Modelling Geomorphological Systems
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Chapter 3

Equilibrium models in geomorphology

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3.1 INTRODUCTION

The terms *equilibrium* and *steady state* imply a particular type of relationship between observable quantities. In this chapter this relationship will be defined and illustrated with several examples of geomorphic models that assume equilibrium. Since most readers already have an intuitive grasp of equilibrium, they may find it surprising that the topic is controversial and requires careful discussion both of the operational definition of equilibrium and of criteria for evaluation of equilibrium or disequilibrium in natural systems. Examples are offered below from simple natural and mathematical systems, and the pitfalls or limitations in the use of equilibrium models are discussed.

3.1.1 Equilibrium Defined

The first hurdle is one of definition. The terms equilibrium and steady state have been given a variety of imprecise, overlapping, and sometimes contradictory definitions (see, for example, the discussions by Chorley and Kennedy, 1971, Ch. 6, and Allen, 1974). Howard (1982) offered a definition that conforms to the spirit of prior usage while offering operational rules capable of quantitative testing. This definition is paraphrased and somewhat amended below as a series of propositions followed by interpretive corollaries and clarifications.

Equilibrium refers to a type of temporal relationship between one or more external variables, or inputs, and a single internal variable, or output, that has the following characteristics:

1. Changes in the inputs must cause measurable changes in the output either immediately or after a finite time. This eliminates the trivial case of inputs that have no effect upon the output.

2. The value of the output at a given time is related by a single-valued, temporally invariant functional relationship to the value(s) of the input(s) at the same time, within a consensual degree of accuracy.
3. The functional relationship should be capable of repeatable testing, either experimentally or observationally.
4. An equilibrium relationship may be limited to certain ranges of the input values and/or to certain rates of change of the input values.

The heart of the definition is the single-valued functional relationship, which incorporates both the notion of time independence in the relationship between inputs and outputs and the possibility that equilibrium can be maintained as both inputs and the outputs change through time (this is analogous to the concept of reversible reactions in chemical equilibria). The concept of an allowable margin of error in the functional relationship moves equilibrium from an unattainable ideal to a useful and quantitatively testable tool.

Although the above definition is fairly self-contained, a number of implications and clarifications need to be explored.

Equilbrium makes no necessary statement as to the chain of cause and effect. The terms input and output suggest a causal dependence, but this is not intended. Inputs and outputs are defined merely as quantities that can be measured more-or-less simultaneously and repeatedly. Similarly, the terms external and internal variables are not meant to imply physical separation or one-way fluxes of mass or energy. In other words, equilibrium as proposed here implies a 'black-box' approach to system identification. In general, an investigator is concerned with the question of cause and effect, and tries to make the external, or input variables the causes and the internal, or output variables, the effect. However, feedback relationships often make such distinctions difficult or impossible. Furthermore, identification of cause and effect generally requires information and hypotheses supplemental to the determination of equilibrium.

Equilibrium is not a property of the system as a whole. No natural system can ever exhibit single-valued functional relationships between all measurable properties at all physical and temporal scales. For example, the velocity of water in a canal may be essentially uniform and steady when measured by a traditional current meter, and related in a predictable manner to the flow depth and the cross-sectional characteristics and gradient of the canal. However, when the flow is measured by hot-wire anemometry, the small-scale rapid turbulent fluctuations lose a single-valued relationship to depth and canal characteristics (although a statistical relationship still applies).

Much of the confusion in the discussion of equilibrium in geomorphic systems arises from the attempt to identify physical systems as being in equilibrium as opposed to exhibiting disequilibrium or possibly 'inherent instability'. In a broad sense, this issue involves our propensity to confuse models and measurements with reality. For example, we generally feel that the 'velocity' of water in a channel flow is a well-defined property of the system. However, from the preceding paragraph, it is apparent that velocity depends not only upon the system but also upon how it is measured, and in particular, upon the characteristic physical and temporal scales of the measuring procedure. Because of these considerations, the definition states that equilibrium criteria must be applied separately to each internal, or output, variable, whereas several external variables may be involved for each internal variable. In general, all temporally varying external variables that may appreciably affect the output variable should be included if a functional relationship is to be identified.

Although equilibrium is manifested by a temporally invariant functional relationship, temporal change in internal and external variables is necessary to identify the relationship and test equilibrium. Thus equilibrium does not imply either unchanging inputs or output. In fact, a completely static relationship is indecisive about the sensitivity of the output to changes in the input. As will become clearer below, the occurrence or lack of an equilibrium relationship depends upon the rate at which the output variable can adjust, or respond, to changes in the input variable.

The selection of input and output variables and their methods of measurement are often determined by a desire to find an equilibrium relationship. All measurements involve temporal and areal/volumetric averaging. The selection of an appropriate scale of averaging is generally influenced by the characteristic response time of the output variables to changes in input as well as other pragmatic or theoretical concerns. Mismatching of spatiotemporal scales for measurement of input and output parameters, or measurements that are made at timescales much longer or much shorter than the characteristic response time will generally complicate or obliterate any possible equilibrium relationships. Returning to the previous example, if velocity is measured by 'instantaneous' hot-wire anemometer readings, no simple hydraulic relationships are likely to be discovered relating the velocity to channel form variables and depth measurements made at larger physical–temporal scales. The importance of timescales to equilibrium concepts is further explored in the following examples, and has been discussed by several authors, including Thornes and Brunsden, 1977; Brunsden, 1980; Cullingford *et al.*, 1980; and Howard, 1982.

3.2 EQUILIBRIUM CONCEPTS AND EXAMPLES

In the following sections the above definition will be applied to various types of system models. The first type of system will be a simple deterministic linear system. The equilibrium concept will then be extended to more complicated systems that involve non-linear relationships, spatial extension, and/or stochastic inputs.

3.2.1 Equilibrium in Deterministic Linear Systems

Howard (1982) illustrated the definition of equilibrium by considering a simple linear system in which an ouput, or response, $y(t)$, is a weighted average of past values of a single input, $x(t)$. The weighting function is assumed to be a negative exponential, so that:

$$y(t)=\lambda \int_0^\infty x(t-\tau)\, e^{-\lambda\tau}\, d\tau \qquad (1)$$

where $T=1/\lambda$ is the characteristic relaxation time of the system (the relaxation time is also sometimes defined as the time, T', such that $e^{-\lambda T'}=0.5$ (Waide and Webster, 1976)). The conditions for equilibrium and the dynamics of such a system can be illustrated by considering the output response to simple inputs such as step changes, sinusoids, and impulses.

The output response to a step change in the input at time t_0 from a value of C_1 to C_2 is, for $t>t_0$:

$$y(t)=C_2+(C_1-C_2)\, e^{-\lambda(t-t_0)}, \qquad (2)$$

assuming $x(t)=C_1$ for all $t<t_0$.

The inputs to many natural systems are quasiperiodic (with, for example, a yearly cycle). Such inputs can be represented conceptually as a sum of sinusoidal inputs with different frequencies superimposed upon an average value. For a single frequency component with an average value:

$$x(t) = \alpha \sin\{\omega t\} + C, \text{ so that} \tag{3}$$

$$y(t) = A \sin\{\omega t - \theta\} + C, \text{ where} \tag{4}$$

$$A = \alpha\beta/(\beta^2 + 1)^{1/2} \text{ and } \theta = \arctan 1/\beta.$$

Also, $\beta = \lambda/\omega$ and it is assumed that the input $x(t)$ has been maintained for all past time.

For this type of input the system response is a constant value with a superimposed sinusoidal component that is a delayed and damped replica of the input. The ratio A/α is conventionally termed the magnitude ratio, and θ is the phase shift (see Pickup and Rieger, 1979). These, in turn, are functions of the parameter β (termed here the *response ratio*) that relates the input frequency to the system relaxation time. The magnitude ratio and phase shift are plotted as a function of the response ratio in Figure 3.1.

Another type of simple input is a linear trend:

$$x(t) = C + \delta t, \tag{5}$$

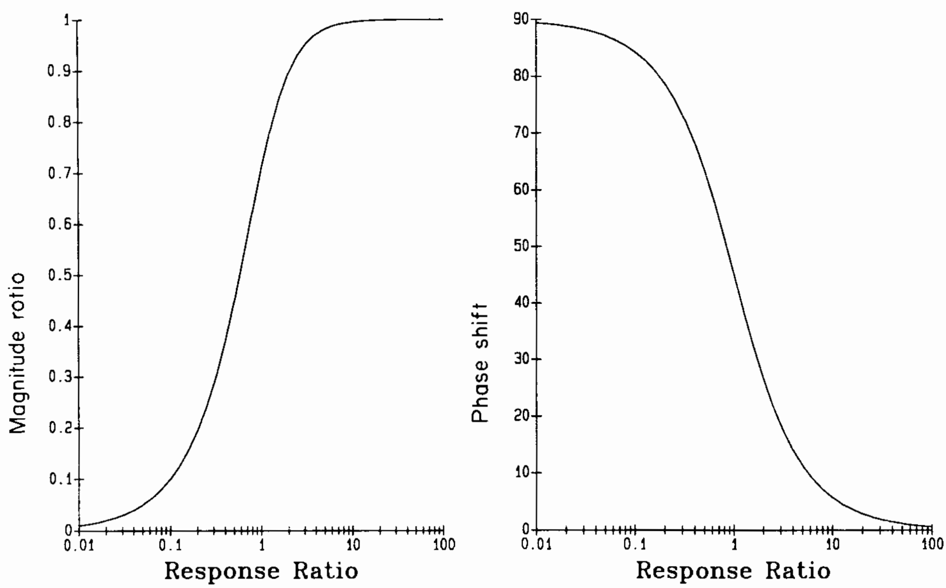

Figure 3.1. The magnitude ratio (A/α) and phase shift (θ, in degrees) plotted as a function of the response ratio (β) for a linear system with finite memory

which has the response:

$$y(t)=C+\delta t-\delta/\lambda. \tag{6}$$

A final simple input is an impulse of magnitude M occurring at time $t=t_0$:

$$x(t)=C+M\Delta(t-t_0), \tag{7}$$

where $\Delta(t-t_0)$ is a unit impulse. The system response for $t>t_0$ is:

$$y(t)=C+M\lambda\ e^{-\lambda(t-t_0)}. \tag{8}$$

The response of our example linear system to an arbitrary input can generally be modelled as a linear combination of such simple inputs.

3.2.2 Criteria for Equilibrium

Equilibrium occurs when the value of $y(t)$ is sufficiently close to the value that it would have if the input variable(s) were constant over an infinite time ($x(t)=C$). This *ultimate* equilibrium value, $Y(t)$, for this system is:

$$Y(t)=\lambda\int_0^{\infty} C\ e^{-\lambda\tau}\ d\tau=C \tag{9}$$

However, as is evident from this equation, an infinite time is required to reach the ultimate equilibrium, but a *consensual* equilibrium can be defined that permits a specified departure from ultimate equilibrium. In this paper the unmodified term equilibrium refers to such a consensual equilibrium.

Two possible definitions of consensual equilibrium for the example system are discussed here. The first definition requires that the response be within a given fraction of its ultimate value, $Y(t)$, such that:

$$|Y(t)-y(t)|\leqslant\epsilon|Y(t)|, \tag{10}$$

where ϵ is the consensual magnitude of acceptable deviation.

For a step change in input (Equation 2) equilibrium will occur for $t\geqslant t_e$, where:

$$t_e=t_0-\ln\{(\epsilon|C_2|)/|C_1-C_2|\}. \tag{11}$$

However, if $t_e\leqslant t_0$ (that is, if $\epsilon\ |C_2|\geqslant|C_1-C_2|$), then equilibrium is considered not to be disturbed by the step change. For a sinusoidal input the deviation of the input and output signals also varies sinusoidally, so that a modification of equation 10 is proposed that uses the maximum deviation during an input cycle as a criterion:

$$\mathscr{M}|y(t)-Y(t)|\leqslant\epsilon\mathscr{A}|Y(t)|, \tag{12}$$

where $\mathcal{M}$ is the maximum, and $\mathcal{A}$ the average value of their arguments over one input cycle. For the input of equation 4 this criterion yields equilibrium for:

$$|\alpha \sin\{\theta\}| \leqslant \epsilon |C|. \tag{13}$$

For small values of the response ratio $\sin\{\theta\} \approx 1$, so that equilibrium will occur only for small amplitude sinusoidal components. For very small response ratios the output responds only to the average value of the input, that is, the high frequency components are filtered out. In such cases it may be warranted to choose a different input variable which is a temporally averaged value of the original variable, since equilibrium might then occur. This prefiltering should have a characteristic response ratio of the same order of magnitude as the system response ratio. Howard (1982) suggested that cyclical components that elicit little system response due to their high frequency can be ignored with regard to the occurrence of equilibrium; the approach recommended here of redefining the input variable by filtering the original variable is conceptually simpler and more consistent.

For some purposes it may be expedient to define equilibrium differently than suggested in equation 10 or equation 12. For example, for systems whose mean value is near zero, an absolute deviation may be appropriate:

$$|Y(t) - y(t)| \leqslant \epsilon, \tag{14}$$

whereas for step changes a proportional response may be informative

$$|Y_2 - y(t)| \leqslant \epsilon |Y_2 - Y_1|, \tag{15}$$

where Y_1 and Y_2 are the initial and final equilibrium response values, respectively. If $y(t)$ and $Y(t)$ are finite and range through several orders of magnitude, then the logarithms of the inputs and output may be used in testing for equilibrium.

The selection of an equilibrium criterion and the limit of consensual error will depend upon the purpose of the study and the nature of the system being examined.

3.2.3 Systems with Stochastic Components

The description of many systems must incorporate random or stochastic components. The implication of randomness to equilibrium concepts depends upon the nature of the randomness and the manner in which it enters into the relationship between inputs and the output.

For some systems the stochastic component can be considered to be part of the input signal, or as an additional, perhaps uncontrolled input signal. For example, random rainfall is an input to synthetic streamflow synthesis.

The status of random inputs relative to equilibrium depends upon the purpose and design of the model. In some cases the random component can be treated as an additional input, or as a component of an input with additional deterministic aspects. Treatment of such random components is conceptually no different than the simple input signals discussed above, in that disturbance from equilibrium depends upon the magnitude of the random component, and the temporal characteristics of the randomness. In particular, the likelihood of disturbance from equilibrium depends upon the strength of the random signal in various frequency ranges. Stochastic components that change slowly compared to the system response time are unlikely to cause departure from equilibrium. Components that change over timescales similar to the response time may cause disequilibrium if they are strong, and very high frequency stochastic inputs will affect the output through an averaged response. Mathematical modelling of system response to stochastic inputs is fairly tractable in the case of linear systems, and is discussed in several texts on stochastic processes, such as Papoulis (1984).

However, in cases where prediction is a goal of the model (rather than, say, identification of the response characteristics of the system), any random input component of large enough magnitude to affect the relationship between the deterministic inputs and the output would be considered to cause disequilibrium.

In some cases the stochastic component may be modelled as being part of the internal functioning of the system, that is, the response of the system to a well-defined input involves some level of unpredictability in the output. One source of such unpredictability might be uncertain knowledge of the initial state of the system, or lack of information on system inputs prior to experimentation or observation. This type of unpredictability, if sufficiently large, does indicate lack of equilibrium, since the criterion is a single-valued relationship between inputs and the output. Alternatively, the system may be very complicated and our model an inadequate description of system response. Complicated responses to simple inputs, for example, thresholds or oscillatory behaviour, generally imply that equilibrium concepts are inapplicable. However, the same physical system when viewed from another perspective, using different input and/or output variables, might exhibit equilibrium. An example from models of meandering streams is discussed below.

Errors in measurement of inputs and/or the output are another source of randomness. In some cases the measurements may appreciably disturb the functioning of the system. Because these are superimposed upon the system being modelled, the scientist would generally prefer to consider such random elements to be irrelevant to the question of equilibrium in the target system. However, and particularly in non-experimental situations, it may be difficult to refine the accuracy and precision of measurement. In fact, it may be impossible to distinguish measurement errors from random components of system inputs or system behaviour, with a resulting uncertainty about presence or absence of equilibrium.

Random components that are due to measurement errors in inputs or outputs have an important distinction from random inputs that affect the output via the system response. The system response transforms the random inputs in the same manner as deterministic inputs. Systems with 'memory', such as the averaging response of equation 1, cause correlation between successive output values, even if the inputs are uncorrelated. Kirkby (1987) shows that this introduces a type of Hurst effect in system response.

Systems that are areally very complex, and are therefore impossible to measure adequately, may be modelled as having spatial randomness. For example, permeability in an aquifer is often modelled as having random components at a spectrum of spatial scales from individual grains to aquifer-wide. However, the permeability can usually be considered to be temporally constant, so that it is possible to model steady-state response to constant recharge. Other geomorphic systems are not spatially fixed (such as meandering streams), so that areal randomness (e.g. of bank resistance) causes temporal randomness.

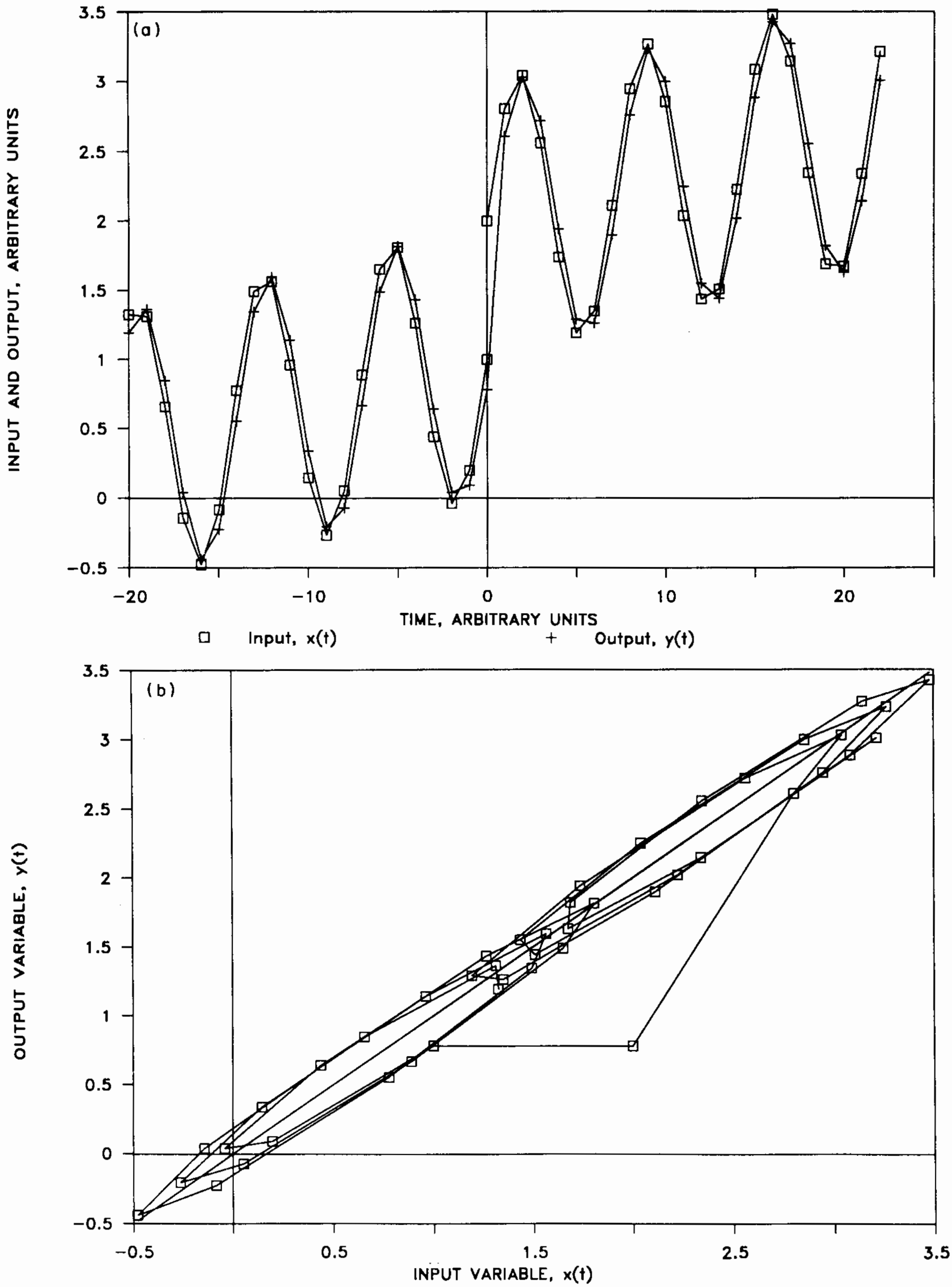

Figure 3.2. Output response for a linear system with finite memory and short relaxation time to an input with a combination of a trend, cyclical fluctuation, and step change: (a) Input and output as a function of time; (b) Output plotted as function of input, showing ultimate equilibrium response (straight line)

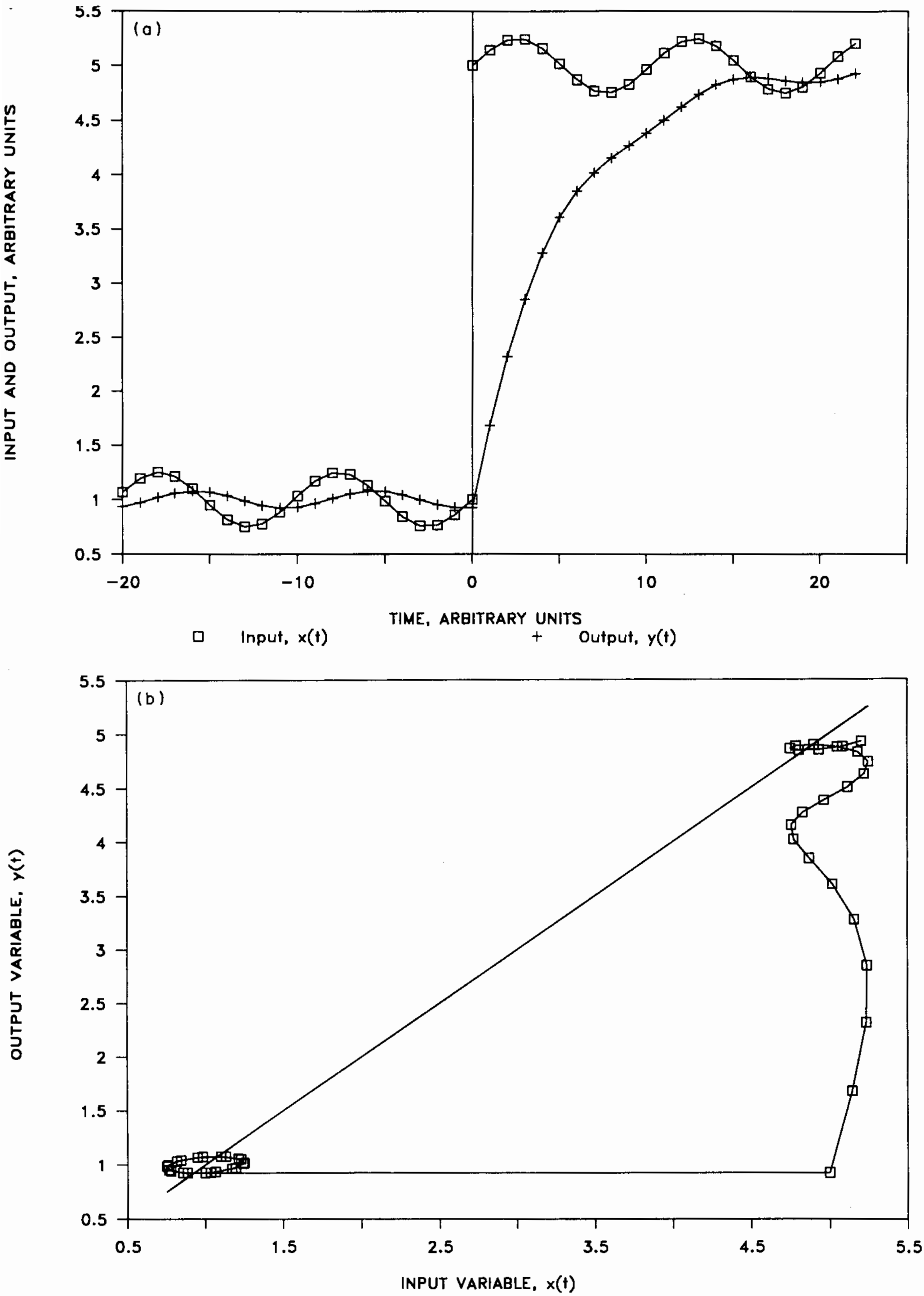

Figure 3.3. Output response to a large step input. See Figure 3.2 for further explanation

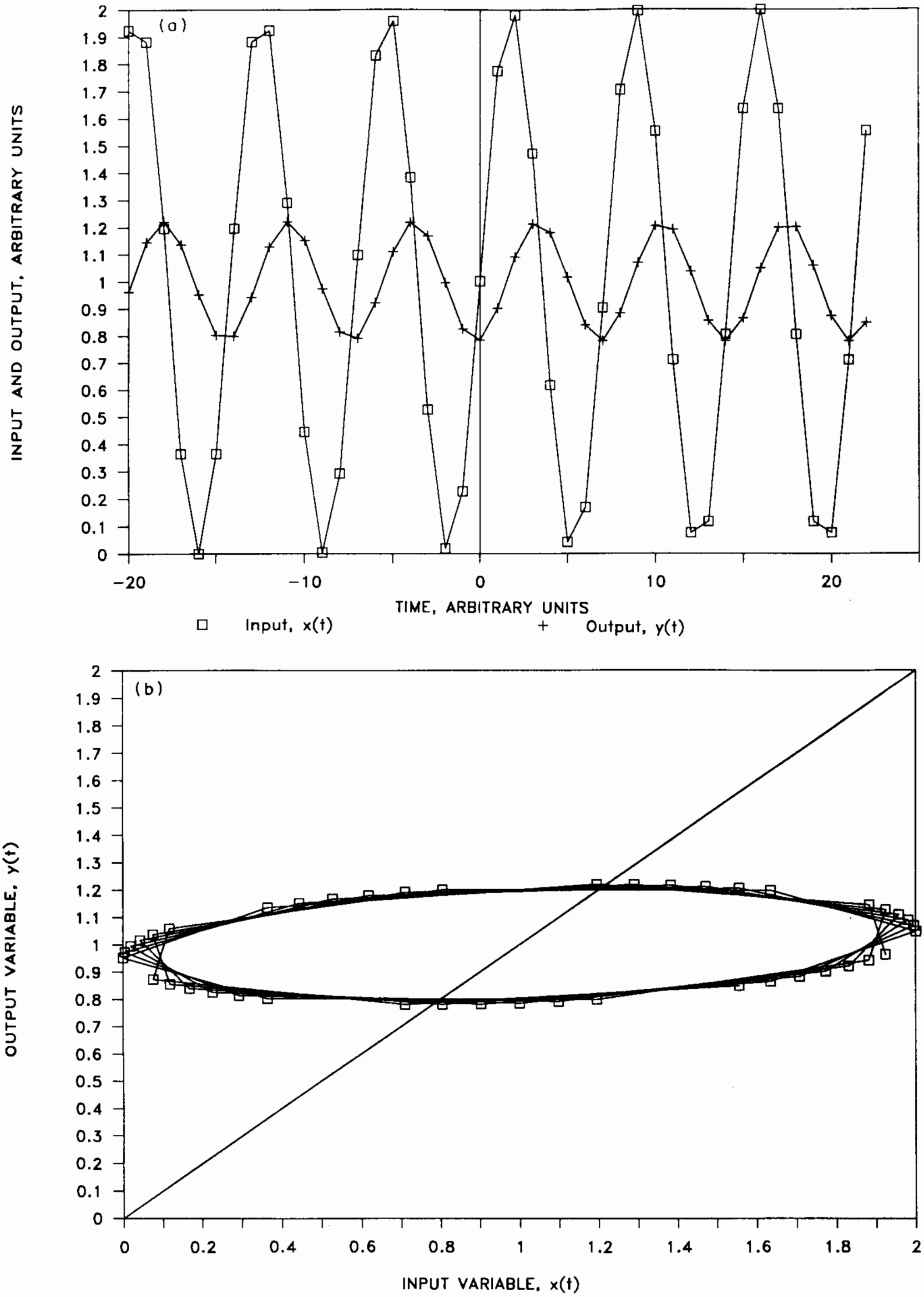

Figure 3.4. Output response to high-frequency cyclical input. See Figure 3.2 for further explanation

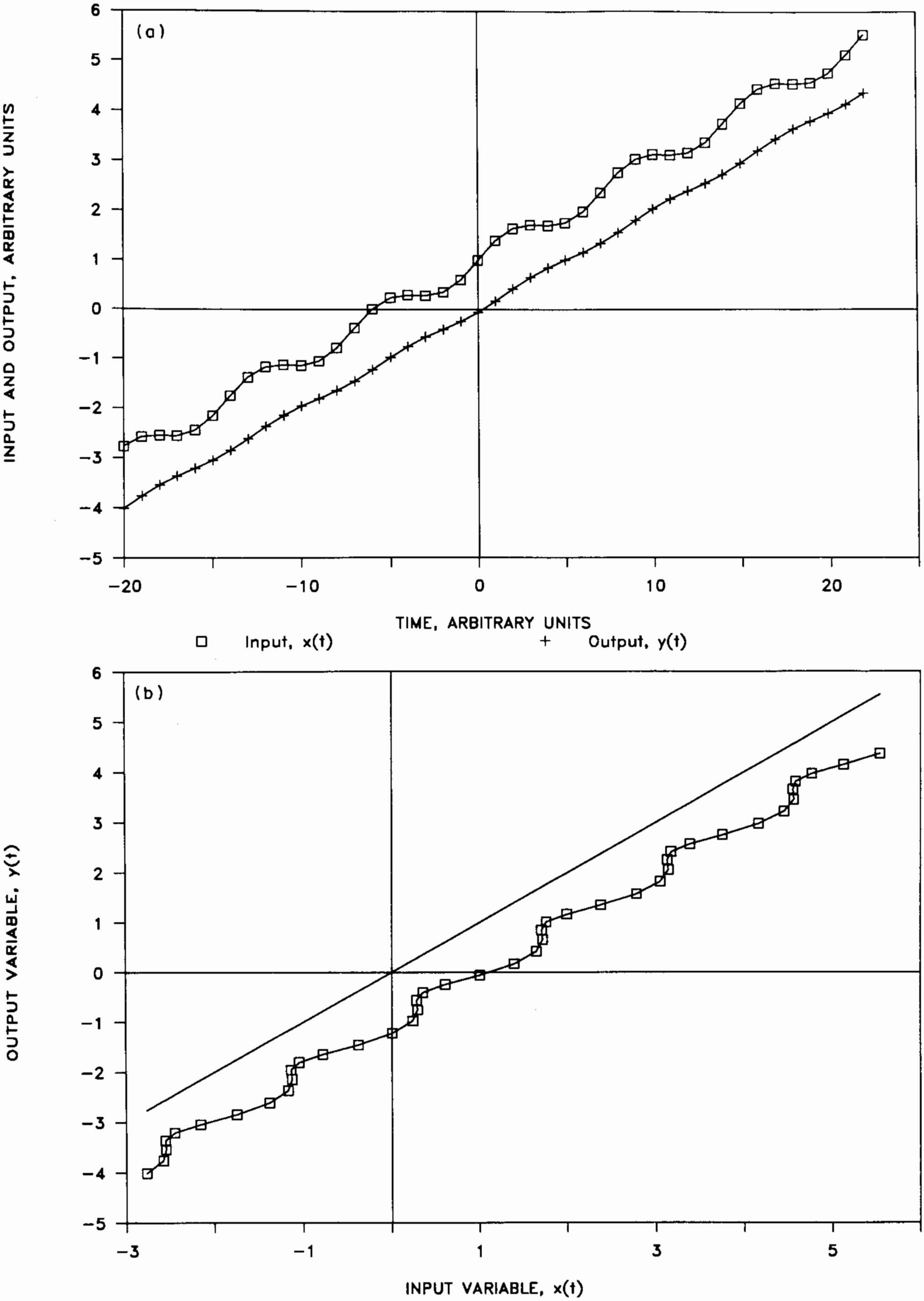

Figure 3.5. Output response to a rapid trend in input. See Figure 3.2 for further explanation

3.2.4 Identification of System Response and Equilibrium

Equilibrium and non-equilibrium behaviour of the simple linear system presented above is illustrated in Figures 3.2–3.5. Figure 3.2(a) shows a system input consisting of a sinusoidal component with a superimposed trend and a step change at time $t = 0$. A short relaxation time for the system has been chosen, so that the output closely follows the input and a plot of the input versus the output (Figure 3.2(b)) shows a nearly linear relationship with the only major deviation occurring immediately after the step change (in this and Figures 3.3b, 3.4b, and 3.5b the ultimate equilibrium value is the straight line of equality between input and output). In Figure 3.3 a large step change is coupled with a slow relaxation time, so that the plot relating input and output is complicated and multivalued. Figure 3.4 illustrates the amplitude damping and phase lag that occur when the system relaxation time is slow compared to the frequency of the input. The input is not strongly related to the input and strong hysteresis is shown in the input–output plot (Figure 3.4(b)). An input characterized by a rapid trend (Figure 3.5(a)) creates a bias in the relationship between the input and output (Figure 3.5(b)).

The simplest scenario for examining equilibrium behaviour occurs when observations of inputs and output are paired but are not sequentially linked. This arises commonly in experimental situations where the inputs are controlled and fixed at constant values and the output readings are taken when all transients have damped out. Each input and output pair result from a separate experimental observation. Many hydraulic and sediment transport relationships have been developed from such experimentation. The equilibrium behaviour of the system can therefore be determined if the experiment is well-defined (lack of uncontrolled inputs, suitable range of input variation, measurements with acceptable errors, sufficient length of system operation under constant input to assure equilibrium, etc.) and if the system is well-behaved (lack of oscillatory or threshold response within the range of input variation, lack of hysteresis, etc.). The main limitation of such experimentation is lack of characterization of transient system response.

Many non-experimental field data in geomorpholgy are analysed in a similar manner. Most hydraulic geometry relationships and morphometric studies fall in this category. Most, however, do not have the observational controls that permit the assertion of controlled, constant inputs and lack of transient responses. As a result, most such relationships exhibit large degrees of scatter. The difficulties of morphometric studies are well known, but a brief review is important to examine the implications for determination of equilibrium.

The causes of scatter are difficult to determine in morphometric and similar studies. One source may be transient response to changing inputs, such as is shown in Figures 3.3–3.5. Uncontrolled inputs are another source, as are measurement errors. If a random input or measurement error component were superimposed upon the responses of Figures 3.3–3.5 and the temporal relationships between observations were obscured, it would be difficult to distinguish between the effects of random components and the transient response to deterministic inputs. Thresholds and oscillatory response are also possibilities. Knowledge of the temporal evolution of a system in many cases permits more definitive modelling of sources of scatter, as will be discussed below.

Two assumptions are commonly made in morphometric studies. One is that underlying trends in the relationship of inputs and the output define the equilibrium response. The trends are commonly identified by statistical regression techniques. The second assumption is that equilibrium behaviour for temporal changes in an input variable can be defined by investigation of spatially separated, but similar systems that exhibit different values of the input variable; in short, this is the technique of the 'substituting of space for time'. These assumptions can be misleading if there exists covariant response of an uncontrolled variable to variations of the input variable. Similarly, the similar spatial systems investigated to determine equilibrium may be conjointly varying in response to a systematic change in the input variables that occurred prior to the sampling. For example, trends in inputs or recent step changes can cause a systematic bias in the response (Figures 3.2 and 3.5). An example of such historical influences is the large increases in channel width that occur in some semiarid streams following major floods (Schumm and Lichty, 1963). Hydraulic geometry studies in such basins will give different relationships if done before versus immediately after such a flood event. Good experimental design can minimize covariant uncontrolled inputs, and historical information about process variations from observations or the sedimentary record can help eliminate or control for large biases.

Considerably greater possibilities for system identification and prediction are possible if the temporal response of the output to changes in the input(s) can be observed. A wide range of time series analytical tools may be applied to determine the relationships between inputs and outputs, including analysis for trends, cross-spectral analysis, and ARIMA and transfer function models discussed by Box and Jenkins (1976). For example, the simple linear system of equation 1 can be represented by the differential equation:

$$y(t) + \frac{1}{\lambda}\frac{dy}{dt} = x(t), \tag{16}$$

which in turn can be expressed as a transfer model difference equation:

$$y_t\left(1 + \frac{1}{\lambda\,\Delta t}\right) - \frac{1}{\lambda\,\Delta t}\,y_{t-1} = x_t, \tag{17}$$

where Δt is the time increment, and the subscripts t and $t-1$ refer to the present and past observation times. Box and Jenkins (1976) detail methods for estimating the coefficients of such difference equations based upon time series observations of x and y, or more generally of the inputs and output. Such models will also estimate a stochastic component that is the unfit residual to the difference model, giving a measure of the degree of fit of the model to the observed process. These time series approaches can distinguish between scatter in the relationship between inputs and outputs that is due to deterministic departures from equilibrium (e.g. Figures 3.3–3.5) from those due to random components. The time series estimation procedures can be used to test the applicability of theoretical models to observed system behaviour or in a more general exploratory mode to identify the most parsimonious mathematical model that adequately describes the system behaviour. The time series approach is, in fact, more general than strictly equilibrium models since transient response also can be estimated.

3.2.5 Examples of Geomorphic Equilibrium Models

Several classes of models that employ equilibrium assumptions or illustrate requirements for equilibrium will be discussed below. Most of these are non-linear models that pertain to spatially extended systems. Some models involve stochastic components. Despite the more complex nature of these models when compared to the previous deterministic linear system, equilibrium behaviour is still a useful concept.

Equilibrium and Grade in Alluvial Streams

Howard (1982) provided an extended discussion of the conditions under which alluvial streams can approach an equilibrium between channel gradient and the hydraulic regime (the water discharge and the size and amount of sediment supplied from upstream). Howard simulated the response of alluvial stream profiles to various temporal histories of variation in the hydraulic regime, such as step changes in discharge or sediment load, pulse inputs of sediment, and sinusoidally varying discharges or sediment loads. He showed that alluvial channels will tend to return to equilibrium following a disturbance such as a pulse input or step change, and provided estimates of how the timescale for reestablishment of equilibrium depend upon the hydraulic regime and the length of the stream or stream network. In a manner similar to the linear system discussed above, cyclical regime changes having a long period compared to the response time of the stream do not cause disequilibrium. Also similarly, stream gradients do not appreciably respond to very high frequency regime changes. Howard also investigated the response of a stream to changes in base level and the timescales of readjustment.

The gradients of dentritic stream systems respond fairly uniformly throughout the network to changes in regime or base level, whereas in long, unbranched streams the gradient response to hydraulic regime tends to propagate downstream, and base level changes propagate upstream. Howard also found that Mackin's (1948, p. 471) classic definition of a graded stream as 'one in which, over a period of years, slope [gradient] is delicately adjusted to provide, with available discharge and with prevailing channel characteristics, just the velocity required for the transport of the load supplied from the drainage basin. The graded stream is a system in equilibrium . . .' is applicable to alluvial stream channels if 'a number of years' is replaced by 'a period of time commensurate with the relaxation time of the gradient'. That is, the timescale of measurement or averaging of input variables should be commensurate with the response time of the system if equilibrium relationships are to be tested or investigated.

Dynamic Equilibrium in Landform Evolution

Hack (1960, 1965, 1975) elaborated the concept of dynamic equilibrium in landform evolution. His hypotheses were never presented in axiomatic or mathematical form. However, the core of his papers is the assumption that *if* (1) a land area undergoes a constant rate of uplift, and (2) geomorphic processes (as affected by climate) remain constant, *then* the geometry of landforms attains a steady-state. Hack then further asserted that certain areas in the southeastern United States fulfilled these conditions for dynamic equilibrium. Dynamic equilibrium is not conceptually different from the general use of equilibrium in this paper, so that the term 'dynamic' is unnecessary baggage. The present discussion will not address the question of applicability of such equilibrium to any given area, but rather clarify the conditions under which equilibrium would occur and present examples of quantitative models of such equilibrium.

Some clarification of the definition given above is necessary, in that the output variable is not clear. The temporal independence of landforms can be made more precise by making the output variable the surface gradient at a particular location. Since the working definition of equilibrium restricts the application of equilibrium to a single output variable which is presumably measured at a given location, the hypothesis also involves the assertion that equilibrium is satisfied at all spatial locations simultaneously. Quantitative dynamic models of slope and channel evolution have been developed in recent years that can be utilized to investigate the geometric properties of equilibrium landforms. A recent example is a paper of F. Ahnert (1987), in which two dimensional slope profile simulation models incorporating process assumptions about mass-wasting and slope wash erosion are started from arbitrary initial conditions and iterated through time under assumptions that the base of the slope erodes at a constant rate. An example of this type of model is shown in Figure 3.6, where it is assumed that creep is the sole erosional process and that the rate of creep

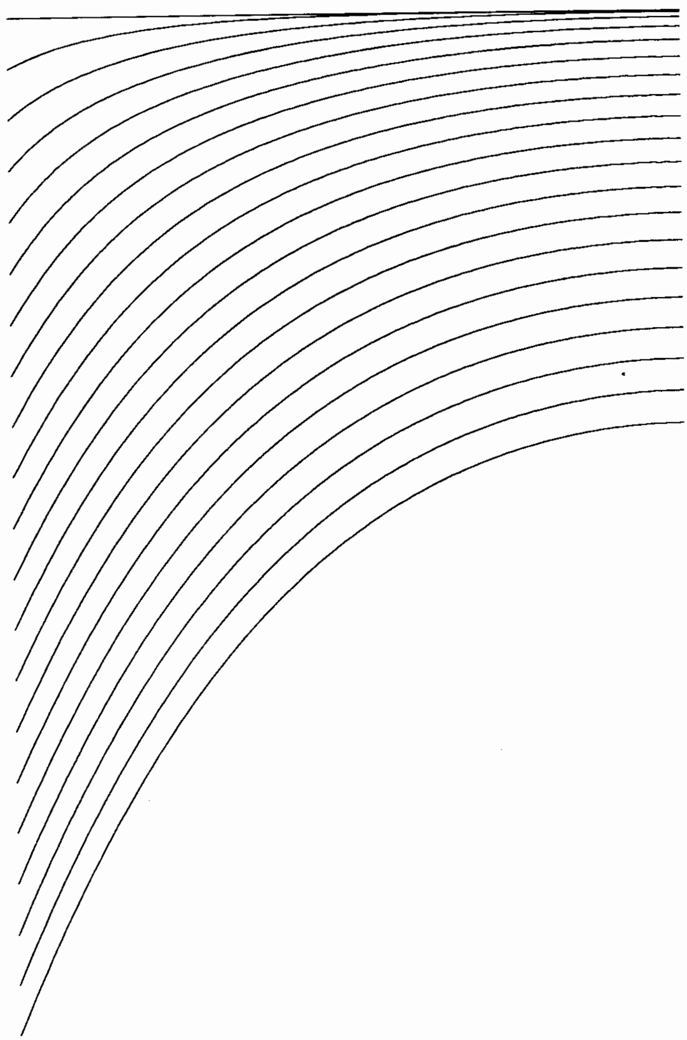

Figure 3.6. Evolution of a two-dimensional slope eroded solely by creep. Initial profile is level, and lower end of slope is eroded at a constant rate. Only the slope surface is shown for each profile

is proportional to the sine of the slope angle. Similarly, Figure 3.7 shows a slope profile in which two processes act: creep and surface runoff. Runoff erosion is assumed to occur at a rate proportional to the shear stress exerted by the runoff on the slope surface. The latter profile shows the inflection point and concave lower slope that is sculpted primarily by runoff (this concave zone typically contains rills and gullies on actual slopes). Such models have limited direct application to natural slopes due to simplified process assumptions, two-dimensional profiles, and the restrictive equilibrium assumptions outlined above. However, they serve several important purposes. Firstly, they demonstrate that fairly realistic assumptions about slope processes are compatible with the development of equilibrium slopes. The temporal and spatial feedbacks tend to create smooth slopes and gradual approaches to equilibrium that lack 'pathological' behaviour such as thresholds, oscillation, metastable states or unbounded change. Secondly, such models give an estimate of the length of time

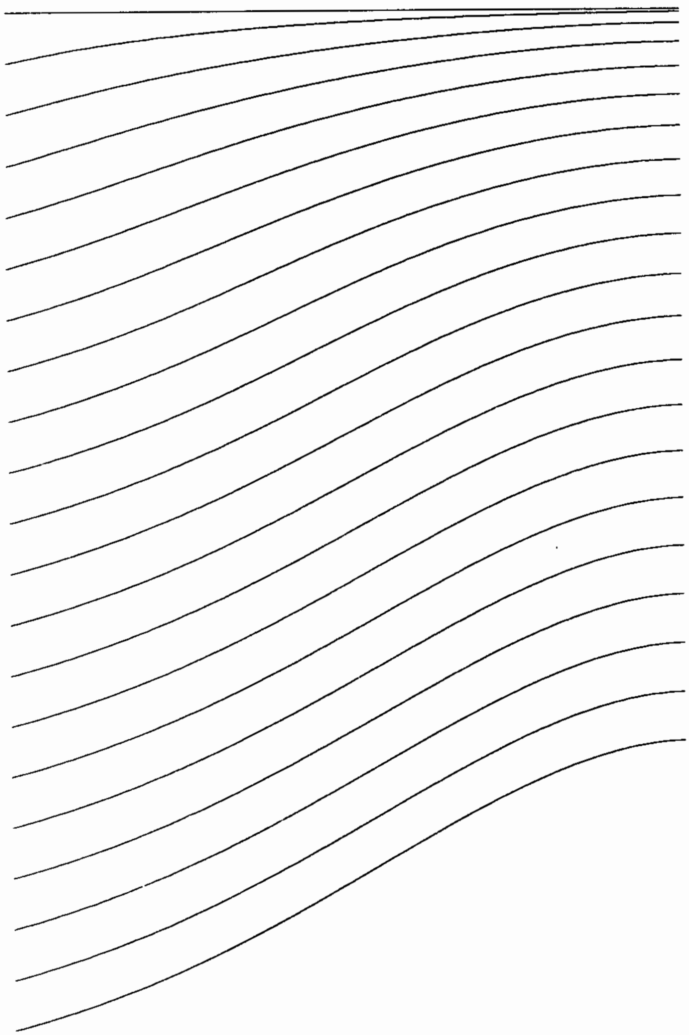

Figure 3.7. Evolution of a two-dimensional slope eroded partially by creep and partially by slopewash. Creep is the dominant process near the divide and slopewash predominates on the lower slope, with the changeover of relative importance occurring near the slope inflection

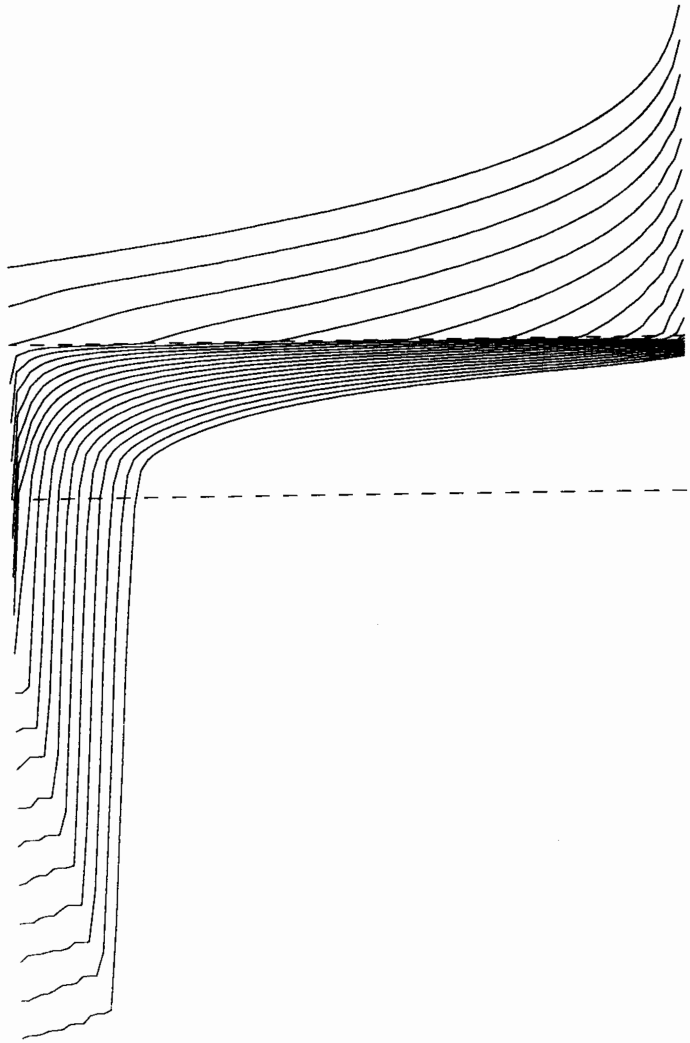

Figure 3.8. Evolution of a bedrock stream channel whose lower end downcuts at a constant rate. The discharge is assumed to increase downstream, so that the initial profile is concave. Bedrock resistance is assumed to be uniform except for the layer between the dashed lines, whose resistance is four times greater. Note that the variable resistance makes the spatial-temporal pattern of gradients and erosion rates non-uniform. The stream profiles exhibit slight wiggling or oscillation at small spatial scales that are an artifact of the simulation which would be removed by shorter temporal increments

(relaxation time) or total amount of erosion that is necessary to approach to a desired approximation of ultimate equilibrium (Ahnert, 1987). The spatial and temporal variation in erosion rates are a good index of the degree of approach to equilibrium. Finally, the equilibrium models help us to understand how the balance of processes controls the geometry of landforms. For example, in the simulations involving both creep and surface wash erosion, the drainage density (or its reciprocal, the constant of channel maintenance) functionally depends on location of the inflection point on the slope. Therefore such slope models can elucidate the process and material parameters that control spatial and temporal variations in drainage density.

The geologic materials being eroded at any given location must remain constant in erosional susceptibility in order for equilibrium to occur. When erosion progresses through layered rock, erosion rates and surface gradients vary through time. An example is shown in Figure 3.8, which is a simulation of channel erosion in which it is assumed that a bedrock-floored channel with discharge increasing downstream is being eroded at a rate proportional to the average boundary shear stress and inversely proportional to the rock resistance (Howard and Kerby, 1983). In this simulation a resistant rock layer inhibits erosion rates (and produces low gradients) above the layer, and produces very high erosion rates and steep gradients below and in the lower portion of the bed. Because the locus of exposure of the resistant bed migrates upstream, channel gradients are clearly not temporally constant.

The extension of such models from illustrating general concepts of slope erosion to application to specific natural slopes is problematic, as is suggested by the wealth of discussion that the Hack papers engendered. The first problem is the applicability of the process assumptions to the natural situation; this is a thorny issue beyond the scope of this paper. A very serious problem is the obvious lack of observation of temporal landform evolution that would be a direct test of equilibrium. Such direct testing could only be undertaken in very rapidly evolving landscapes, such as some badlands. Most of Hack's arguments in the southeastern United States were based upon substitution of space for time; the difficulties of this approach have been mentioned above.

Aside from temporal changes caused by the exposure of differing rock types, changes in climate are most likely to cause disequilibrium. Short-term climatic changes, and even individual storms, modify slopes and channels locally. This suggests that the definition of slope and channel gradients that would be tested for long-term equilibrium should involve spatial and temporal averaging at a scale equivalent to the natural relaxation timescale. In fact, some landforms exhibit threshold behaviour such as landslides at short timescales. Informative examples occur on the steep slopes of basaltic rocks on the Hawaiian Islands. Many slopes on the tradewind sides of the islands are very steep, with gradients averaging more than 60°. The abundant rain and easy weatherability of the basalt encourage rapid plant growth and soil development. But whenever an appreciable soil thickness develops, soil avalanches occur, triggered by the frequent intense rains, and the process begins anew. Therefore, over short timescales the slope behaviour is clearly episodic, but long-term average erosion rates and overall slope profiles may possibly maintain equilibrium.

The assumption of constant rate of tectonic uplift is probably untestable in most areas. However, a more tractable assumption may be substituted: the hypothesis that the regional rate of base level lowering has remained constant through time. In areas spatially and vertically remote from the oceanic base level, erosion rates and the overall relief are essentially unaffected by relative land–sea changes over timescales less than several million years. In such remote areas the timescales for readjustment of slope and headwater channel form are shorter than the timescales affecting overall relief. This should be true in areas where the local relief is much smaller than the elevation above base level. Thus individual basins should be undergoing erosion rates that are areally and temporally fairly uniform. But these erosion rates may differ significantly in contiguous basins if they drain through much different paths to the ocean. An example is the considerable relief difference across the Blue Ridge Scarp in southern Virginia and North Carolina (Hack, 1973).

In conclusion, the concept of dynamic equilibrium in landforms is of problematic applicability to natural landscapes, but it is an important and informative concept in theoretical modelling.

Equilibrium Models of Aeolian Dune Form and Migration

In contrast to the dynamic equilibrium of large regions, the application of equilibrium hypotheses to the form of sand dunes is better substantiated and process modelling is more realistic. Theoretical and empirical studies have concentrated primarily upon barchan dunes (Howard *et al.*, 1978; Howard and Walmsley, 1985; Wipperman, 1986), although some efforts have been made on field measurements and sand transport modelling on longitudinal dunes (Tsoar, 1983b) and other transverse dune forms (Tsoar, 1983a; Lancaster, 1985; Livingstone, 1986).

The barchan dune is an isolated dune forming on flat desert surfaces in essentially unidirectional winds. These dunes have been observed to migrate over long distances with little change in size or shape. Therefore, to a first approximation the barchan dune is a sedimentary landform on which the spatial pattern of erosion and deposition are balanced to maintain a constant geometry (so long as the coordinate system is considered to follow the migrating dune). This maintenance of form is of considerable help in testing theoretical models of dune evolution in that form equilibrium implies a specific areal variation in relative erosion and deposition rates over the dune that can be compared with model predictions. Furthermore, the barchan dunes migrate rapidly enough in most deserts that migration rates have been related to wind climate and dune size. The equilibrium assumption for natural barchan dunes requires that temporal variations in wind speed can be accounted for by an equivalent constant wind speed and that direction fluctuations are small. Because different wind speeds will, in general, be associated with somewhat different dune geometries, this may result in somewhat biased predictions. Some natural dunes will obviously satisfy these requirements more closely than others.

Howard *et al.* (1978) measured the areal variation in wind speed and wind direction over a natural and a model barchan and used sediment transport relationships corrected for slope effects to predict areal variations in sediment erosion and deposition rates. The predicted rates agreed in broad pattern with the rates predicted by the assumption of equilibrium. The limiting factor in this modelling approach was that the predictions were limited to the particular dune geometry for which wind measurements were taken, so that it is difficult to assess the reasons for the residuals in the prediction of erosion/deposition rates. Such residuals could have arisen from inappropriate model assumption about sediment transport mechanics, from errors in measurement of wind speed and dune geometry, and from the effects upon dune form of the naturally varying wind speeds and directions (a unidirectional oncoming wind of a fixed velocity was assumed in the modelling).

A more comprehensive modelling effort requires prediction of near-surface wind speeds and directions for a dune form of arbitrary geometry. This permits a more general simulation model in which the wind field is calculated followed by estimation of erosion and deposition are according to the sediment transport assumptions (thereby modifying the dune form), and the iterations of wind calculation and erosion/deposition are continued until an equilibrium form is obtained. Such an approach has recently been used by Howard and Walmsley (1985) and Wipperman (1986), with encouraging results. Such models require efficient algorithms for calculating the wind field as well as a powerful computer. The Howard and Walmsley simulations were troubled by numerical instability but nonetheless showed that a rounded pile of sand subject to a unidirectional wind began to take a barchanoid form. The Wipperman simulations were more successful, progressing to a realistic barchan form, but they employed a somewhat arbitrary scheme of sediment redistribution. When these models are perfected, it will be possible to test the implication of model assumptions and parameter variations on dune form, which should give insight into the factors controlling the size and shape of natural dunes, including the effects of variation of wind speed and direction.

In the dune simulation models, the occurrence of equilibrium offers a powerful test of model assumptions. Once the flow and sedimentary portions of the model are validated, the model can be utilized in more general, dynamic simulations.

Equilibrium Model of Saltation

Aeolian processes offer another example of equilibrium modelling at a shorter temporal and spatial scale that illustrate the coupling of deterministic and stochastic elements. Over the past 15 years several models of the process of sediment transport by saltation have been developed. A recent model by Ungar and Haff (1987) illustrates the necessity for assumption of one or more types of equilibrium in order to make theoretical modelling tractable. They make an elegant statement of their rationale for simplified steady-state modelling.

> 'The full theory of saltation is too complicated to have yet yielded to analysis, but from a simplified theory like that presented here it can clearly be seen how different parts of the saltation mechanism fit together. In particular one is interested in divining in a consistent way the self-limiting nature of the process. Such a schematic model, however, should not be judged too critically on its ability to make accurate numerical predictions, since they inevitably depend upon finer details which can be present only in a full theory. In as much as such a theory is lacking at the present time, the only way to achieve accuracy in prediction is through the use of semi-empirical formulae. The wholly empirical method has its place in practical applications, but it obtains its power at the expense of understanding. Therefore it is desirable to eschew that approach in favor of the more informative but less accurate model described in detail below . . .' (Ungar and Haff, 1987, p. 290).

One difficulty in developing a general analytical model of saltation arises from the intimate interaction between the near-surface wind and the saltating particles, such that the wind profile near the surface is modified from the usual logarithmic profile by the saltation process (Bagnold, 1973). Even more difficult to model is the impact of saltating grains on the surface and the reinjection of grains into the air.

The primary constraint that Ungar and Haff utilize in constructing their equilibrium model is that the saltation process be self-replicating, that is, on the average, the number and trajectories of grains ejected from the surface and accelerated by a constant wind shear should be reproduced following impact with the surface. Ungar and Haff present a general mathematical formulation of this constraint, but their model utilizes a particularly simple scenario that satisfies this condition. In particular, based upon empirical observations that impacts generally produce a single high-velocity rebound into the saltation population and a number of low-velocity grains that constitute the creep load, they assume that each impact generates a single new saltating grain with a velocity determined by the impact velocity, and that the population of saltating grains are all ejected with the same velocity, with the underlying assumption that the behaviour of an average or typical grain is representative of the population. Furthermore, they assume that ejection of grains in steady-state saltation is due solely to grain impact and not affected by surface wind forces.

The model employs several additional assumptions, such as a steady wind, the applicability of a mixing-length turbulent flow model, neglect of lift and spin forces on particle trajectories, lack of collisions or other direct interactions between saltating grains, a flat and unrippled surface, *inter alia*. However, these are not critical to the discussion of equilibrium aspects of the model, so they will not be discussed further.

The wind profile and saltation load for a given wind shear, particle density, and particle size are calculated by an iterative procedure subject to empirical constraints on the average saltation height and impact threshold (cessation of motion) shear velocity. Despite the rather oversimplified assumptions, the model predicts near-surface wind velocities similar to those actually observed and a reasonable dependence of transport flux upon shear velocity.

Models of Stream Meandering

Meandering is a quasi-oscillatory response of a stream that can occur in a constant hydraulic regime (e.g. the experimental studies of Friedkin, 1945). The temporal record of channel position or curvature at a given location is clearly not an equilibrium response. Furthermore, meandering in natural streams also involves complicated spatial variations because the meanders are somewhat irregular. This irregularity can be modelled by stochastic elements. Thus it may seem surprising that meandering can be an equilibrium response given a stable hydraulic regime for a suitable choice of input and output variables.

Models that employ stochastic or random elements are largely uninformative about single instances, but they become deterministic in predicting the properties of large samples and populations. For example, the kinetic theory of gases envisions random particle collisions, but it explains the pressure, volume, and temperature law for gas behaviour. In a geomorphic context, Shreve's (1966) theory of stream networks hypothesizes that any individual stream network of a given size (number of first-order tributaries) drawn randomly is equally likely to exhibit any of the topologically distinct network configurations for that size. Nevertheless, as the size of a sampled network increases, or as the number of sampled stream networks of a given size increases, averaged properties such as bifurcation, length, and area ratios tend to approach stable (equilibrium) values (Shreve, 1966, 1969).

A similar situation occurs for the quasi-oscillatory behaviour of meandering. The size and shape of individual meander loops is subject to wide variance, but size and shape statistics collected over large samples of loops approach stable population distributions if the environmental controls over meandering (the hydraulic regime and valley characteristics) remain constant along the sampled stream so that the statistics are stationary (Howard and Hemberger, in preparation; O'Neill, 1987). The change in spatial scale from output variables describing individual loops to variables measuring averages over many loops is an example of selecting the appropriate scale for testing for equilibrium.

Some progress has also been made in modelling of meandering in streams. Ferguson (1976) proposed a stochastic differential equation model for generating meandering stream patterns:

$$\theta + \frac{2h}{k}\frac{d\theta}{ds} + \frac{1}{k^2}\frac{d^2\theta}{ds^2} = \epsilon, \tag{18}$$

where θ is the local direction angle of the stream path, s is the location along that path, k is a wavenumber parameter, h is a shape, or damping parameter, and ϵ is a random forcing function, assumed to be normal Gaussian noise (Ferguson, 1976). Thus the important elements of the model are an oscillatory component, a 'memory' of past θ values which decays proportionally to h, and a random forcing component. The model can also be expressed as a second-order autoregressive model, and time series techniques can be used to estimate, or fit, the model parameters to natural streams as well as to test for spatial non-stationarity in meander characteristics. Howard and Hemberger (in preparation) and O'Neill (1987) show that the Ferguson model generates statistical meander properties similar to those of natural streams. However, the Ferguson model is not a kinematic model, since the stream pattern is generated sequentially downstream and migration behaviour is not simulated. Thus it is a model which is applicable only in 'explanation' of population meander characteristics.

More recently, kinematic models of meander movement have been proposed (Parker, 1984; Howard, 1984; Howard and Knutson, 1984; Ferguson, 1984). These models are based upon models of flow in curved channels and assume that bank erosion is proportional to bank shear stress. Such models can be used in a predictive sense for a given channel segment. They also can be used to predict population values of meander statistics (Howard and Hemberger, in preparation). Interestingly, present versions of these kinematic models are less successful than the disturbed periodic model in predicting population meander statistics (Howard and Hemberger, in preparation). The models show that the sporadic occurrence of cutoffs is as, or more, important than variations in bank resistance in creating irregularity in meander loops.

3.3 DISCUSSION

The definition of equilibrium given above suggests that the primary indication of equilibrium behaviour is a temporally invariant, single-valued relationship between the output and one or more input variables, with the provision that the relationship can manifest a consensual degree of variation. There are several ways in which the presence or absence of equilibrium may be of interest to the scientist. If the behaviour of the system is not well known, disequilibrium behaviour helps to reveal the characteristics of the system (that is, to illuminate the black box). If the goal is prediction, lack of equilibrium limits forecasting skills. Sometimes the system is well enough understood that it could be theoretically modelled, but lack of appropriate field data (or high cost of obtaining the data or running the model) prohibits application of the general models. In this case, the theoretical model can be examined to determine under what conditions an assumption of equilibrium is appropriate. In other cases, such as the saltation model, theoretical models may only be tractable if some type of equilibrium is assumed. In such models the relevance or accuracy of the equilibrium assumption needs to be determined. Testing of systems (experimentally or observationally) under controlled conditions can reveal the equilibrium behaviour of the system even if dynamic modelling is not possible.

Choice of variables characterizing inputs and outputs for natural systems depends upon the timescales of change and relaxation times of external parameters and system responses. Inputs or system behaviour that fluctuates rapidly relative to the timescale of interest are commonly assumed to be representable by an average, and possibly equilibrium value. For example, hydrogeochemical models of watersheds commonly assume chemical equilibrium. Similarly, long-term models of climatic change assume short-term thermal equilibrium at the Earth's surface. Models of channel scour and stream profile evolution in alluvial streams assume steady-state sediment transport and a dominant discharge that is a weighted average of natural discharges (Howard, 1982). Similarly, a model of valley development by groundwater sapping assumes effective groundwater flow can be represented by steady-state flow (Howard, in press). Such assumptions can, of course, be misleading or biased, and require justification.

Systems responding to slowly-changing inputs can often be assumed to remain in equilibrium. The most critical timescales of change of inputs are those commensurate with the response time of the output variable. Large trends or cyclical inputs occurring over this intermediate timescale cause disequilibrium, as do large step changes or impulses in inputs that have occurred recently as compared to the output response time.