regimes, but are subsequently subjected to identical regimes, this view implies that the morphologies will not necessarily converge, even over the long run. For example, Schumm (1968) suggests that the Murrumbidgee River in Australia responded to change in hydraulic regime by selecting changes of channel width and sinuosity which absorb the change in regime without aggradation or entrenchment. But either of two situations may have occurred which could explain the lack of aggradation or entrenchment: (1) The pattern of alteration in the hydraulic regime may have been by chance just such that change in channel width and sinuosity offset the necessity for change in valley gradient; or (2) The paleochannels had not reached their ultimate equilibrium morphology due to the short duration of the climatic variations, but because of their short response time, channel width and sinuosity were strongly affected. Because of the long time scale for aggradation and entrenchment, over the short run the valley gradient is an independent variable (Schumm and Lichty, 1965) which has an effect upon channel width and sinuosity.

Leopold and Bull (1979, p. 195) argue that 'base levels of a river, whether ultimate base level (the ocean) or local (lake, dam, or resistant bedrock in the channel) affect the vertical position of the longitudinal profile only locally' and that 'base level and the profile of the graded stream are not closely related'. However, their argument seems unclear, for they recognize that channel gradients have an equilibrium value determined by the hydraulic regime (p. 182, 184). But if the gradient, S, at each point on a stream channel has an equilibrium value determined by the hydraulic regime imposed by slopes within the drainage basin, then the channel elevation, E, and the gradient have an integral–differential relationship:

$$E(x) = \int_0^x S(x)\,\mathrm{d}x + E_0; \text{ or } \mathrm{d}E/\mathrm{d}x = S, \tag{20}$$

where the distance x is measured from the downstream base level at elevation E_0.

Leopold and Bull cite several field examples in support of their contention that base level has a limited upstream effect, but their documentation is inconclusive. For example, sedimentation in small headwater debris dams commonly creates backfill gradients which are approximately half those of the original channel gradient, so that the new profile and the original profile intersect upstream, and no sedimentation has been noticed above this intersection, even many years after the apparent completion of deposition behind the dam. Leopold and Bull (1979, p. 192) attribute the lower gradient of deposition to a smaller hydraulic roughness in the channel over the sediment wedge than within the natural channel due to the absence of riffle–pool sequences and central bars. However, assuming both the fill and original bed sediment are of the same grain size, then it is reasonable to expect that the channel over the fill will eventually develop similar roughness and thus similar gradients as the natural channel, but the time scale of this adjustment (secondary response) may be many times that of the original filling.

Many geomorphologists, including Davis (1902), Gilbert (1877), Knox (1975), Mackin (1948), and Leopold and Bull (1979), feel that grade implies a balance between erosion and deposition. For example, Leopold and Bull (1979, p. 194) state that 'the difference between a graded and non-graded river is essentially whether, over a period of time, the channel aggrades (non-graded), degrades (non-graded), or remains at the same elevation (graded)'. Ultimately, the choice of input and output variables to be examined for equilibrium is a matter of preference, based primarily upon utilitarian concerns. Although upstream bed elevations are very sensitive to gradient changes due to their cumulative response (equation (20)), the hydraulic regime determines channel gradient (equation (12)), not bed elevation. The time scales of changes of bed elevation and gradient are different, so that the implications of grade would be different for the two choices of output variables. For example, bed elevation may fluctuate widely over short time scales due to short-term scour and fill, whereas the channel gradient remains unchanged. Similarly, slow bed erosion in response to lowered base level can occur without change of gradient. Since gradient is more directly related to the hydraulic regime than bed elevation, equilibrium relationships involving gradients are simpler than those using bed elevation.

Finally, some geomorphologists suggest that grade is a multivariate response, such that 'a graded stream is one in which over a period of years, slope, velocity, depth, width, roughness, pattern and channel morphology delicately and mutually adjust to provide the power and efficiency necessary to transport the load supplied from the drainage basin without aggradation or degradation of the channels' (Leopold and Bull, 1979, p. 195). Although these various channel properties do respond in a coordinated way to change in hydraulic regime or base level, the concept of equilibrium or grade is difficult to apply unless it is related to a single system property (output). One major problem with the use of multiple outputs is that the occurrence of equilibrium is severely restricted when the various outputs have different characteristic time scales of response. For example, some forms of bed roughness (bedforms) have a very short response time, whereas gradient has a very long time scale. Thus equilibrium, if applied to all channel properties simultaneously, could only occur if the controlling factors had no appreciable cyclical components over the entire range of resonse time scales.

DISCUSSION

A concern with equilibrium seems to be central to a relatively few sciences, particularly chemistry, ecology, and geomorphology. In chemistry equilibrium and steady state are used to denote a system adjustment to constant external conditions, and the concepts of consensual limits of error and the effects of different types of input signals are not generally utilized.

In electrical and mechanical engineering, where the dynamics of systems is most systematically studied, the concept of equilibrium is relegated to the restrictive usage of system specification using the balance of forces. The lack of use of the present equilibrium concept seems to be due to the mathematically exact modelling of systems structure and response; the equilibria defined here are just special cases of the more general specification of system dynamics in response to arbitrary inputs. This mathematical systems approach has been applied to biological systems of limited complexity (Milsum, 1966) and to the modeling of ecosystems (Patten, 1971; 1972), and similar approaches have been used in watershed modelling.

However, in geomorphology we are burdened by severe limitations in attempting to mathematically model processes in natural landscapes. Characterization of the complex interactions between processes, materials, and landforms is still primitive, and generally limited to special or idealized situations, as in the case of the sand-bed alluvial stream model discussed above. The long characteristic time scale of response of geomorphic systems limits our insight into their structure and interactions and makes difficult the verification of model assumptions. The uniqueness of geomorphology and ecology in their preoccupation with equilibrium concepts is not, I feel, an indication of a lack of utility of the concept. Rather, it indicates a scientific approach limited on one hand by the complexity of natural systems but balanced by insight gained by modelling and experimentation under idealized (often equilibrium) conditions, for example, in flume studies of sediment transport. One or another type of equilibrium is often assumed to characterize a geomorphic response, often implicitly. Therefore the application of models based upon the assumption of equilibrium requires an inquiry into the degree to which such conditions are approximated in the natural counterparts and into the potential errors resulting from the application of the models to disequilibrium systems.

Several approaches may be used to determine the degree of equilibrium, if any, between an output variable and the input(s):

1. If the system structure can be mathematically modelled, then, given the temporal history of the inputs, the variations in the output can be predicted and compared with the actual response. Due to the slow characteristic time scales of geomorphic systems, this approach can rarely be used, but it is often employed to validate (or in some cases to calibrate) watershed models.
2. If the output and inputs can be temporally monitored, then a single-valued functional relationship between input and output indicates an equilibrium (see the Introduction for further definition and discussion), assuming that a causal relationship between inputs and outputs is known to exist and that the system can be determined to be 'well-behaved'.

Because of the long time scale of response in geomorphic systems, it is generally necessary to employ indirect and weaker methods for testing for equilibrium:

3. Approach 1 above can be used to predict the present value of the system variable, given the present and inferred past values of the inputs, under the assumption that the output is sensitive to the inputs and that the predicted relationship is not satisfied 'accidentally' (see Introduction). Because of the quantitative limitations of geomorphic models, this type of approach is hazardous.
4. The 'substitution of space for time' method may be used to check for a single-valued functional relationship between the input(s) and the corresponding output. This approach is useful but potentially misleading. For example, a recent climatic change may be causing a rapid trend in the geomorphic response, but the spatial variation in the response may still show a consistent but biased relationship to the spatial variations in the present inputs.

Even where it may be impractical to verify the occurrence of equilibrium in a natural system, the modelling of equilibrium behaviour, either mathematically or conceptually, has a heuristic value in the description of the limiting behaviour of system response. The assumption of equilibrium generally allows solution to models that may not be practical in general, and it clarifies the interactions present in complex systems. In passing it may be noted that geomorphologists are generally resigned to a low degree of quantitative prediction (a large scatter), indicating that the consensual departure from the ultimate value in geomorphic equilibria (equation (3)) should realistically be 10 to 20 per cent rather than the 1 to 5 per cent used in this paper.

The characteristic relaxation times (time scales) of various components of geomorphic systems are important indices of the types of equilibria that might occur and are also important for the selection of independent and dependent (input and output) variables (Schumm and Lichty, 1965). The central role of relaxation times has been emphasized by several authors (Howard, 1965; Chorley and Kennedy, 1971; Allen, 1974; Thornes and Brunsden, 1977; Wolman and Gerson, 1978; Bull, 1979; Brunsden and Thornes, 1979; Pickup and Rieger, 1979; Cullingford, Davidson, and Lewin, 1980). Determination of the relaxation time of a geomorphic attribute indicates those frequency components of input variations for which there can be equilibrium (for very low and very high frequencies compared to the response time), and the recovery time following disturbances such as step changes and impulses.

Unfortunately, the determination of the response time is not always simple. The rate of change of a system property is the most easily observed characteristic of its response, but the rate depends not only upon the relaxation time but also upon the magnitude of the change provoking the response and the elapsed time since the disturbance (see equation (4)). Also complicating the determination of relaxation time is its dependence not only upon the internal structure of the system, but also upon the magnitude of the input variables, and, to a lesser degree, upon the magnitude of the change (equations (16)–(18); also see Chorley and Kennedy, 1971; Allen, 1974). However, it is sometimes possible, empirically or theoretically, to estimate the relaxation time, as has been done here for sand-bed alluvial channels.

As an example of one practical application of relaxation time, the very long time scales for gradient adjustment of large river systems (>10,000 years) implies that for engineering purposes a channel reach can be defined with the input of water and sediment being taken at the upstream end of the reach (rather than delivery from slopes) and, for most proposes, the gradient of undisturbed channel sections can be considered to be invariable and the total load (excluding the wash load) can be estimated by a suitable transport formula.

This paper necessarily leaves many loose ends for future discussion. The equilibrium concepts proposed here need to be tested in further applications and refined and amended through the fertilization by concepts and experience of other researchers. Among the issues which I perceive to need further discussion are the role of secondary responses and thresholds in equilibria, methods for the verification of the occurrence of equilibrium in natural systems, the incorporation of stochastic inputs, and the role of large magnitude events (for example, Beven, in press; Brunsden and Thornes, 1979). The system responses discussed in this paper (primarily channel gradient) exhibit a highly damped reponse to input variations, but many geomorphic responses (bedforms, meanders) are oscillatory, and the role of such responses in equilibrium requires clarification (temporal averages of oscillatory responses should exhibit the various types of equilibria discussed here).

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