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Sufficient Conditions for River Meandering: A Simulation Approach

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A digital simulation model produces realistic meandering patterns starting from an initially straight channel with small random perturbations. The model assumes that nominal migration rates increase as channel curvature increases but reach a maximum when bend radius divided by channel width is about 3, with decreasing rates for sharper bends, in accord with observations of Nanson and Hickin (1983) and others. However, it is also necessary to assume that local migration rates are a weighted average of local and upstream nominal rates, with the weighting decreasing upstream. The model includes provisions for neck cutoffs and may be used to forecast future patterns of channel migration in meandering streams.

INTRODUCTION

The meandering of rivers has been a central concern in geology and civil engineering for many years, not only because channel migration has practical implications in land use, sediment budgets, and navigation, but also because explanation and prediction of the meandering process has remained elusive. A wide range of theoretical approaches have provided insights into the origin of meandering, but no single theory based on fundamental principles of fluid flow, sediment transport, and erosion is likely to predict the richness of natural meandering. Thus a suite of approaches is necessary. In this paper a set of operational rules, or conditions, are defined which are sufficient to create a realistic meandering pattern.

The complexly evolving geometry of meandering channels makes it difficult to predict the effect of theoretical model assumptions on resultant channel form. For example, *Abrahams and Mark* [1982] differ with *Begin* [1981] about the distribution of bend radii that would result if Begin's theoretical function describes channel migration. In the present paper a simulation approach is employed to test model assumptions. Generally, extant theoretical models of meandering have represented channel form as an exact periodic function, such as the sine-generated curve [e.g., *Ferguson*, 1973]. Channel migration has been discussed in terms of enlargement or translation of such curves, either in a theoretical context [e.g., *Allen*, 1977, 1982; *Ikeda et al.*, 1981], as a basis for quantification of migration rates [*Daniel*, 1971], or to investigate sedimentary processes in meandering streams [*Bridge*, 1975]. Although theoretical studies are illuminating the basic mechanics of meandering [e.g., *Ikeda et al.*, 1981; *Parker et al.*, 1982], four features of natural meandering streams prohibit the use of analytical models for long-term prediction of meandering. First, natural meander patterns are irregular, although they have strong periodic components. Second, channel migration rates are nonlinearly related to channel curvature [*Hickin and Nanson*, 1975]. Third, cutoffs drastically alter the channel pattern. Fourth, the erosional resistance of channel banks can vary spatially [e.g., *Fisk*, 1947]. Conversely, digital simulation models can accomodate these features. The simulation approach has been partially anticipated by *MacLennan* [1979] and *Parker* [1982].

In the model presented here, a stream is represented by a series of nearly equally spaced points representing its centerline. Other stream properties, such as width, depth, channel shape, and flow properties are not modeled explicitly; rather, they are accounted for indirectly through their influence on bend migration rates. The simulation proceeds by repeatedly cycling along the channel with the points being moved (corresponding to channel migration) as a function of local and upstream channel curvature and the temporal increment, as discussed below. Thus the model represents the stream in discrete spatial (points) and temporal (iterations) increments.

The task addressed here is to determine the simplest set of rules governing the spatial pattern of bank erosion which will produce a realistic meandering pattern starting from an arbitrary initial channel pattern. A stringent test of a candidate model is its ability to develop a meandering pattern starting from a nearly straight initial stream. Several sets of model assumptions are able to continue the enlargement and translation of preexisting meanders for short time periods but cannot develop meanders from straight channels. Therefore most simulations started from an initially straight, horizontal stream with superimposed small random vertical perturbations. The perturbations provide the seed disturbances upon which the model assumptions operate systematically to produce a meandering pattern. Small variations in channel pattern corresponding to the initial perturbations occur in even the straightest of natural streams.

RATE LAW GOVERNING BANK EROSION

The rate of lateral migration in meandering channels depends upon bank resistance, flow characteristics, and sediment transport. In order for a channel to migrate, erosion on one bank (generally the outside bank in a bend) must be balanced with deposition on the opposite bank. If point bar deposition does not keep up with opposite side bank erosion, the gradual widening of the channel will decrease shear stresses until migration ceases. Conversely, if bank erosion rates are very slow, opposite side deposition will narrow the channel to the point where increased velocity prohibits further deposition. Thus bank migration rate may be controlled either by bank erodibility or by the quantity of sediment in transport. Bank erodibility has been cited as a controlling factor by *Fisk* [1947],

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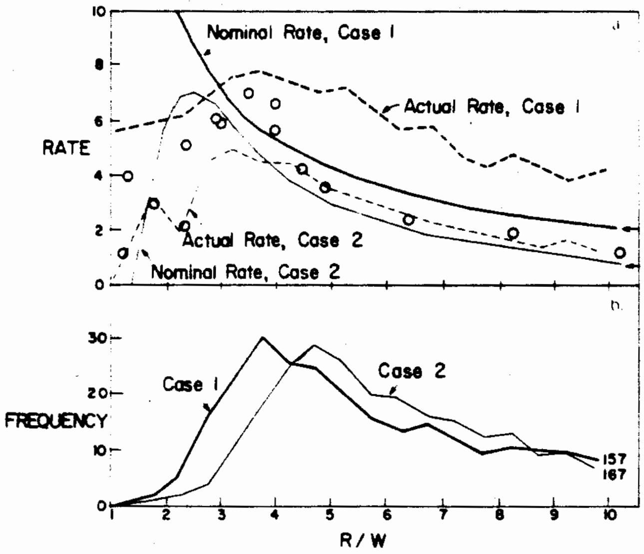

Fig. 1. (*a*) Input parameters and statistical properties of simulated streams shown in Figure 2. Nominal rate curves show values of R_0 as a function of R/W which were input to (1) during the simulations. Actual rate curves show average migration rates as a function of R/W for simulations. Circles indicate migration rates observed by *Nanson and Hickin* [1983]. Rate scale is arbitrary. Arrows to right show average actual rates for R/W values greater than 10. (*b*) Relative frequency of occurrence of R/W values for simulated streams shown in Figure 1. Numbers to right indicate frequency of R/W values greater than 10.

Nanson and Hicken [1984], *Rohrer* [1984], *Grissinger and Murphey* [1984], and others, whereas the role of sediment supply has been emphasized by *Nanson and Hickin* [1983], *Neill* [1984], and *Simons and Julien* [1984].

Bank erodibility is likely to be the controlling factor for narrow channels with strongly cohesive banks (the composition of the lower portions of the banks is most critical in determining erosional resistance, as discussed by *Nanson and Hickin* [1984]) and for meanders incised in bedrock. Such channels roughly correspond to the "sinuous canaliform" classification of *Brice* [1984]. On the other hand, in wide channels where banks are low and weakly cohesive, the rate of throughflow of bed and suspended load should limit lateral migration to the rate of deposition on the point bars, corresponding to Brice's "sinuous point bar" channels.

For both of these cases the migration rate depends functionally upon the degree of flow asymmetry across the cross section and, in particular, upon the contrast in shear stresses between opposite banks. In the case where the rate of sediment deposition is the limiting factor, cross-channel bed slope and secondary circulation set up by the flow asymmetry are also important. Although in natural channels the flow patterns and corresponding stresses and bed topography are spatially and temporally complex, the regularity of meander patterns and cross-sectional asymmetry in meandering channels suggests that migration rates averaged over a period of years can be related to simple descriptions of channel flow properties.

The present model incorporates a number of simplifying assumptions about the characteristics of the simulated channels, of which the most important are the following.

1. Bank erodibility is assumed to be uniform along the channel.
2. Width-average bed and suspended sediment load is assumed to be uniform downstream.
3. Migration is assumed to proceed slowly enough that the effects of individual flow events can be modelled as a continuous process over time scales long enough to cause pronounced channel migration.
4. The channel has a spatially and temporally constant width with no anastomosing.
5. The flow and sediment input to the reach from upstream is statistically stationary.

A FIRST-APPROXIMATION MODEL

A first approximation to a rate law governing bank erosion is based upon the measurements of *Hickin and Nanson* [1975] and *Nanson and Hickin* [1983] and theoretical approaches of *Allen* [1977] and *Begin* [1981], who related bank erosion rates to local channel curvature. Flow asymmetry should increase with channel curvature for given channel dimensions and discharge, leading to greater bank erosion and point bar deposition. *Nanson and Hickin* [1983] summarized detailed measurements of channel migration rates in the Beatton River, Canada as a function of bend radius R normalized by the channel width W (R/W is the inverse of dimensionless channel curvature); they found that migration increases to a maximum as R/W decreases to a value near 3 but drops rapidly for smaller values. Other researchers [e.g., *Brice*, 1974*a*; *Allen*, 1982] have shown migration rates increasing as R/W decreases from large values but have not conclusively found an optimal R/W value with decreased migration rates for very sharp bends. However, relatively small or inwardly directed migration rates for sharp bends are also suggested by other evidence: (1) field evidence for relatively slow bank erosion rates in very sharp bends [*Carey*, 1969; *Hickin*, 1974, 1978; *Woodyer*, 1975; *Nanson*, 1980], (2) a greater dissipation of turbulence in sharp bends, possibly reducing relative shear on the walls [*Bagnold*, 1960], (3) most rapid flows being located close to the inner bank in sharp bends [*Leschziner and Rodi*, 1979], (4) erosional attack of the inner bank in sharp bends [*Jackson*, 1981], and (5) theoretical arguments [*Allen*, 1977; *Begin*, 1981].

A functional dependence of migration rate on R/W similar to *Nanson and Hickin*'s [1983] observations has been incorporated into a simulation model, with the exact form of the dependence being adjustable as an input function (e.g., Figure 1*a*). However, when erosion rates depend solely upon local values of R/W, the model does not produce realistic meandering; rather, when this rate law is applied to the randomly perturbed straight channel described above, the channel pattern breaks up into three-point bends alternating positive and negative curvature. This pattern persists until migration decreased R/W to its limiting value where erosion ceased ($R/W \sim 1$). If the model is applied to an established meandering planform, the bends continue to enlarge but lack downstream translation, but the relatively straight portions of the channel become unstable as before. A simulation model with similar assumptions and likewise unstable results was programmed by *MacLennan* [1979]. Thus models which suggest that migration rates can be described solely by local curvature [*Hickin and Nanson*, 1975; *Allen*, 1977; *Begin*, 1981; *Nanson and Hickin*, 1983] appear to be insufficient. The migration rate at a given location seems not to be solely a function of R/W at that same point but depends also upon upstream (and to a lesser degree, downstream) channel geometry. The reason for this is the finite length of channel required for the development of the flow asymmetry, superelevation, and secondary circulation in channel bends that directly and indirectly produces bend migration. This spatial lag between bend geometry and flow properties accounts for the fact that maximum rates of shear and bank erosion are usually displaced downstream from the locii of maximum bend curvature [e.g., *Friedkin*, 1945; *Engelund*, 1974; *Hickin*, 1974; *Hooke*, 1975; *Dietrich et al.*, 1979].

A SECOND-APPROXIMATION MODEL

A modified approach permits relationships between erosion rates and channel curvature similar to that observed by *Hickin and Nanson* [1975] and *Nanson and Hickin* [1983] but in addition accounts for the fact that erosion rates are influenced by upstream channel geometry through a weighting procedure. At each node of the stream the nominal migration rate R_0 is determined by the R/W value for that node using a piecewise linear fit to an assumed relationship, as discussed further below. These nominal rates are given the sign of the local channel curvature (positive for clockwise curvature, facing downstream). The adjusted migration rate R_1 is determined by weighting the nominal migration rates for the present and upstream locations:

$$R_1(s) = \Omega R_0(s) + \left[\Gamma \int_0^{\infty} R_0(s - \xi) G(\xi)\, d\xi\right]\left[\int_0^{\infty} G(\xi)\, d\xi\right]^{-1} \quad (1)$$

where $R_0(s)$ and $R_1(s)$ are the adjusted and nominal migration rates at the location s (measured downstream from the origin of the simulated stream), $R_0(s - \xi)$ is the nominal migration rate at a distance ξ upstream from s, Ω and Γ are weighting parameters, and $G(\xi)$ is an upstream weighting function. At least two forms of upstream weighting can produce stable meandering. In the first, the weighting parameter Ω is zero, Γ is unity, and $G(\xi)$ takes the form

$$G(\xi) = e^{-\alpha\xi} \cos(\beta\xi - \gamma) \quad (2)$$

where α, β, and γ are adjustable parameters that determine the wavelength, shape and other aspects of meander geometry. The significance of these parameters is discussed below. G. Parker (1983, personal communication) has shown that his theoretical model [*Ikeda et al.*, 1981] implies a weighting procedure with $\Omega = -1$ and

$$G(\xi) = e^{-\alpha\xi} \quad (3)$$

where α is equivalent to the same parameter in (2). The parameter Γ in the *Ikeda et al.* [1981] model is related to the degree of cross-channel bottom slope and has a value of about 2.5 for most meandering alluvial streams.

The integration in (1) is indicated to proceed infinitely far upstream; in the simulations the upstream weighting is terminated when $G(\xi)$ is negligibly small or when the upstream end of the stream is reached.

Exponential Decay Terms

The exponential decay term in (2) and (3) and the parameter α model the diminishing influence of the channel geometry on local migration rates at greater distances upstream. The upstream effects are great for closeby portions of the stream and negligible for portions lying far upstream. The downstream decay of the effects of local boundary conditions in fluid flow is due to frictional dissipation. This dissipation can be heuristically modeled by the gradual deceleration of turbulent channel flow without a downstream channel gradient. The rate of energy loss per unit length of flow per unit bed area in a wide channel is equal to the average boundary shear stress. If a coefficient of friction, C_f, is defined to be the square of the ratio of the shear velocity to the average velocity, then the average velocity V at a given location s is

$$V = V_0 e^{-C_f s/D} \quad (4)$$

where V_0 is the velocity at $s = 0$ and D is the average channel depth. For a given channel curvature, bank erosion is assumed to be proportional to the average shear stress, so that the relative erosion rate in the decellerating flow is proportional to

$$\tau/\tau_0 = e^{-2C_f s/D} \quad (5)$$

In a natural channel where the frictional energy losses are balanced by the channel gradient, the upstream weighting parameter in (2) and (3), α, is postulated to be proportional to the fraction of the shear stress "inherited" from that distance upstream, that is

$$\alpha = k2C_f/D \quad (6)$$

where k is a scaling parameter of order unity. The model of *Ikeda et al.* [1981] predicts a value of unity for k. Data from natural streams [*Williams*, 1978] suggests that the upstream distance to half value of the function on the right side of (5) varies from a fraction of a stream width to a few tens of widths (using the width as a scaling parameter).

Preferred Wavelength of Meandering

Both (2) and (3) imply an optimal wavelength for meandering, in accordance with abundant empirical evidence for a dominant wavelength [e.g., *Leopold and Wolman*, 1960; *Carlston*, 1965; *Nagabhusanaiah*, 1967; *Ackers and Charlton*, 1970; *Chitale*, 1973; *Ferguson*, 1975, 1977; *Hickin*, 1977] and theoretical models that suggest such a wavelength [e.g., *Hansen*, 1967; *Parker*, 1976; *Ponce and Mahmood*, 1976; *Ikeda*, 1980; *Hayashi and Ozaki*, 1980; *Ikeda et al.*, 1981; *Parker et al.*, 1982]. When meandering develops from a nearly straight stream, the first stages of meandering formed using either (2) or (3) are describable by the following equation:

$$y = Ae^{vt} \cos(\lambda x - \omega t) \quad (7)$$

where x and y are Cartesian coordinates (x downstream). The parameter v is the rate of increase of the amplitude of the meanders, ω is the downstream rate of migration, and λ is the initial wavenumber of meandering. This nearly sinusoidal growth and downstream translation is well illustrated by the first few hundred simulation iterations in Figure 2. During the initial stages of meander growth the nominal migration rate R_0 is essentially proportional to the channel curvature η, given by

$$\eta = -A\lambda^2 \cos(\lambda x - \omega t) \quad (8)$$

Also, for low-amplitude meandering, bank erosion is directed nearly normal to the stream course ($\pm y$ direction). Then (3) implies a wavenumber given by [after *Ikeda et al.*, 1981]

$$\lambda = \alpha(\Gamma^{1/2} - 1)^{1/2} \quad (9)$$

where the parameter Γ is given by

$$\Gamma = [(A + 2) + F^2]/2 \quad (10)$$

where F is the Froude number of the flow and A is an empirical constant related to cross-sectional bed gradient with a value of about 2.9. Similarly, the cosine term in (2) results in an amplifying effect for meander frequencies close to the parameter β, because for such a value the upstream changes in sign of the weighting function are phased with sign changes of the nominal migration rates. The coefficients A, v, λ, and ω are complicated functions of α, β, and γ. The predicted value of λ is close but not equal to the forcing frequency β. In particular, large phase lags (large γ) make $\lambda > \beta$.

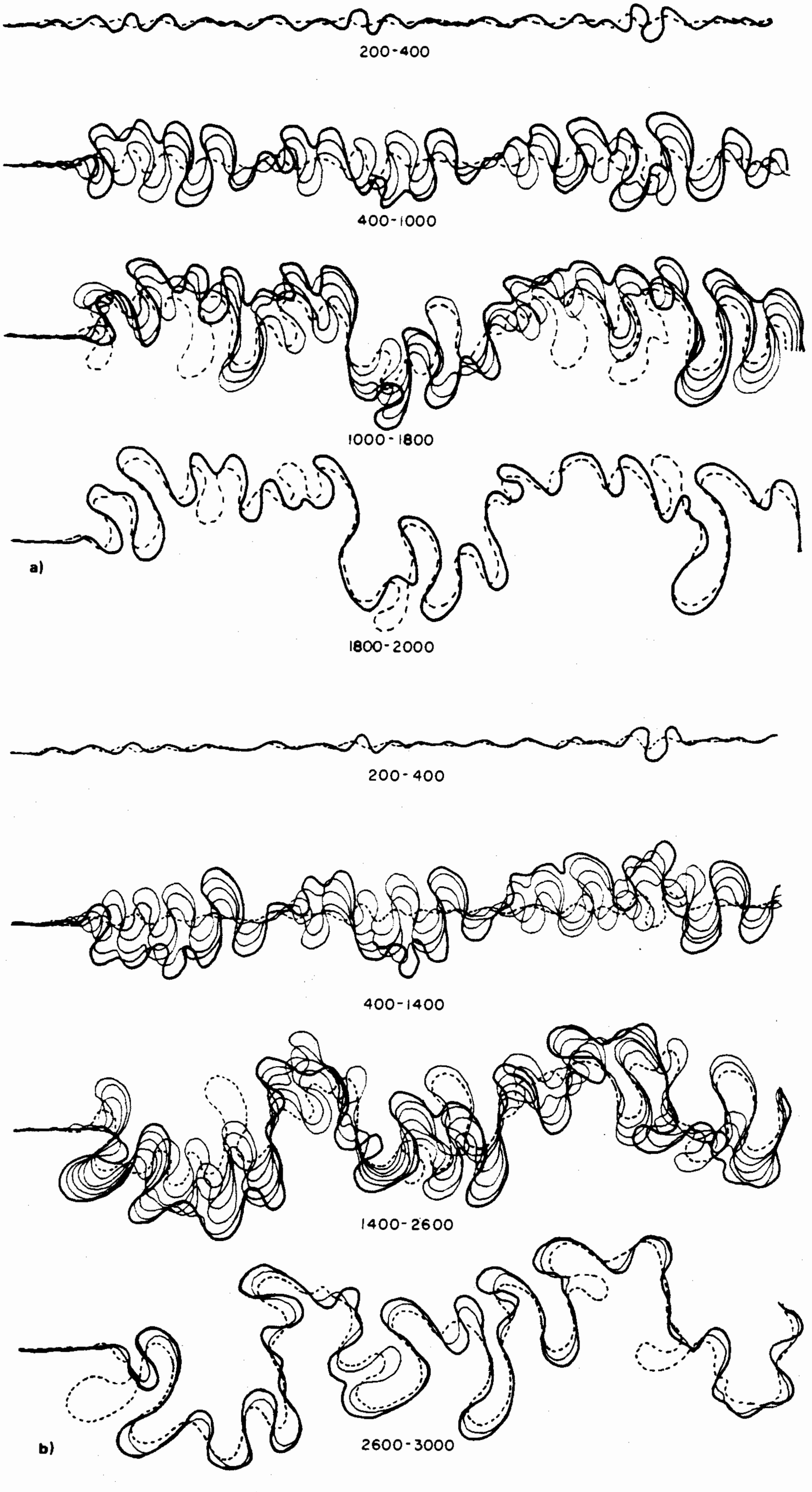

Fig. 2. Successive centerlines of simulated streams, displayed in increments of 200 iterations. Downstream to right. Sets of curves identified by inclusive iteration numbers. First centerline of sequence identified by dashed line, last centerline by bold line. (*a*) Case 1. (*b*) Case 2.

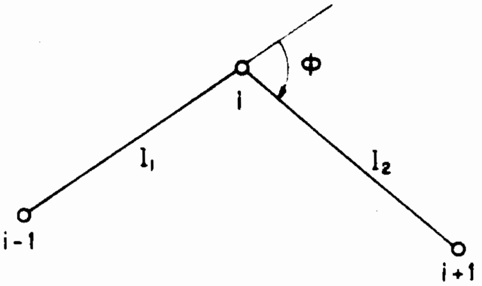

Fig. 3. measurement of channel curvature using stream nodes representing the channel centerline. The present node is *i*, and *i*-1 is the preceding upstream node. The angle ϕ is measured in radians.

Sinuosity Effects

The time scale for development of full meandering in natural rivers is tens of years to a few thousand years, depending upon valley gradient, bank erodibility, and the hydraulic regime. On the other hand, due to the large amount of sediment that must be eroded or deposited to change the valley profile, at least 10 times as long a period may be required to produce appreciable changes in valley gradient in response to changes in channel sinuosity. As a result, the valley gradient can generally be assumed to be constant over the time scales required to create an equilibrium meandering pattern. For example, *Schumm* [1968] showed that past changes in the hydraulic regime of the Murrumbidgee River in Australia required readjustments of channel cross section and sinuosity (now recorded in abandoned paleochannels) without appreciable change in valley gradient. Therefore, as the sinuosity of a channel increases, the average velocity and shear stress decrease for a given bend geometry, reducing bank erosion rates [*Allen*, 1977]. This has been incorporated in the simulation model by scaling the overall rate of channel migration to the sinuosity as follows:

$$R_1' = R_1 \mu^{\varepsilon} \tag{11}$$

where R_1 is the adjusted rate of migration from (1), R_1' is the actual migration rate, μ is the sinuosity, and ε is a parameter. The presently assumed value of ε, $-2/3$, is derived from the following reasoning. The bank erosion rate is assumed to be proportional to the average bed and bank shear stress for a given bend geometry. The average channel gradient is inversely proportional to the sinuosity for constant valley gradient. Following *Ikeda et al.* [1981], C_f and the channel width are assumed to be independent of channel gradient. Under these assumptions the average shear stress is proportional to the $-2/3$ power of the sinuosity. Similarly, the flow depth increases as the 1/3 power of sinuosity, which affects the value of the parameter α in (2), (3), and (6). Note that these sinuosity corrections affect all points of the channel proportionally and that the preferred wavelength (measured along the pathlength) increases with sinuosity.

OTHER MODEL FEATURES AND ASSUMPTIONS

The simulation proceeds by repeatedly cycling downstream through the linked list representing the stream. Each point is moved, corresponding to bank erosion and channel centerline migration, by an amount equal to the calculated value of R_1' times the time increment. Local curvatures used in the weighting function are calculated as (Figure 3)

$$W/R = W\eta \approx (W\phi)/(l_1 + l_2) \tag{12}$$

The channel is shifted normal to the stream centerline in the direction given by the sign of R_1' (positive to the left, facing downstream). Due to the weighting function (2), the direction of migration may locally be contrary to the direction of channel curvature.

As natural channels increase in sinuosity, the channels may become locally straightened by cutoffs. Two intergrading types of cutoffs occur in natural channels, chute and neck. Neck cutoffs occur when the local sinuosity becomes so extreme that adjacent loops intersect one another, causing the channel to abandon the former loop, and resulting in an oxbow lake when sedimentation closes the ends of the former loop. Neck cutoffs are dominant in most of *Brice*'s [1984] sinuous canaliform rivers, particularly those with narrow channels, well-vegetated banks, and low gradients [e.g., the White River of Indiana [*Brice*, 1974*a*, *b*], the Beatton River of British Columbia [*Hickin and Nanson*, 1975; *Nanson and Hickin*, 1983], and the Saline River of Kansas (W. Dort, unpublished material)]. Neck cutoffs have been incorporated in the model such that when separate portions of the channel centerline approach closer than a predetermined distance (about one channel width), the cutoff occurs by deleting the abandoned loop from the linked list (Figures 2 and 4).

Chute cutoffs occur when a new channel is created across the inside of a meander bend. Such chutes generally follow swales between scroll bars in recently deposited point bar deposits. The chute cutoffs occur most frequently where bend curvature is strong and at times of high or flood discharges [*Fisk*, 1944, 1947; *Friedkin*, 1945; *Bridge*, 1975; *Callander*, 1978; *Thorne and Lewin*, 1979], and they seem to be encouraged by wide channels, poorly cohesive, weakly vegetated banks, and high gradients and generally fall into Brice's sinuous point bar classification. Rivers in which chute cutoffs dominate have low sinuosity compared to channels dominated by neck cutoffs. Chute cutoffs are not explicitly modelled in the present version of the model, although they may be incorporated in future versions. However, neck cutoffs occur occasionally at strongly curved bends during the simulations at locations where the length of the cutoff channel is equivalent to only a few channel widths; such short neck cutoffs would probably occur as chutes in a natural stream. Also, in the present model bank erosion rates can be directed towards the inside of a bend if the curvature becomes very great ($R/W \sim 1$); concave bank deposition or inner bank erosion have been observed in sharp bends in natural streams [*Carey*, 1969; *Woodyer*, 1975; *Jackson*, 1981], and such locations would also be likely sites of chute cutoffs. Thus the present model may indirectly account for certain types of chute cutoffs.

The nominal migration rates are assumed to be a function of R/W. The simplest assumption, adopted by *Ikeda et al.* [1981] and *Parker* [1982], is that the nominal rate increases proportionally to dimensionless curvature (W/R) (case 1, Figure 1*a*). *Engelund* [1974] and *Ikeda et al.* [1981] show that the ratio of bank shear stress to width-averaged shear stress is proportional to dimensionless curvature for small values of W/R. This assumption produces stable and fairly realistic meandering (Figure 2*a*). Interestingly, the average observed actual migration rates (from (1) and (13)) for simulations using this linear dependence show a slight maximum at an R/W value close to the optimal value observed by *Nanson and Hickin* [1983] (case 1, Figure 2*a*). A closer fit to the Nanson and Hickin data can be obtained by assuming a nonlinear dependence of nominal rates to W/R (case 2, Figure 1*a*).

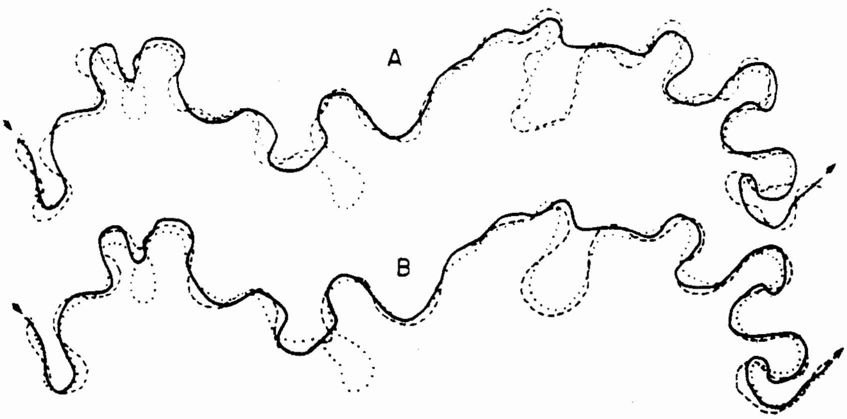

Fig. 4. Simulated and actual changes in channel pattern of the West Fork of the White River, Indiana (natural channel patterns from *Brice* [1974*a*]). Dotted lines are 1937 natural channel, solid lines are 1966 actual channel, and dashed lines are simulated 1966 channels. (*a*) Simulation using (2) with $0.69W/\alpha = 4$, $\pi W/2\beta = 3$, $\gamma = 0^\circ$ (*b*) Simulation for $0.69W/\alpha = 5$, $\pi W/2\beta = 3.5$, $\gamma = -45^\circ$.

Various such dependencies can be assumed, with marked effect on the actual migration rates and on the pattern of the resulting meandering (e.g., Figure 2*b*). However, the empirical weights and corresponding curvatures observed by *Nanson and Hickin* [1983] are average values over a large segment of individual loops. Thus the empirical rates correspond neither to the local R_1 convolution values (equation (1)) nor to the R_0 weights but should lie between, because the R_0 and R_1 values would be equal for a long bend of constant curvature.

No theoretical justification exists at present for a particular form for the nominal rates versus W/R dependency; a suitable form can be determined by trial and error until a good fit to the *Nanson and Hickin* [1983] data occurs and/or simulations with statistical properties close to natural meandering result.

Further implementation details of the simulation model are reported by *Howard* [1984]. *Parker* [1982, 1984] has programmed a simulation model similar to that reported here using (3) as the weighting function. His model differs from that reported here by not incorporating provision for cutoffs and in the assumption that the nominal migration rate R_0 is linearly proportional to dimensionless curvature.

RESULTS OF SIMULATIONS

The simulation model has been tested using both (2) and (3). Equation (3) has been used for the simulations illustrated in Figure 2, whereas (2) was used for the simulations of Figures 4 and 5. Simulations have been made for a variety of input parameters. The shape of the curve relating the nominal migration rate versus R/W has also been varied. Many features of the generated stream patterns resemble those of natural meandering streams both in visual appearance and in statistical properties. Due to the paucity of appropriate comparative data from natural streams, the results reported here are preliminary and may be altered as more appropriate input parameters and model structure are developed.

Most simulations started from an initial model stream that was straight except for small random perturbations. Bank erosion and cutoffs have been modeled deterministically in simulations to date, with bank erodibility considered to be uniform. Despite the lack of random factors (except for initial conditions), the evolutionary history of the simulated streams is rich and varied (Figure 2); much of this richness is due directly and indirectly to cutoffs.

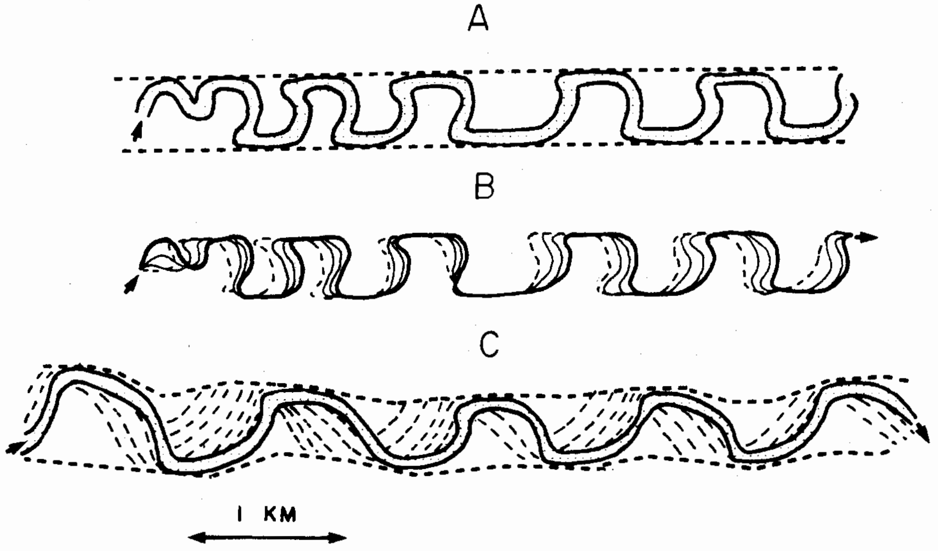

Fig. 5. Confined meandering. (*a*) Simulated channel with resistant valley walls shown by dashed lines. (*b*) Successive simulated centerlines for channel shown in (*a*). (*c*) Channel centerline of a confined meandering stream showing scroll bars (Beaver River, Canada; drawn from a photograph from *Allen* [1982]).

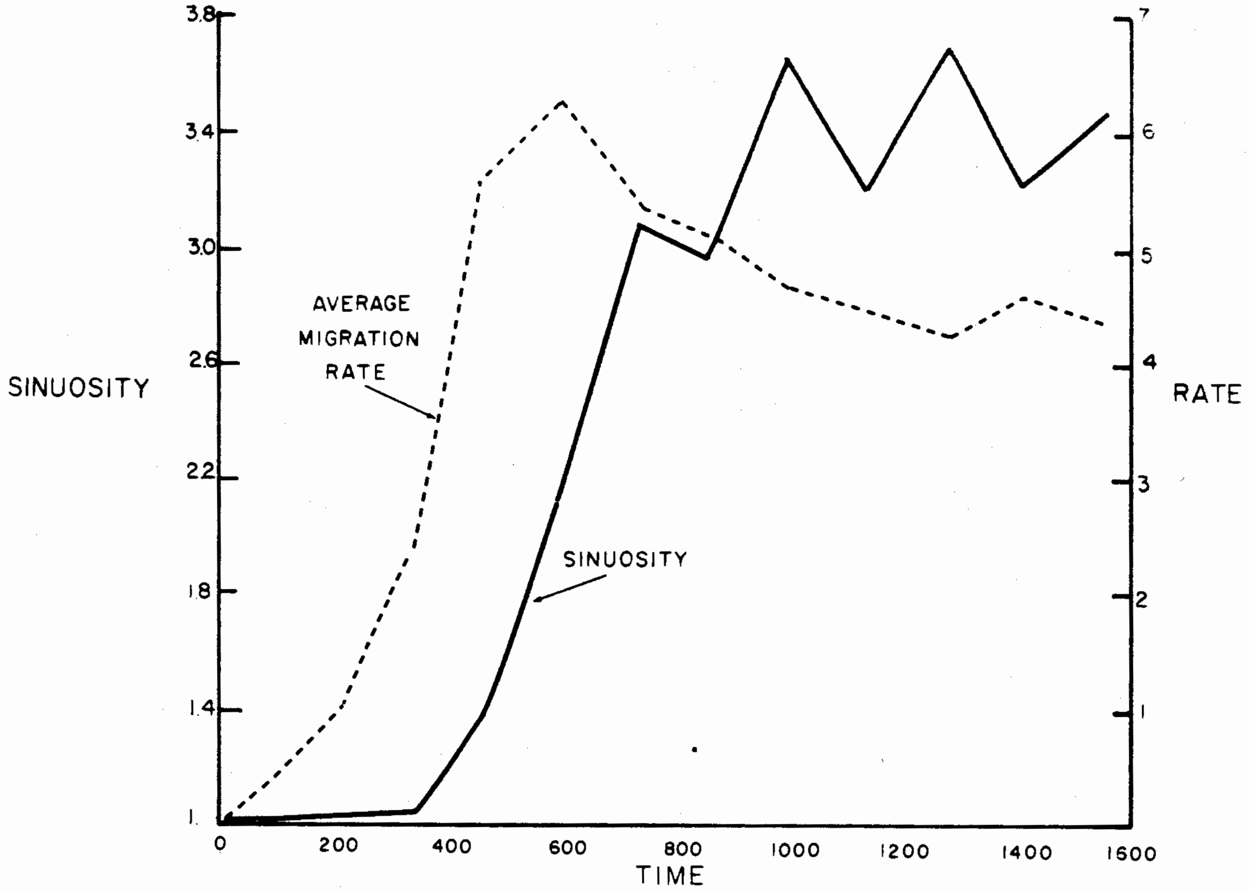

Fig. 6. Sinuosity and average rate of bank migration as a function of time for the simulation shown in Figure 1*a*. Note that sinuosity measurement does not take into account a meander belt axis [*Brice*, 1974*b*; *Allen*, 1982], so that it is more properly a measure of tortuosity [*Allen*, 1982].

Initial development of meandering occurs slowly, due to the slow migration rates associated with bends of low curvature (Figure 6). After about 300 years of model erosion (using bank erosion rates scaled to measurements of *Nanson and Hickin* [1983]) the sinuosity increases rapidly, but after about 900 years the sinuosity stabilized and fluctuates about a value of 3.4 (Figure 6). Three concurrently acting factors account for this: (1) migration rates reach a maximum at a certain R/W value (Figure 1), (2) increased sinuosity decreases channel gradient, thus decreasing the average migration rate, as discussed above, and (3) neck cutoffs begin to occur after the sinuosity exceeds 1.5. Most of the short-term variations in sinuosity in Figure 6 are due to cutoffs balanced by gradual meander enlargement.

Because of the upstream weighting function (1) the adjusted migration rates are generally different from the nominal rates. However, on the average, the adjusted rates turn out to follow the nominal rates closely except for strongly curved bends, as indicated in Figure 1*a*, where the average adjusted rates for the simulations in Figure 2 are plotted versus R/W and compared to the nominal rates.

The distribution of number of occurrences versus R/W follows a skewed distribution (Figure 1*b*). Most freely meandering rivers exhibit a similar skewing [*Brice*, 1974*a*; *Hickin*, 1977].

Successive centerlines for two simulations are presented in Figure 2. These simulations illustrate several consistencies in meander evolution in streams generated with the model as it is presently realized:

1. Low-amplitude meanders migrate (translate) rapidly downstream, but as their amplitude increases, the translation diminishes, so that the inflection points (crossovers) between loops become relatively fixed. This pattern is evident in natural meandering streams [*Brice*, 1974*a*, *b*] and in laboratory experiments [*Friedkin*, 1945].
2. As meanders increase in amplitude, the meander loops for most values of the input parameters are skewed upstream (Figure 2), as has been noted by *Brice* [1974*a*], *Nanson* [1980], *Parker et al.* [1982], and *Carson and Lapointe* [1983] in natural meandering streams. This is partly a result of the most rapidly migrating portions of the long loops being upstream from the center of symmetry and partly due to upstream rotation of arms of the meander due to erosion of the downstream portions of the previous loop. In the model this results from stretching of the meander loop during enlargement interacting with the phasing effect of the upstream weighting function.
3. When the path length through a meander loop becomes very long, shorter wavelength meanders may initiate on the nearly straight meander arms. These superimposed low-amplitude meanders initially migrate rapidly downstream, just as the original meander did at its inception. These new meanders have a wavelength close to the optimum implied by (1). Thus after meandering becomes well-developed, a range of superimposed wavelengths occurs. As a result, larger meander loops may acquire complex geometry, such as the "T" shapes noted by *Brice* [1974*a*, *b*] and *Hickin* [1974] in natural streams. This growth and birth process has an analog in death: cutoffs, as discussed next.
4. Cutoffs occur surprisingly late in the development of a

typical loop, after it has become quite elongate, due to the relative immobility of the loop arms. When a cutoff does occur, the sharp bend at the cutoff often enlarges rapidly into a new loop, which migrates somewhat downstream from the site of the cutoff and encroaches on the next loop downstream. *Handy* [1972] and *Kulemina* [1973] noted accelerated erosion downstream from cutoffs, attributing them to locally increased gradient, while *Nanson and Hickin* [1983] felt that the increase in sediment supply resulting from the cutoff enhances erosion rates downstream. However, in addition to these mechanisms, downstream growth may become accelerated simply due to the phasing effect of (1) and (2) because of the shorter pathlength through the cutoff loop. Often the growth of the new loop at the cutoff may shortly cause cutoff of the next loop downstream.

5. Cutoffs upset the left-right balance of the channel and locally shift the "center of gravity" of the channel, as noted by *Nanson* [1980]. This results in large-scale wandering of the channel system distinct from the meanders generated by loop growth.

6. Because of the dependency of local bank erosion on upstream channel morphology, the patterns discussed above are not universal. For example, occasionally a new loop may not form at the site of a cutoff; rather, the remnant bend may become distorted, resulting in further straightening by cutoff or inward migration.

Many meandering streams are confined by resistant valley walls. This was simulated by setting a valley width such that when a stream node reached the set width, any component directed against the valley walls was set to zero. The result is strongly asymmetrical meanders which have a very sharp bend on the upstream side and arms which are almost entirely concave upstream (Figure 5). This pattern is commonly found in natural streams with restricted meander width (Figure 5*c*) [*Lewin*, 1976]. Of course, the confining walls restrict meander migration to translation rather than growth, which is evident in the successive simulated centerlines (Figure 5*b*) and in the scroll bars shown in the photograph of the channel drawn in Figure 5*c*.

Unlike many slow-acting geomorphic processes, meandering in many rivers occurs rapidly enough that appreciable change in channel pattern has occurred over the period of historical record. This allows a more direct and sensitive testing of the simulation model, in which an early stream pattern is entered into the model to see how accurately the present meander pattern is predicted. As an example, the 1937 channel of the White River upstream from Edwardsport, Indiana, as shown by *Brice* [1974*a*] was input to try to predict the 1966 channel (Figure 4). The simulations predict two of the three cutoffs (the unpredicted cutoff occurs across a wide neck and may be artificial) and, particularly for the model parameters in Figure 4*b*, the broad pattern of channel migration. An optimal set of model parameters could be found by trial and error.

DISCUSSION

The simulation model described above duplicates many aspects of natural meandering. Two model assumptions provide the basis for the realistic simulations: (1) nominal bank erosion rates increase as bank curvature increases, but reach a maximum with decreased rates for very sharp curvature, and (2) actual bank erosion rates are a weighted average of local and upstream nominal rates, with the weights decreasing upstream. The first feature has been anticipated by *Hickin and Nanson* [1975], *Allen* [1977], *Begin* [1981], *Nanson and Hickin* [1983], and others, but the necessity for a weighting function had not been appreciated except by implication in the model of *Ikeda et al.* [1981]. Although some form of weighting of upstream and possibly downstream channel geometry seems necessary for realistic meander simulation, (1) to (3) are probably not the ideal descriptors of such weighting. Similarly, the best shape for the nominal rate curve is likely somewhat different than portrayed in Figure 1. In fact, the best form for these functions may vary among different fluvial environments. This is the reason for the use of the word "sufficient" rather than "necessary" in the title of this paper; other weighting procedures and possibly strikingly different assumptions might also successfully model stream meandering. However, it seems likely that such models would have assumptions roughly paralleling the two summarized above. Similarly, "conditions" has been used rather than "causes" in the title because not all model assumptions have been linked firmly with basic mechanics of fluid movement, transport, and erosion.

The model is presently being refined and elaborated to simulate meandering more realistically. This involves comparison of model predictions with natural meandering streams, both in the static sense of producing stream planforms that statistically resemble the range of natural meandering and in the kinematic sense of being able to duplicate the sequence of bank erosion and cutoffs that have occurred in natural streams with adequate historical record. Refinements of the model will include (1) selection of appropriate values of adjustable model parameters, (2) possible modification of the weighting procedures based on new theoretical or empirical information, (3) provision for spatially and temporally variable bank resistance, and (4) incorporation of procedures for modelling of chute cutoffs.

Simulation modelling not only increases understanding of the causes and conditions of meandering but also has practical implications in predicting the future course of meandering in natural streams (sites of most rapid bank erosion or likely cutoffs), as shown by *Parker* [1982].

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