

Simulation Model of Stream Capture

ABSTRACT

Because the importance of the process of capture, or piracy, in the formation of stream networks is difficult to evaluate by field or map studies, an indirect approach is used in this paper to investigate capture, through the use of a simulation model involving capture within rectangular stream networks on a square matrix. The simulation rules make the probability of capture of a stream by a lower adjacent stream proportional to the advantage in gradient of the potential path of capture between the streams compared to the present gradient of the higher stream. Stream elevations are assumed to be defined by the same type of pattern observed in natural stream networks, that is, a linear relationship between the logarithms of gradient and drainage area. The slope of this relationship, Z , is variable in nature and is the main adjustable parameter in the simulation model. Simulation of capture must start from assigned initial network patterns; random walk networks and parallel drainage are among those used for initial networks.

For a given value of Z , the statistical properties of networks (for example, stream numbers, length and area ratios, and shape factors) formed after repeated captures are nearly the same for a wide range of assigned initial networks. However, when the value of Z changes during capture, the statistical properties of the resultant networks may depend upon the type of change, so that properties may be partially inherited from earlier stages of basin evolution.

Both the networks simulated by capture and natural networks have similar slight deviations from topological randomness. The capture simulations more closely predict many properties of natural networks than do completely random methods of simulation, such as the random walk. In addition, several parameters in the capture-simulated networks exhibit a consistent trend with respect to the parameter Z that appears to occur also in natural networks. These correspondences between the capture model and natural networks suggest that capture may be an important natural process. However, cap-

ture should have its greatest relative importance in early stages of drainage basin evolution.

INTRODUCTION

Many features of stream networks have been closely predicted by theoretical and simulation models involving random processes (for example, the topological theory of Shreve, 1966, 1967, 1969, and the simulation models of Leopold and Langbein, 1962; Schenck, 1963; Smart and others, 1967; Howard, 1971). The success of random models in predicting stream topology and order-ratio statistics probably arises because of the multiplicity of causes involved in the development of natural stream networks (Krumbein and Shreve, 1970, p. 40; Howard, 1971).

Observed deviations of the numerical properties of stream networks from present random theories and simulations (Smart and others, 1967; Smart, 1969, p. 1770-1771; James and Krumbein, 1969, p. 550-551; Krumbein and Shreve, 1970, p. 78) may be due to systematic influences on the development of stream networks. Capture, or piracy, could be one such influence.

Although numerous instances of inferred past or impending capture are common in the geologic literature, their numbers are small compared to the total number of streams (Small, 1970, p. 236), and most of these captures are due to structural and stratigraphic causes (for example, the drainage diversions along the Catskill Escarpment, New York).

The rarity of present captures is due to the rather direct courses that most streams follow from their origin to their junction with another stream or their termination, coupled with the large junction angles between streams (adjacent streams are seldom closely parallel, and large streams are separated from one another by low-order streams; *see* Lubowe, 1964). But the rarity of captures in long-established networks may be due, in part, to frequent capture early in the development of the drainage network when indirect courses, determined by ir-

regularities on the original surface, were straightened by capture, or an originally parallel drainage net on a sloping surface was converted into a dendritic network by abstraction (Gilbert, 1877, p. 141; Horton, 1945, p. 333-349; Small, 1970, p. 242). In either case the numerical properties of these networks may have been influenced by capture.

The importance of capture in natural stream networks is difficult to evaluate through field or map study because of the slowness with which stream networks evolve and the subtlety of the evidence for past captures (Small, 1970, p. 236-250). Therefore, an indirect approach is used here. First, a simulation model of stream capture is developed, and the effects of the parameters of the model upon the drainage basin properties are examined. The main parameter of the model is the exponent of proportionality between stream gradients and the corresponding drainage area. This parameter also can be measured in natural stream networks, so that the correlation between variations in this parameter and changes in stream network properties can be compared in natural and simulated streams.

THE CAPTURE MODEL

The computer model simulates the evolution of drainage basins by successive captures of one stream by another, starting from an assigned initial drainage pattern. The probability of capture is determined by a function which depends upon the gradient relationships at the potential site of capture.

Model Network

The drainage network is represented by stream segments ordered on a 40×40 matrix. One stream segment originates from each position in the interior of the matrix (a total of 38^2 positions). Matrix points on the four edges are *drainage exits*; that is, no streams originate from these points, and all streams terminate at one of these locations. The stream segments are constrained to flow either east, north, south, or west from each interior position. Because of these restrictions, all networks have an equal total length of streams (1444 units). Figure 1 shows typical drainage patterns developed on this matrix. For simplicity of calculations each stream segment is assumed to receive drainage from one unit area of surrounding slope (that is, a uniform drainage density of unity is postu-

lated). The restrictions to four flow directions and the assumption of uniform drainage density have been used in previous simulation models (Leopold and Langbein, 1962; Schenck, 1963; Smart and others, 1967; Howard, 1971).

Probabilistic Modelling of Capture

In natural stream networks, capture occurs through a large number of processes, for example, by headward erosion and reduction of divides, by subterranean capture or abstraction, and by breaching of a divide by a meandering stream (Crosby, 1937; Thornbury, 1969, p. 147-154; Gilbert, 1877, p. 141; Lauder, 1968; Small, 1970, p. 236-250). All of these processes produce *discrete* capture, for part of a stream network changes its point of entry into the rest of the network (or becomes tributary to a different network). Although these processes differ in detail and will vary in importance with relief and rock type, in each case the path of capture is steeper than the original stream course. At the immediate site of potential capture of one stream (the *captive*) by another (the *captor*), three situations may occur ("captor" and "captive" are adopted from Gresswell, 1967, p. 211):

1. Capture is *impossible*: The gradient of the potential path of capture is uphill (negative).
2. Capture is *disadvantageous*: The gradient of the captive is greater than that along the potential path of capture.
3. Capture is *advantageous*: The gradient of the captive is less than that along the potential path of capture.

In natural stream networks capture would be expected if, and only if, capture were advantageous. The ratio R of the gradient along the potential path of capture G_c to that of the captive at that point G_d conveniently measures the possibility of capture, for capture is impossible if R is negative, it is disadvantageous if R lies between zero and unity, and the potential path of capture becomes increasingly advantageous as R ranges above unity.

A probabilistic approach to capture is necessary in the simulations because interstream slopes are not modelled explicitly. In natural streams the breaching of a divide, on the surface or subterraneously, is a prerequisite to capture. The model assumes that, within a natural stream system in a region of fairly uniform drainage density, divide relief, and geology, the frequency of discrete capture is an increas-

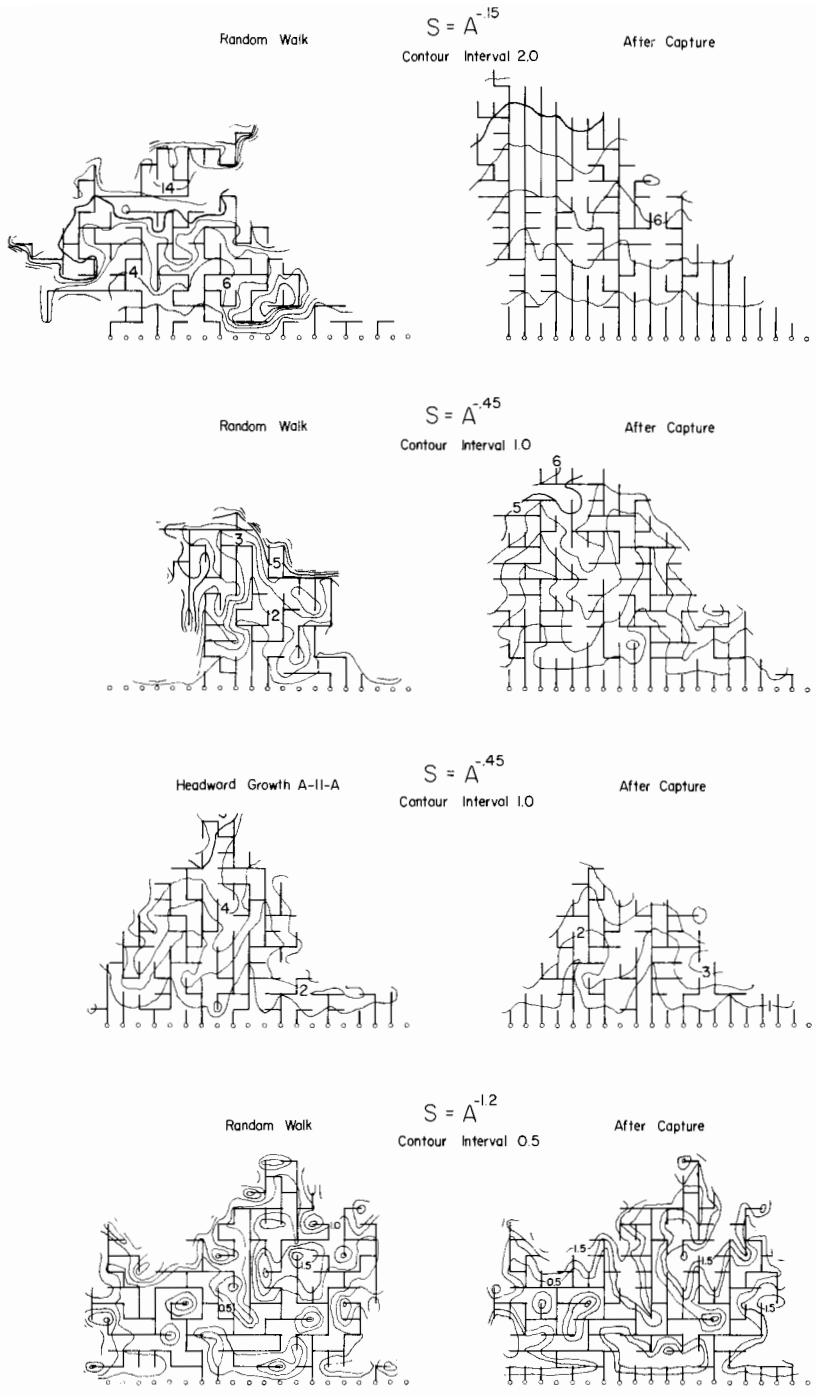


Figure 1. Initial stream matrices and changes produced by capture. Rectangular connected lines are stream channels, circles are drainage exits (stream termi-

nations), and curved lines are contours. Only those networks which are tributary to the drainage exits on the right half of the lower matrix edge are illustrated.

ing function of the ratio R' of the average gradient between the adjacent streams Gc' to the downstream gradient of the captive Gd (the prime marks indicate the use of the average gradient between streams rather than the gradient at the immediate point of capture). Higher values of the ratio Gc'/Gd in natural streams should correlate with divides asymmetrically close to, and only slightly higher than, the captive, so that breaching of the divide is likely. By analogy, in the capture simulations the probability of capture P is assumed to increase with the ratio, R' with the following functional form:

$$P = \begin{cases} 1 - e^{-W(1-R')}, & R' \geq 1 \\ 0, & R' < 1 \end{cases} \quad (1)$$

where e is the base of natural logarithms and W is the probability parameter. This functional form was selected to allow captures only in situations where the average gradient between streams would be advantageous and to maintain P within the range of zero and unity.

A probabilistic approach to capture is necessary not only because slopes are not explicitly modelled, but also because the model streams are constrained to uniform spacing, whereas natural streams are not so regular. Thus two streams with the same elevation difference between them will have differing values of R' depending upon the interstream distance, and hence, differing frequencies of capture. The differences in spacing between natural streams are assumed to be nonsystematic (random) in areal distribution, so that the effects of these differences upon capture can be simulated by the random component of the model.

One stream network may enlarge at the expense of shrinkage of another through gradual migration of divides without discrete capture (Small, 1970, p. 240-242). However, the concomitant growth of one network and withdrawal of another can be modelled by discrete shifting of the headward ends of one stream network to another by assigning a higher probability of capture to a faster rate of divide migration. It will be assumed that the rate of migration of divides would correlate with the ratio R' , where Gd is the gradient of the headward end of the higher stream (the captive and Gc' is the gradient between the headward ends of the higher and lower (captor) streams.

Stream Elevations in the Model Networks

In order to simulate capture within a matrix network, the elevations along the streams must

be assigned initially, and assumptions must be made about the evolution of these elevations as a result of capture. Most natural stream networks developed on rocks of uniform lithology exhibit a strong relationship between stream gradient S and the drainage area A which is closely described by:

$$S = KAZ, \quad (2)$$

where Z and K are parameters (for example, Fig. 2). The strong correlation between drainage area and the discharge and sediment load within streams accounts for this systematic relationship.

The initial gradients within the stream networks acted upon by capture are assumed to comply with equation 2. This suffices to determine all elevations within the network with the further assumption that the elevation of all drainage exits is zero.

Furthermore, after every few captures within the network, the basins of both the captor and captive streams are assumed to regrade in accordance with equation 2. This is assumed to

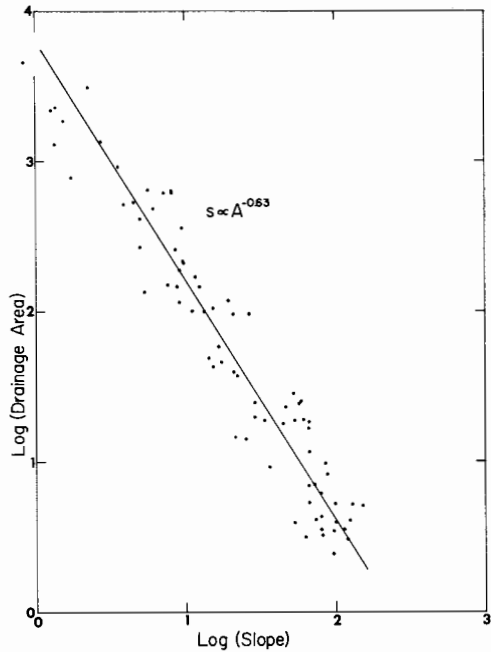


Figure 2. Relationship between stream gradient, S , and drainage area, A , for stream networks in the Berlin quadrangle, West Virginia, showing the least-squares linear regression line for gradient. Drainage area and gradient are shown in arbitrary units.

correspond to natural networks in which the captures occur infrequently enough that almost complete regrading (due to addition or withdrawal of drainage area) occurs between captures.

Regrading generally increases the elevations of beheaded channels relative to the drainage exits (the gradient increases due to loss of drainage area). Such an increase of relative elevation might occur in natural networks without aggradation if the average amount of erosion within the network between captures were greater than the relative changes in elevation occasioned by capture.

Thus the assumptions regarding elevation changes limit the model to analogy with infrequent captures within eroding natural basins characterized by constant parameters K and Z . Few natural areas are likely to have had a corresponding history of infrequent captures during continuous erosion in homogeneous rock. However, if the process of capture in natural streams is equifinal, so that the pattern of drainage networks continuously adjusts toward a unique equilibrium with the gradient relationships, then the capture model can be used to investigate equilibrium drainage patterns in natural networks, if the other assumptions of the model are reasonable. The question of equifinality is discussed below (see Effects of Initial Network Configuration).

Initial Networks

Because the capture model involves modifications within pre-existing networks, the initial configuration of the network on the matrix must be assigned. A wide range of initial drainage patterns was assumed in order to evaluate the effects of initial conditions upon subsequent capture.

Most capture simulations started from sample networks generated by one of three random simulation models: the random walk model (Smart and others, 1967) and the A-II-A and A-II ($P = 0.5$) headward growth models (Howard, 1971). The statistical characteristics of all three models are summarized in Howard (1971). Some properties of these networks are indicated in Figure 3, with the processes of generation abbreviated by RW, A-II-A, and A-II, respectively. In the capture simulations investigating the effects of the parameter Z , one of each of these random networks was used as an initial network, and, in addition, one simulation at each value of Z started from a network com-

posed of the 38 parallel streams formed by setting all interior segments to flow southward (abbreviated PAR in Fig. 3).

Process of Capture

Because the capture model is unavoidably complex, the model rationale follows an exposition of the simulation rules. Briefly stated, capture occurs according to the following scheme: a point in the interior of the matrix (a potential captive) is selected at random, and its capture by one of the surrounding streams occurs (or fails to occur) with a probability proportional to the function P defined above. Additional matrix points are examined, and occasional captures occur, until the process is terminated. Specifically, the process proceeds as follows:

1. All interior matrix locations (potential captives) are examined in a random order, without replacement, for the possibility of capture. After this examination, if capture is to continue, a new random listing of the matrix points is constructed and examined in sequence. Thus capture occurs in generations.

2. Having selected a potential captive, the probability of capture of the stream by the four surrounding matrix locations to the east, west, north, and south (potential captors) is evaluated by the function P . This probability is, of course, zero if the stream now flows directly to the potential captor. Junction of more than two streams at a single matrix location was made illegal in order to be able to unequivocally estimate the order of branching within the network; therefore, the probability of capture by a potential captor is equal to zero if the captor already receives two incoming streams.

3. If none of the four probabilities of capture is greater than zero, the next matrix location in the random listing is examined. If only one path of capture with nonzero probability exists, it is examined for the occurrence of capture (next step); if two potential paths of capture exist, then one of them is selected at random.

4. A random number from a uniform distribution between zero and unity is generated and compared with the probability of capture; if the random number is greater than the probability of capture, then no capture occurs, and a new matrix point is selected.

5. If the random number is less than the probability of capture, the direction of the captive stream segment is changed to drain into the captor location; the drainage area and elevation of the affected parts of the network are adjusted

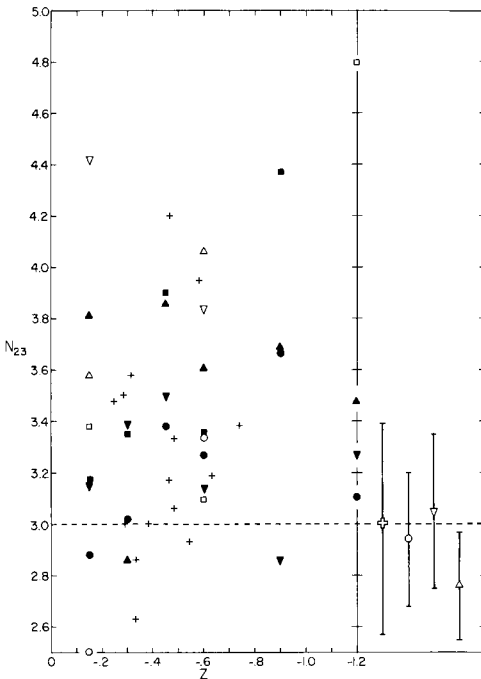
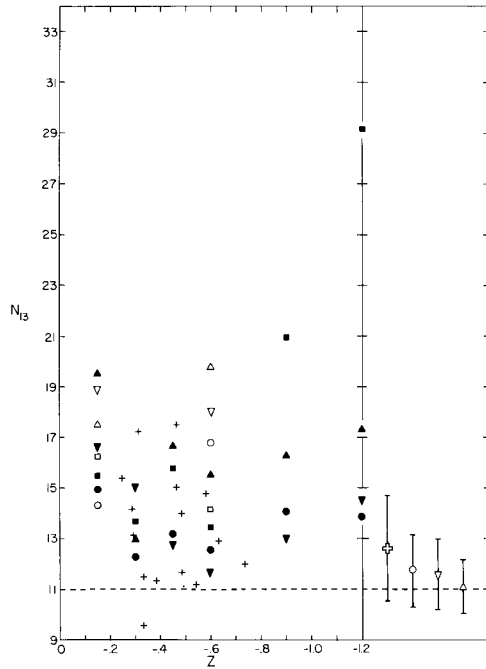
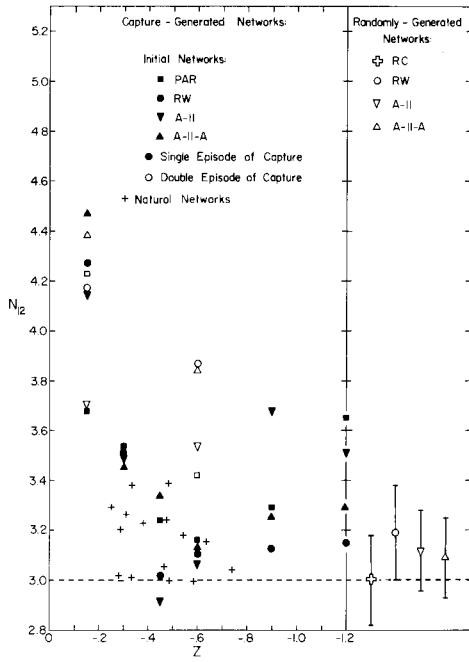


Figure 3. Average dimensionless properties of capture-generated and natural networks plotted *versus* the area-gradient exponent, Z . Average properties for double episodes of capture (*see* text) are plotted at the final value of Z . Also shown are the average and 95-percent confidence limits for the population mean for several types of random network models. Each point for natural and capture-generated networks is the average of about 120 second-order networks or 25 third-order networks, but 95-percent confidence limits are omitted for clarity. Horizontal dashed lines show the expected value for an infinite topologically random channel network. The probability parameter, \mathcal{W} , has a value of 0.5 for all capture simulations pictured here.

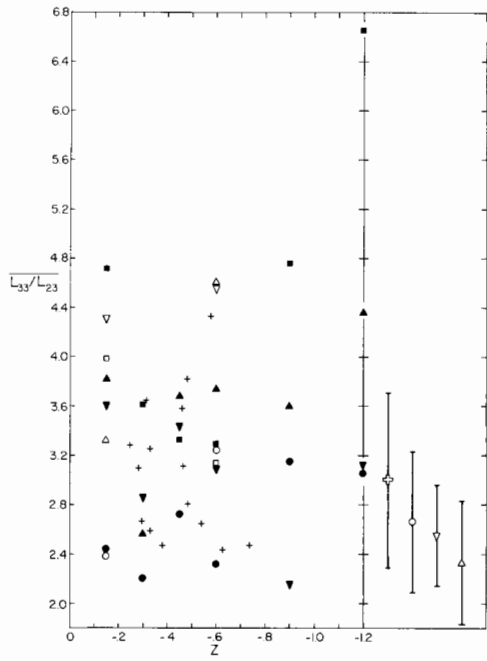
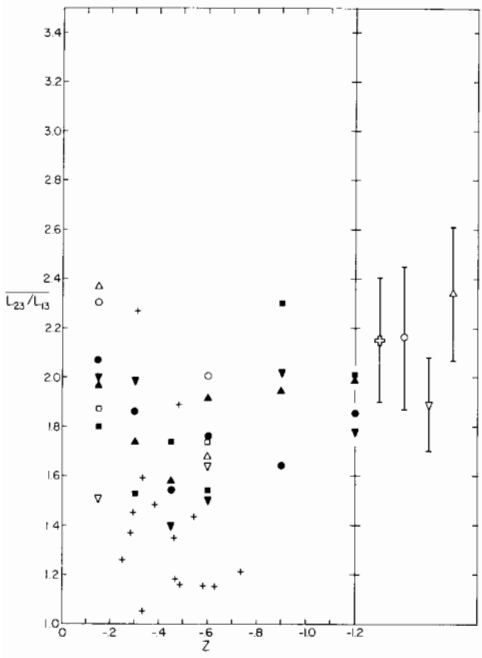
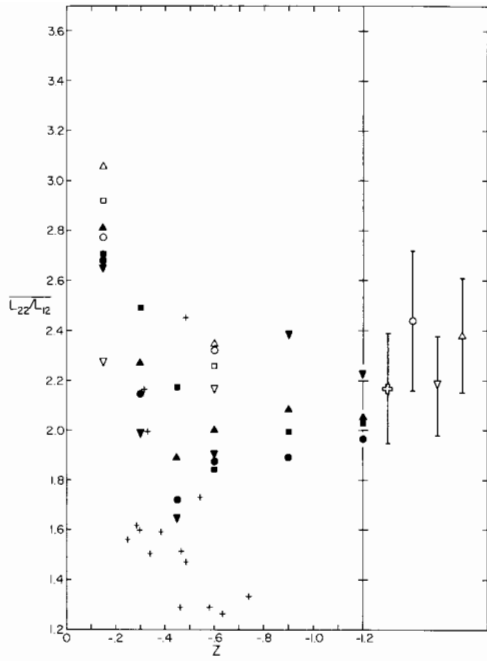
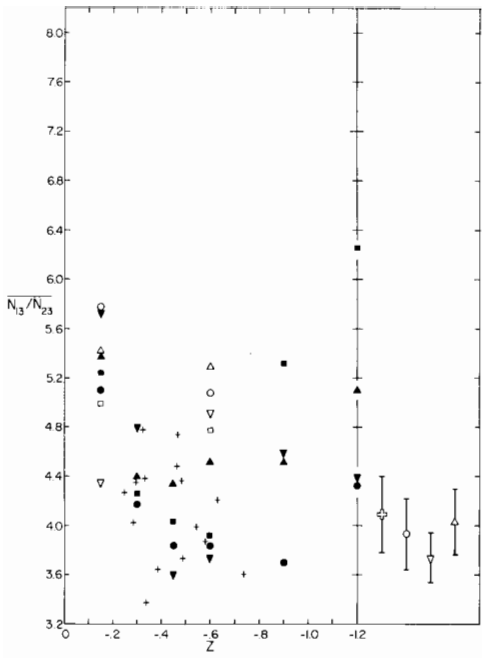


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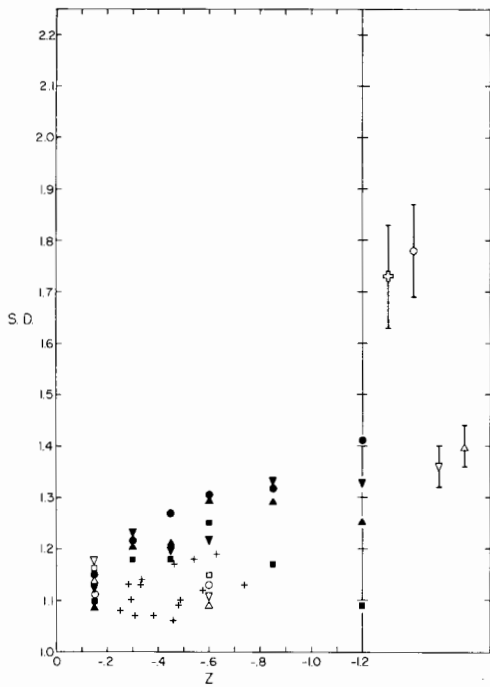
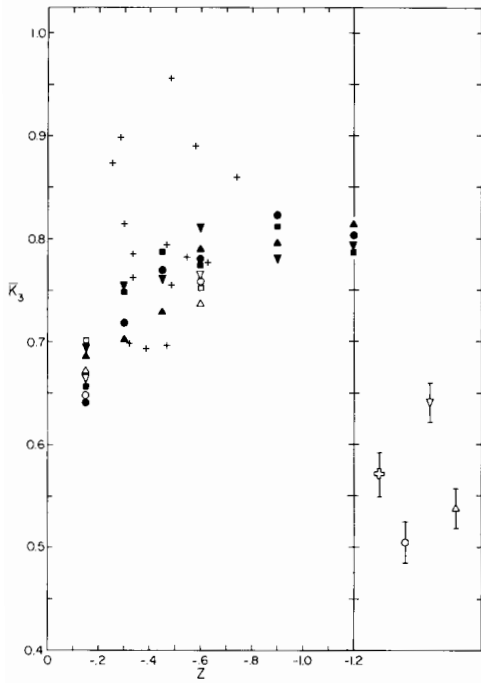
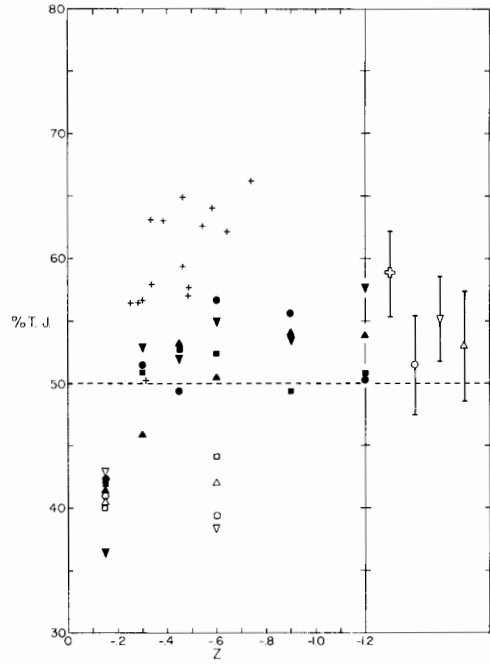
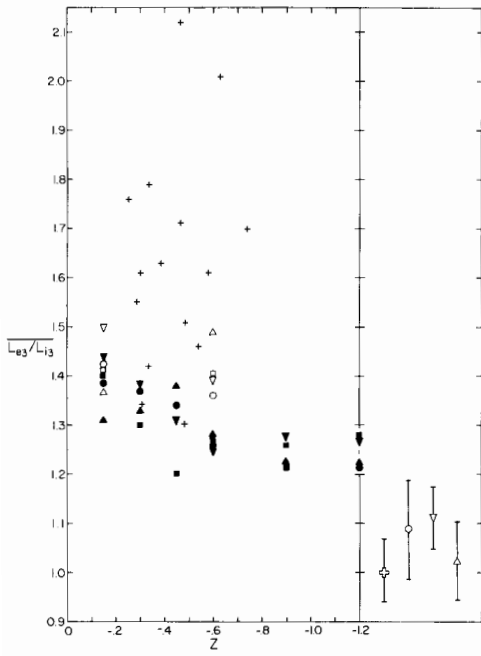


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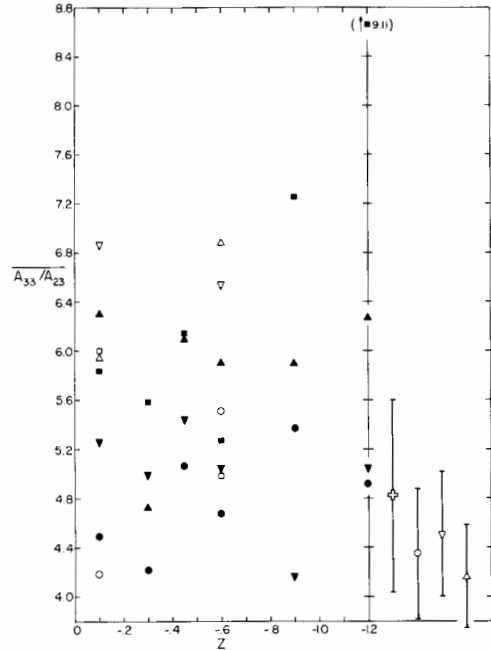
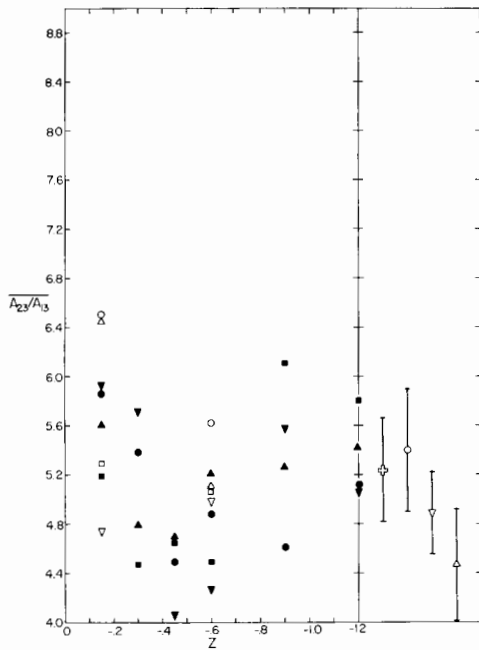
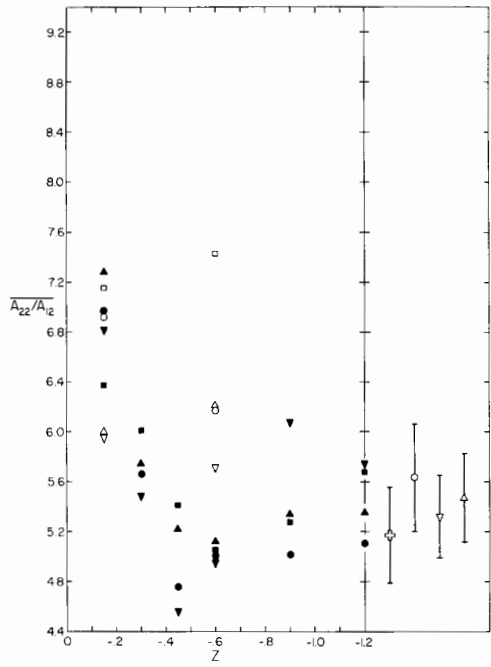
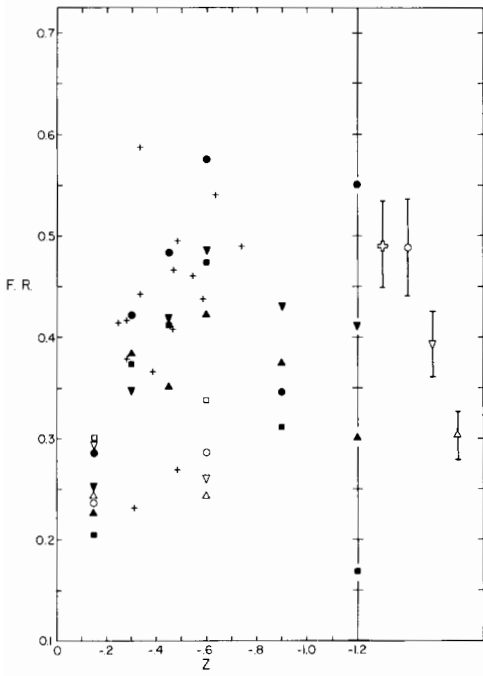


Figure 3 (continued).

to compliance with equation 2; another matrix location is selected, and the process is continued until capture is terminated when only a fixed number of interior matrix locations are sites of potential capture, or when an arbitrary number of captures is performed.

In all simulations in which the exponent Z (equation 2) was negative, the number of captures within each generation decreased with successive generations, on the average. If the process of capture were continued indefinitely in such cases, the number of possible captures would reach zero, but the last few captures, occurring in situations with low probability, would consume much computer time. Because the percentage change of network properties that would be produced by these few captures is small, the process was terminated when capture remained possible at only 80 of the 1444 interior matrix locations (about 5.5 percent). Termination before completion of all possible captures is likely to correspond to the scattered occurrences of impending capture in ancient natural networks. Fifteen to twenty generations were usually required for termination.

Re-evaluation of the drainage area and elevation of each location which would be affected by a capture (using equation 2) requires considerable computer time due to the difficulty of identifying affected portions of the captor and captive basins. Therefore, the drainage areas and elevations of the entire matrix are re-evaluated only after one-fifth of all possible captures within the matrix actually occur.

Because drainage area and elevations are not evaluated after every capture, a correlation is introduced between the probability of successive captures. In most cases this correlation is small because the captures occur in headwater areas or involve only channel straightening, resulting in only small additions or deletions of drainage area to affected basins; therefore, changes of gradient are small, except in the immediate vicinity of the capture. Correspondingly, captures might occur frequently enough in rapidly evolving natural drainage networks so that complete regrading does not occur between captures.

The method of sampling of interior matrix points assumed in this model (that is, random sampling without replacement during successive generations) more closely approximates the natural process, in which every stream location is continuously subject to possible capture, than would a simpler sampling procedure, such

as random sampling with replacement. In random sampling with replacement, some matrix points would be sampled several times before others would be sampled once; therefore, a random component would be introduced into the model which would not depend upon the gradient relationships within the basin. In the adopted sampling procedure, the temporal random component within the model resides almost solely in the probability function (although the stochastic sampling within generations introduces desired randomness in areal sampling which minimizes directional biases and interactions).

MEASURED PARAMETERS

All streams, natural and simulated, were analyzed according to the Horton-Strahler method of stream ordering (Strahler, 1952, p. 1120), and information on stream numbers, stream lengths, and drainage areas were tabulated by order. The portion of these data that are presented here is determined by two considerations.

Firstly, the simulated networks can be compared with natural networks only by enumerative or dimensionless properties. Hence only such parameters as stream numbers, length, and area ratios are discussed here.

Secondly, comparisons between natural and simulated networks of these dimensionless and enumerative parameters for basins of a given order are reasonable only if the simulated networks are sufficiently large that the size of basins of that order is not limited by the dimensions of the matrix. For example, basins of order 4 or greater may be unrepresentatively small, or missing, on stream networks generated on a 40×40 matrix. However, this size of matrix appears to be sufficiently large so that the average statistical properties of second- and third-order simulated basins are nearly equal to those of the same order generated on a very large (nearly infinite) matrix by the same process:

1. Simulated networks developed by purely random processes on a 40×40 matrix (for example, by random walk or headward growth) are close to being topologically random (defined *in* Shreve, 1966, p. 27; *see* tests for randomness *in* Howard, 1971). In addition, the average values of topological properties of all second- and third-order basins formed by these random processes on a 40×40 matrix are very close to the expected values in infinitely

large topologically random networks (defined in Shreve, 1967, p. 178; tests in Howard, 1971). Similarly, the total numbers of first-, second-, and third-order streams formed by random processes on a 40×40 matrix decrease by a factor of about 4 between order as expected in infinite topologically random networks (Shreve, 1967, p. 182; Howard, 1971).

2. The average topological properties of second- and third-order basins generated by the capture process on a 40×40 matrix, in general, deviate from expected values for infinite topologically random networks in proportion to the chi-square goodness-of-fit statistics from tests for topological randomness on the same matrices (Fig. 4).

3. The average topological properties of second- and third-order basins formed by the capture model deviate from those expected for infinite topologically random networks by having an excess of contributing lower order streams. If the second- or third-order basins were limited in size by the dimensions of the matrix, an average deficiency of contributing streams would be likely (and actually occurs for statistics of fourth- and fifth-order streams).

4. An increase in matrix size to 50×50 in randomly generated networks (increasing the number of stream segments by about 1.6 times) has little effect upon the topological and dimensionless ratio properties of second- and third-order networks.

Therefore, all second- and third-order basins generated by capture on the 40×40 matrix were sampled, and the average values of several enumerative and dimensionless properties were calculated. The following notations were used:

\bar{N}_{jk} Average number of j th-order streams in all k th-order networks developed on the 40×40 matrix.

$\bar{N}_{jk}/\bar{N}_{(j+1)k}$ The average, for all k th-order networks on the matrix, of the ratio of the number of j th-order streams to the number of $(j+1)$ -order streams in each network (the ratio is calculated within each network, and then averaged between networks).

$\bar{L}_{(j+1)k}/\bar{L}_{jk}$ The average of the ratio of the average length of $(j+1)$ -order streams to the average length of j th-order streams in all k th-order networks. (The lengths are first averaged by order within the network, the ratios are

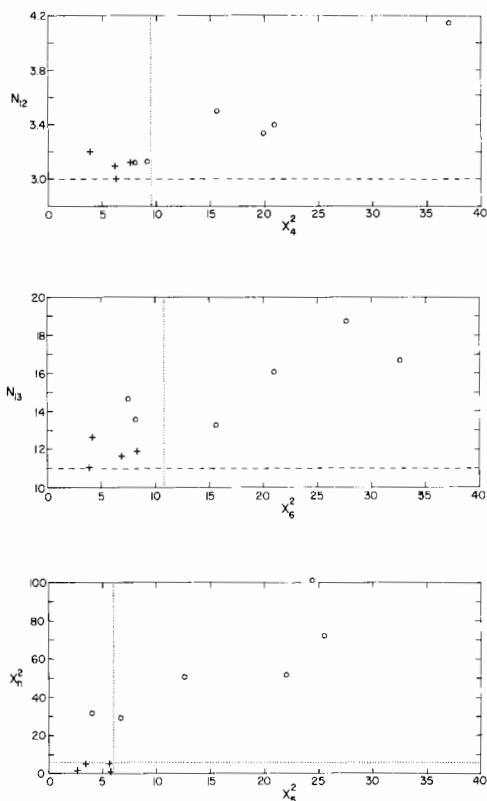


Figure 4. Relationships between chi-square statistics showing departure from topological randomness (horizontal axis) and departures from expected values for properties of infinite topologically random networks (vertical axis) for networks generated on a 40×40 matrix. Each circle shows values for all appropriate networks generated at a single value of Z in capture-generated networks, while plus-signs indicate values for randomly generated networks (RC, RW, A-II, or A-II-A models). Dashed lines show the expected values for infinite topologically random networks, while dotted lines show critical values in chi-square tests. The subscripts for chi-square tests for topological randomness indicate the number of sources in tested networks (4-6). The X_n^2 statistic results from tests for the expected ratio of 4 between total numbers of stream networks of successive orders in an infinite topologically random network. This test was applied to the total number of first-, second-, and third-order networks generated on 40×40 matrices. Scatter in the diagram is produced primarily by two factors: (1) the topological tests and values plotted on the two axes are derived from different populations within the same matrices (for example, networks with 6 sources and third-order basins); (2) the number of sampled networks varies somewhat between the plotted points. Nevertheless, the graphs clearly show that matrix networks which are most nearly topologically random also approach most closely to the expected properties of first-, second-, and third-order networks in an infinite topologically random network.

taken, and the average ratio is calculated between networks.)

$\overline{A_{(j+1)k}}/A_{jk}$ The average ratio of average contributing drainage areas, as with length, above.

$\overline{L_{ek}}/L_{ik}$ The average of the ratio of the average length of exterior links (first-order streams) to the average length of interior links (portions of streams between junctions) in all k th-order networks. Statistics are presented for third-order networks only.

S.D. *Stream directness*, that is, the ratio of (1) the length along a stream from the farthest point of the network divide to the mouth of the network to (2) the straight-line distance. This parameter, measured only on third-order networks, is similar to sinuosity, except that it considers larger scale wanderings, that is, the "straight-line distance" does not follow the general valley trend as is done when measuring sinuosity.

%T.J. *Percentage of trans junctions*, that is, the percentage frequency of occurrence, when following the path of greatest magnitude upstream, of the entrance of successive tributaries from opposite sides of the main channel (James and Krumbain, 1969, p. 547). Measurements were continued upstream to the beginning of second-order streams, whereas James and Krumbain (1969, p. 549) terminated their measurements upon reaching magnitude 10.

\overline{K}_k The average, for all k th-order networks, of the total number of links (exterior and interior) in the network times the drainage area of the network, divided by the square of the total length of channel within the network (after Shreve, 1967, p. 185). Measured only on third-order networks.

F.R. The *form ratio* is defined as the ratio of the total network area to the square of the distance from the network mouth to the farthest point of the divide (after Horton, 1932, p. 351), and gives a relative measure of the compactness of the drainage basin. This ratio was measured and averaged for all third-order networks.

RESULTS

Effects of Model Parameters

The effect of the area-gradient exponent Z

was investigated by subjecting four types of initial networks to capture at several values of Z . The probability parameter was fixed at a value of 0.5.

Values of Z greater than zero (gradient increasing downstream) produce unstable networks in which the number of captures per generation reaches a steady state, because capture increases the relative elevation of the captor at the point of capture, making recapture by adjacent streams with smaller drainage areas advantageous.

When Z equals zero, capture eventually forms straight, unbranched streams leading from symmetrical divides, because only the length of flow, and not the size of the stream, determines the elevation differences within the network. Indirect courses, therefore, are straightened.

For Z less than zero (as in natural stream networks), gradient decreases downstream. Such stream networks exhibit an "economy of scale": addition of new drainage area by capture results in a general lowering within the network after re-evaluation of elevations. The portion of the stream network beheaded by capture is concomitantly raised in relative elevation, and, therefore, is susceptible to further capture, reinforcing the tendency for the concentration of drainage into a few large basins. Generally, capture with negative Z forms branched networks.

However, the geometry of the generated networks varies with the value of Z (Fig. 1), although in all cases capture generates a stream system with few strong interstream gradients. For slightly negative Z , stream networks are elongate and nearly parallel, and the drainage patterns are fairly regular and symmetric. Networks are more pear shaped, or irregular, with more negative Z . Greatest modification of the original drainage pattern occurs for the less negative values of Z ; this is reflected both in the degree of alteration (Fig. 1) and in the total number of captures (Fig. 5).

The average dimensionless network properties generally show strong dependence upon the value of Z prevailing during capture (Fig. 3). Many of the relationships are not monotonic; several show a maximum or a minimum near a Z of $-1/2$. The reasons for most of the observed functional relationships are uncertain; deduction of these relationships from the basic premises of the model probably would be hopelessly complex in view of the three-dimensional

structure of the drainage networks and the inclusion of stochastic processes. In general, properties involving first- and second-order streams (for example, \overline{N}_{12} , $\overline{L}_{23}/\overline{L}_{13}$) show stronger variation with Z than do relationships between second- and third-order streams (for example, $\overline{A}_{33}/\overline{A}_{23}$).

The effect of variation in the probability parameter (W in equation 1) upon network properties was investigated at three values of Z (-1.5, -3, -6). The effects of increasing W from 0.5 to 30 upon network properties is generally slight or nonsystematic (Fig. 6). Because fewer advantageous captures are bypassed at higher values of W , future simulation models might employ high values of the probability parameter, or, in the limiting case, automatically allow any advantageous capture to occur (subject to the other restrictions of the model, such as the two-junction rule).

Effects of Initial Network Configuration

Of considerable importance in the interpretation of stream patterns is the degree to which the process of capture destroys the initial structure of the network, that is, the lack or presence of inherited features in capture-generated networks. The degree of inheritance depends both upon the geometry of the initial network and the value of the exponent Z .

As noted previously, courses of main streams and divides remain only slightly changed for highly negative Z , but are greatly altered for less negative values. Statistical properties of

capture-generated networks differing only in initial source network often show greater variation the more negative the value of Z , reflecting greater inheritance of features from the original network (Fig. 3). Finally, the number of captures during the simulation decrease as Z becomes more negative (Fig. 5).

As might be expected, the initial networks which are developed by random processes (random walk and headward growth models) and simple geometrical models like the parallel drainage, in general, are modified extensively by capture, because the gradient relationships within the network play no part in their formation. The headward growth networks, however, are less modified by capture than are the random walk models, for they are already highly symmetrical and regular (Figs. 1 and 5).

The degree of inheritance was further investigated by subjecting a single network to two successive episodes of capture at different values of Z . The four networks subjected to capture at Z of -6 were further modified at Z of -1.5. Similarly the four networks which were formed by capture at Z of -1.5 were subsequently subjected to capture at Z of -6. The statistical properties of the resulting networks after the double episodes of capture are shown in Figure 3 at the final value of Z .

The simulations with a sudden change in the value of Z within the network are not proposed to be realistic representations of erosional history; changes in Z within natural networks would be gradual. However, in order to simulate capture with a gradual change in Z , the simulation rules must specify the rate at which capture occurs relative to the rate at which Z changes. The step change investigated here is one end of this spectrum; the sole purpose of these simulations is to investigate further the importance of inherited network features in the capture model.

Stream networks first subject to capture at negative values of Z near zero are only slightly modified if Z becomes more negative, due to low probabilities of capture within the network (Fig. 3). However, if the value of Z is changed from more to less negative, the statistical properties of the networks are altered to values close to those resulting from a single episode of capture of the initial network at the final value of Z , due to a large number of additional captures. This failure of symmetry and lack of equifinality (independence from initial conditions) is one of the most striking results of the capture simu-

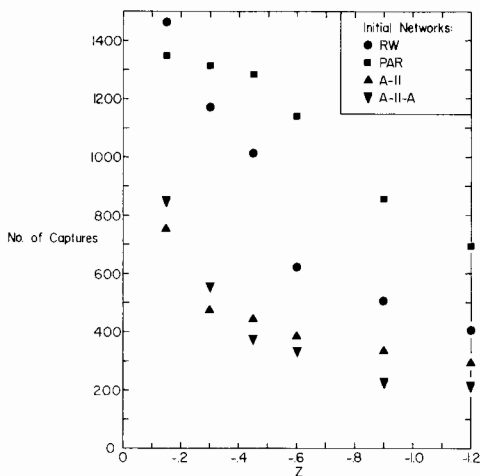
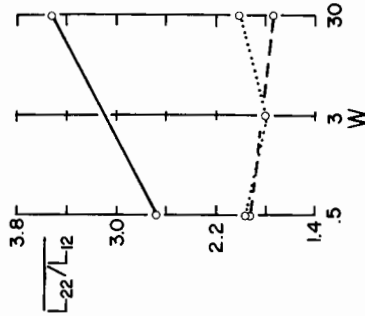
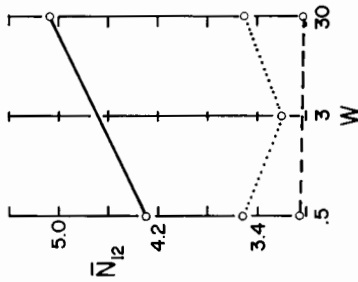
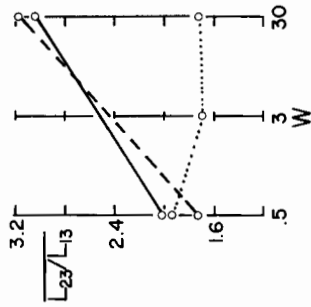
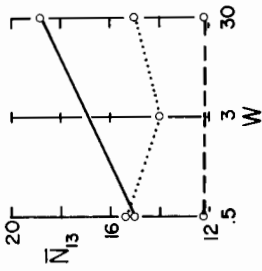
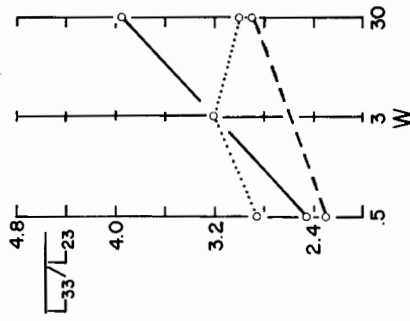
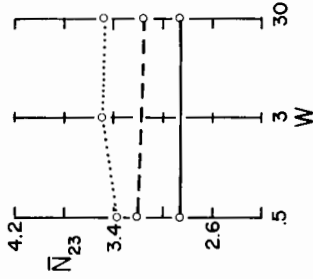
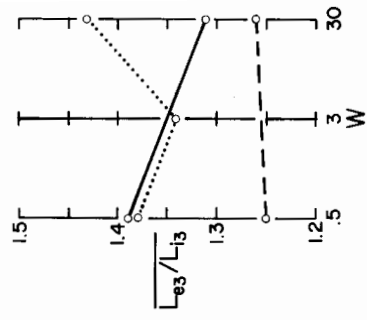
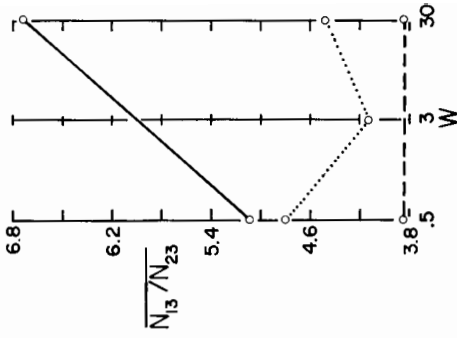


Figure 5. Number of captures occurring during simulations on 40×40 matrices. The various symbols indicate the type of initial networks.



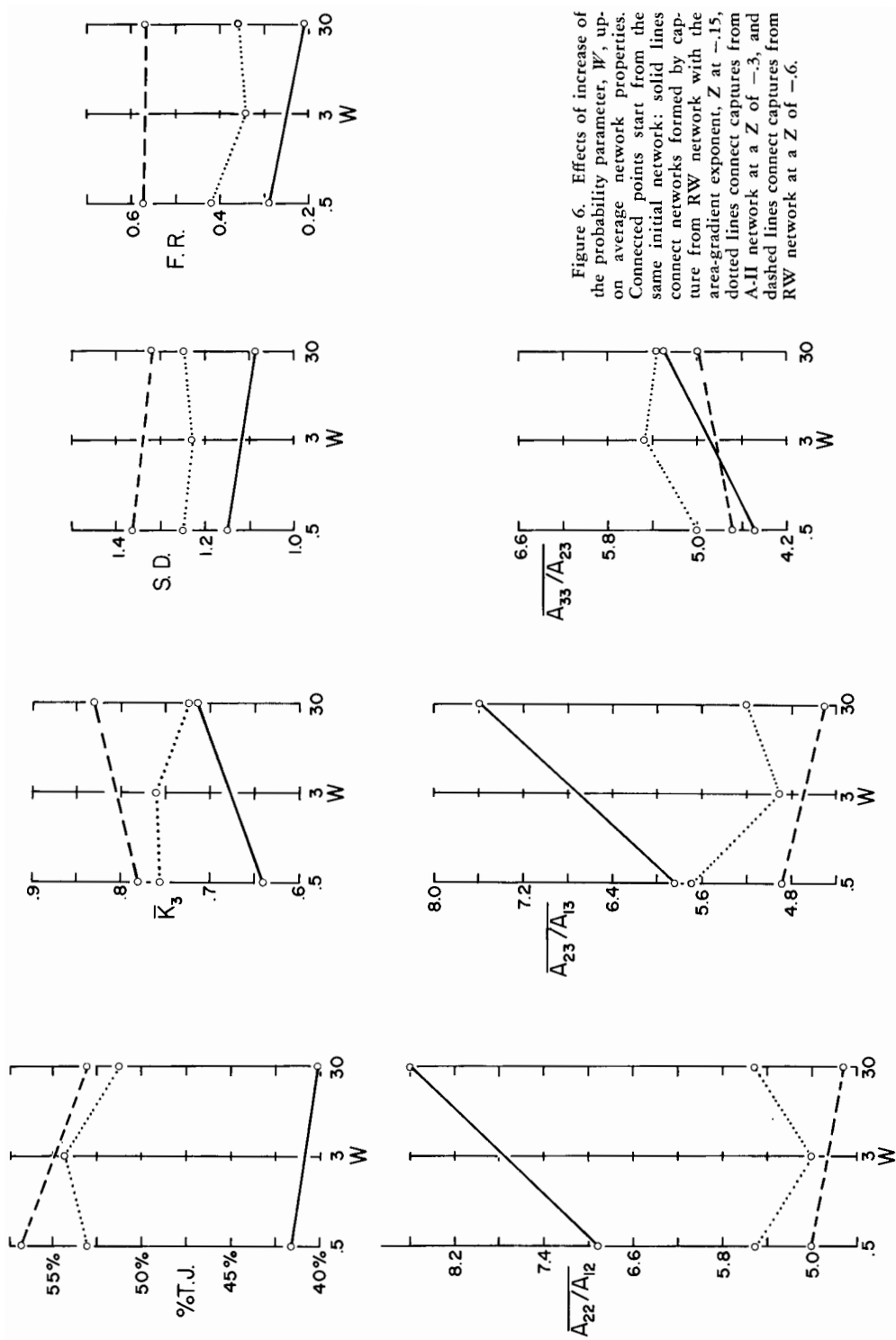


Figure 6. Effects of increase of the probability parameter, W , upon average network properties. Connected points start from the same initial networks: solid lines connect networks formed by capture from RW network with the area-gradient exponent, Z at -15 , dotted lines connect captures from A-II network at a Z of -3 , and dashed lines connect captures from RW network at a Z of -6 .

lations. Were the process of capture equifinal, then the postulated history of changes in the parameter Z (for example, a step change or a gradual change) would be unimportant if a sufficient period of capture occurred at the final value of Z ; the lack of equifinality indicates that more research should be done on the influence of the evolution of drainage basin relief on the planimetric and topologic properties of stream networks, both simulated and natural.

The above results indicate that, for a given value of Z in the simulation model, there exist a large number of networks that are relatively immune to capture under advantageous conditions (that is, they are *stable*). Those stable networks which result from extensive capture within initially unstable networks have statistical properties which are generally similar. However, other stable networks exist, such as those formed by two steps of capture at successively more negative Z , which have considerably different statistical properties. The evolution of certain natural drainage basins may parallel the change in Z in the simulation model from less to more negative, for example, in drainage basins formed by dissection of nearly flat or undrained land. In such cases inheritance of original features of the drainage network might be important, assuming that natural basin evolution shows a similar lack of symmetry and equifinality as does the capture model.

Comparison with Natural Streams and Random Models

For comparison with the capture simulations the statistical properties of second- and third-order networks in 15 areas in the United States were measured from 1:24,000 scale topographic maps. Measurement of stream network properties was done by a single operator who was unacquainted with the results of the capture simulations. The drainage network was drawn as completely as possible, using aligned sharply bent contours as criteria for smaller tributaries. The 15 areas were not selected entirely at random, for the following criteria were used: (1) existence of topographic maps at 1:24,000 scale with a small enough contour interval to outline clearly the course of small stream channels; (2) absence of obvious geologic control by differences in lithology and structure and lack of Pleistocene glaciation; areal uniformity of drainage density and a dendritic drainage pattern were the main criteria; (3)

diversity in locations; (4) adequate representation of the drainage area-stream gradient relationship by equation 2. Only one area was rejected by this restriction. (5) Finally, the areas were selected so that the exponent Z had as wide a range of values as possible.

On each selected topographic map one or two high-order drainage networks (fifth or sixth order) were drawn and ordered. Statistical properties of all second- and third-order basins from these networks were sampled. The points for average values of drainage basin properties in Figure 3 are based upon a sample of about 120 second-order and 25 third-order networks from each area.

Therefore, a similar sampling procedure was used in both capture simulations and natural networks (exhaustive sampling without replacement). Because adjacent networks probably interact to some extent in both natural and simulated networks, the assumption of independence used in most statistical testing may not be strictly valid. However, the statistical tests which are conducted below concern the adequacy of the theory of topological randomness (Shreve, 1966, p. 27), which assumes independence between adjacent streams. Systematic methods of sampling, such as those used here, may allow a more sensitive testing of Shreve's hypothesis.

Comparison of the natural networks with the properties of the networks simulated by capture posed two problems: (1) The dependence of most of the statistical properties upon Z in the simulation model is obviously nonlinear in the range of Z found in the natural networks (Fig. 3). (2) The generated networks do not duplicate exactly the values of Z found in the natural networks.

The procedure adopted for comparison was to superimpose the measured properties of natural networks upon the results of the simulations (Fig. 3). Although the number of sampled basins whose properties were averaged for plotting in Figure 3 was nearly the same for simulated networks as for natural, the latter show greater variation and less clearly defined trends with respect to the parameter Z . This greater variation within natural networks may be caused by such factors as: (1) small errors in estimating the parameter Z in natural basins from a finite sample; (2) the natural variance of gradient for the same drainage area in natural basins (Fig. 2); (3) systematic or random influences upon network development not ac-

counted for in the simulation model, such as structure, lithologic differences, microclimatic variations, or geomorphic processes other than capture; (4) if the natural networks show a similar inheritance of properties from earlier stages in drainage basin evolution as occurs in the simulation model, then differences in the past evolution of relief within the basin would produce variation in the statistical properties plotted in Figure 3 among networks with the same present value of Z .

The degree of explanation of natural stream networks provided by the capture simulations is best evaluated by comparison with theoretical and simulation models involving random processes, for example, the random topology theory of Shreve (1966, 1967, 1969) and the random walk and headward growth simulation models.

The capture-generated networks more closely predict the dimensionless and enumerative properties of second- and third-order natural networks within their common range of Z than do the randomly generated networks. This is especially clear in the case of the following network properties: \bar{K}_3 , S.D., $\bar{L}_{23}/\bar{L}_{13}$, \bar{N}_{13} , and $\bar{L}_{e3}/\bar{L}_{i3}$ (Fig. 3). However, the randomly simulated networks are more successful than the capture simulations in predicting the %T.J. statistic.

Because many of the dimensionless properties of the capture-simulated networks exhibit a maximum or minimum at Z of about $-1/2$, these properties show little correlation with respect to Z within the range of Z of natural networks. The corresponding properties of natural networks likewise exhibit little systematic trend with Z . However, for those dimensionless properties of the capture simulations which do show a clear trend in this range of Z (%T.J., F.R., \bar{K}_3 , and S.D.), the natural networks appear to follow a similar trend (neither the theory of topological randomness nor the random models of generation would predict systematic relationships between network properties and the parameter Z).

The natural and capture-generated networks can be tested against the hypothesis of topological randomness by tests of goodness-of-fit to the predicted frequency of topological or ambilateral classes (Smart, 1969) for networks with 4, 5, and 6 first-order tributaries (Table 1). Both the natural and capture-generated networks in the same range of Z fail these tests for topological randomness, and both show a general excess

of observed over predicted number of networks in second-order classes (Table 1). However, natural and simulated networks correspond less closely in the relative abundances of third-order networks with 5 and 6 first-order tributaries. By comparison, the randomly generated networks (A-II, A-II-A, and RW) approach much closer to topological randomness in the same tests (Howard, 1971, Table 4).

The natural networks sampled in the present study appear to deviate more strongly from topological randomness than do samples reported by other authors, possibly due to unintentional biases in sampling or measuring or to the selection rules discussed above. For example, the large number of basins sampled by Smart (1969) closely approach topological randomness. Because Smart's basins were not always selected within areas of uniform drainage pattern, inhomogeneities of bedrock and structure may have considerably affected stream patterns, thus introducing random (unaccounted-for) variation in network topology which would mask any effects of systematic process, such as capture.

On the other hand, Krumbein and Shreve (1970, p. 78) narrowly accepted the hypothesis of topological randomness in a goodness-of-fit test by ambilateral classes for 153 basins of magnitude 5 in an area of fairly uniform lithology in eastern Kentucky (goodness-of-fit statistic of 5.9 with a 95 percent critical value of 6.0). A sample of size comparable to that used in the similar test in the present paper (370 basins, Table 1) might have resulted in rejection.

Additionally, the average magnitude of 90 third-order basins in the Krumbein and Shreve study (1970, Table 2.2 gives \bar{N}_{13} as 13.7) is considerably greater than the expected value of 11 for an infinite topologically random network, and is similar to average values observed in the present study (grand mean for 342 third-order basins is 13.4).

One intriguing feature of the capture-generated networks is the close approach to randomness in the narrow range of Z near $-1/2$ (Table 1). In this same range many basin properties reach a maximum or minimum (Fig. 3), and the enumerative properties of second- and third-order basins most closely approach those of the random models and of the expected values for infinite topologically random networks.

The deviations of the properties of capture-

TABLE 1. CHI-SQUARE TESTS FOR TOPOLOGICAL RANDOMNESS OF NATURAL AND CAPTURE-GENERATED NETWORKS WITH 4, 5, OR 6 SOURCES*

		4 Sources by Topological Classes					
		Capture-Generated Networks					
Z = - .15	No. Observed	43	33	16	21	5	118 [‡]
	No. Expected	23.6	23.6	23.6	23.6	23.6	118
	Chi-Square	15.95 [†]	3.74	2.45	.29	14.66	37.08 (9.49) [§]
Z = - .3	No. Observed	35	39	20	35	15	144
	No. Expected	28.8	28.8	28.8	28.8	28.8	144
	Chi-Square	1.33	3.61	2.69	1.33	6.61	15.58 (9.49)
Z = - .45	No. Observed	46	35	40	33	22	176
	No. Expected	35.2	35.2	35.2	35.2	35.2	176
	Chi-Square	3.31	.00	.65	.14	4.95	9.06 (9.49)
Z = - .6	No. Observed	36	37	31	36	18	158
	No. Expected	31.6	31.6	31.6	31.6	31.6	158
	Chi-Square	.61	.92	.33	.61	5.85	8.01 (9.49)
Z = - .9	No. Observed	46	31	43	43	14	177
	No. Expected	35.4	35.4	35.4	35.4	35.4	177
	Chi-Square	3.17	.55	1.63	1.63	12.94	19.92 (9.49)
Z = -1.2	No. Observed	37	28	45	33	11	154
	No. Expected	30.8	30.8	30.8	30.8	30.8	154
	Chi-Square	1.25	.25	6.55	.16	12.73	20.93 (9.49)
Z = - .3 to - .6, lumped.	No. Observed	117	111	91	104	55	478
	No. Expected	95.6	95.6	95.6	95.6	95.6	478
	Chi-Square	4.79	2.48	.22	.74	17.24	25.47 (9.49)
		Natural Networks					
$\bar{Z} = - .44$	No. Observed	119	96	118	122	78	533
	No. Expected	106.6	106.6	106.6	106.6	106.6	533
	Chi-Square	1.44	1.05	1.22	2.22	7.67	12.16 (9.49)

generated networks from topological randomness and from randomly generated networks is due to the restrictions placed upon the occurrence of capture, that is, the provision that capture occurs only under advantageous conditions. If this condition is relaxed, so that captures occur randomly within the network irrespective of the gradient relationships at the site of capture, the statistical properties of the generated networks (random capture networks, abbreviated RC in Fig. 3) are similar to those produced by other methods of random simulation.

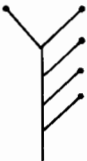
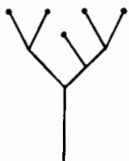
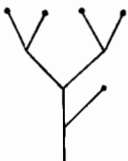
Random capture networks are formed by modifying an arbitrary existing network by selecting a matrix location in the interior of the matrix at random. One of the two possible directions of capture is selected at random, and the stream at the selected location is allowed to

be captured by the adjacent stream in the chosen direction, unless a loop would be formed in the resulting network. A new matrix location is selected, and the process is continued until the visual appearance of the original network is greatly altered (about 10,000 captures on a 40 x 40 matrix). The resulting networks resemble those generated by the random walk process.

CONCLUSIONS

Many of the dimensionless properties of natural streams are better simulated by the capture model introduced in this paper than by random models, suggesting that capture processes may be important in stream network development, especially in the early stages. However, the evidence is insufficient to be considered proof for several reasons:

TABLE 1. (continued)

5 Sources by Ambilateral Classes					
					
Capture-Generated Networks					
$Z = -.15$	No. Observed	74	14	2	90
	No. Expected	51.43	25.71	12.85	90
	Chi-Square	<u>9.91</u>	5.34	9.16	24.41 (5.99)
$Z = -.3$	No. Observed	76	17	9	102
	No. Expected	58.28	29.14	14.57	102
	Chi-Square	<u>5.39</u>	5.06	2.13	12.57 (5.99)
$Z = -.45$	No. Observed	75	22	11	108
	No. Expected	61.71	30.86	15.42	108
	Chi-Square	<u>2.86</u>	2.54	1.27	6.67 (5.99)
$Z = -.6$	No. Observed	74	29	10	113
	No. Expected	64.57	32.28	16.14	113
	Chi-Square	<u>1.38</u>	.33	2.33	4.05 (5.99)
$Z = -.9$	No. Observed	87	13	10	110
	No. Expected	62.85	31.43	15.71	110
	Chi-Square	<u>9.28</u>	10.80	2.07	22.15 (5.99)
$Z = -1.2$	No. Observed	89	13	8	110
	No. Expected	62.85	31.43	15.71	110
	Chi-Square	<u>10.88</u>	10.80	3.78	25.43 (5.99)
$Z = -.3$ to $-.6$, lumped.¶	No. Observed	225	68	30	323
	No. Expected	184.56	92.28	46.12	323
	Chi-Square	<u>8.86</u>	6.39	5.64	20.89 (5.99)
Natural Networks					
$\bar{Z} = -.44$	No. Observed	241	82	47	370
	No. Expected	211.4	105.7	52.9	370
	Chi-Square	<u>4.14</u>	5.32	.64	10.10 (5.99)

1. The degree of improvement in explanation of natural network properties provided by the capture model over random models is generally slight enough that the improvement might have arisen from random factors in selection of natural networks.

2. Systematic processes other than capture in natural networks might produce the observed deviations of the stream properties from random models. One such process might be modification of junction angles (Schumm, 1956, p. 617-620).

3. Correlations of capture-generated network properties with the exponent Z may result from the geometrical structure of the model rather than from capture related assumptions, for example, from the equidistant spacing of streams or from the direction-of-flow restrictions.

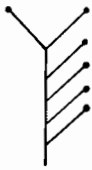
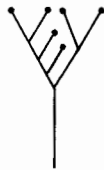
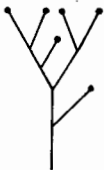

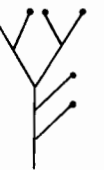

ACKNOWLEDGMENTS

This study was supported by a National Science Foundation Institutional Grant at the University of Virginia. Early studies were supported by the Geography Department at the Johns Hopkins University. The author wishes to acknowledge the helpful comments by R. L. Shreve and J. S. Smart. The tireless efforts of James Cole in analysis of natural stream networks are appreciated greatly.

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TABLE 1. (continued)

		6 Sources by Ambilateral Classes						
								
		Capture-Generated Networks						
Z = - .15	No. Observed	52	8	12	1	2	1	76
	No. Expected	28.95	14.47	14.47	7.23	7.23	3.62	76
	Chi-Square	<u>18.36</u>	2.89	.42	5.37	3.79	1.89	32.73 (11.07)
Z = - .3	No. Observed	43	12	12	5	7	2	81
	No. Expected	30.86	15.42	15.42	7.71	7.71	3.86	81
	Chi-Square	<u>4.78</u>	.76	.76	.95	.07	.89	8.21 (11.07)
Z = - .45	No. Observed	34	17	10	2	10	4	77
	No. Expected	29.33	14.66	14.66	7.33	7.33	3.66	77
	Chi-Square	<u>.74</u>	<u>.37</u>	1.48	3.88	<u>.97</u>	<u>.03</u>	7.48 (11.07)
Z = - .6	No. Observed	44	13	16	0	8	1	82
	No. Expected	31.23	15.61	15.61	7.81	7.81	3.90	82
	Chi-Square	<u>5.22</u>	.44	<u>.01</u>	7.81	<u>.00</u>	2.16	15.64 (11.07)
Z = - .9	No. Observed	50	14	10	1	8	1	84
	No. Expected	32.00	15.99	15.99	8.00	8.00	4.00	84
	Chi-Square	<u>10.13</u>	.25	2.25	6.12	.00	2.25	21.00 (11.07)
Z = -1.2	No. Observed	57	15	10	2	7	0	91
	No. Expected	34.66	17.33	17.33	8.66	8.66	4.33	91
	Chi-Square	<u>14.40</u>	.31	3.10	5.12	.32	4.33	27.58 (11.07)
Z = - .3 to - .6, lumped.¶	No. Observed	121	42	38	7	25	7	240
	No. Expected	91.42	45.70	45.70	22.85	22.85	11.42	240
	Chi-Square	<u>9.57</u>	.30	1.29	10.99	<u>.20</u>	1.71	24.08 (11.07)
		Natural Networks						
$\bar{Z} = - .44$	No. Observed	148	35	38	31	21	9	282
	No. Expected	107.4	53.7	53.7	26.8	26.8	13.6	282
	Chi-Square	<u>15.34</u>	6.51	4.59	<u>.64</u>	1.27	1.46	29.80 (11.07)

* For explanation of tests by topological and ambilateral classes see Smart (1969).

† Underlined chi-square values due to excess of observed over expected.

‡ Marginal totals in this column.

§ Critical value at 95% level of significance.

¶ All networks with given number of sources from the 12 40 × 40 matrices generated with a Z of - .3, - .45, and - .6.

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MANUSCRIPT RECEIVED BY THE SOCIETY JULY 10, 1970

REVISED MANUSCRIPT RECEIVED DECEMBER 17, 1970