# *Optimal Angles of Stream Junction: Geometric, Stability to Capture, and Minimum Power Criteria*

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*Abstract.* The angles of junction of streams adjust by erosion and sedimentation at the junction until the water surfaces and floodplains merge accordantly with each other and with the stream segment continuing downstream. A simplified geometric model of the zone of merger predicts angles of junction determined by the gradient relationships at the junction that are close to those in badland rill and gully networks. Merging streams with these predicted angles of junction should not be subject to discrete capture by each other or by their downstream continuation. Evaluation of angles of junction with regard to minimum rate of work performed by gravity produces normative equations for these angles that are unexplainedly similar to those predicted by the geometric model within the range of rates of downstream decrease of gradient typical of natural streams.

The regularity of the angular relationships at stream junctions was recognized early in the development of geomorphology; *Playfair* [1802, pp. 113–114] notes that '. . . this law is in general observed, that where a higher [steeper] valley joins a lower [gentler] one, of the two angles which it makes with the latter, that which is obtuse is always on the descending side; a law that is the same with that which regulates the confluence of streams running on a surface nearly of uniform inclination'. However, *Horton* [1932, pp. 358–360; 1945, pp. 349–350] was apparently the first to offer a quantitative model and rational explanation for angles of junction.

Horton supposed that tributaries to high order streams originate as rills on a planar hillslope abutting the stream. If the rill follows the line of greatest declivity on the slope, its angle of junction to the main stream (Figure 1*a*) depends on the ratio of the gradient (slope tangent) of the main stream $S_m$ to that of the hillslope $S_n$ by:

$$\cos Z = S_m/S_n \qquad (1)$$

where $Z$ is the angle of junction, measured on a horizontal plane [*Horton*, 1945, p. 349]. Playfair's generalization that $Z$ is acute is implied in equation 1. A map analysis by *Lubowe* [1964] shows that angles of junction are inversely related to the ratio $S_m/S_n$ as predicted by equation 1, and, by means of Lubowe's technique for gradient measurement, that the predicted angles are generally close to those observed in natural networks.

However, Horton's model now appears to be deficient in two respects: first, in the assumed natural processes determining angles of junction, and second, in the mathematics of the model:

1. Horton believed that angles of junction are determined at the time of origin of the stream network. However, *Schumm* [1956, p. 618] observed that angles of junction in badland topography change with the gradient ratio. Similarly *Morisawa* [1964, pp. 351–352] noted individual changes in angles of junction in newly formed stream networks on an upraised lake floor, although the average angle of junction did not change significantly. These observations imply that angles of junction are a dynamic rather than a static feature of the landscape. The drainage networks examined by *Lubowe* [1964] are probably very old, and the stream gradients have probably evolved through time; therefore a strong, systematic relationship like the one observed between angles of junction and present stream gradients would be unlikely if the angles of junction are remnant features.

2. According to equation 1, two streams of

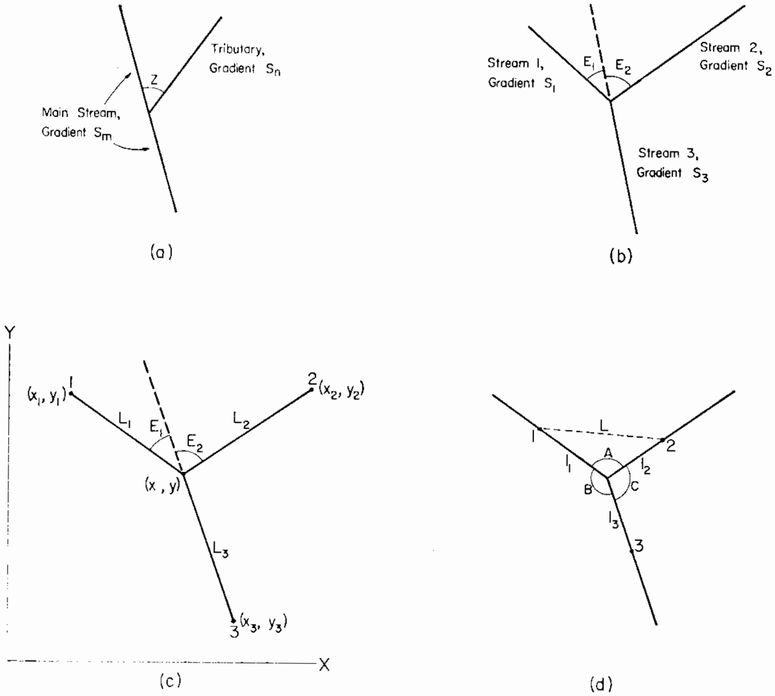


Fig. 1. Plan views of stream junctions illustrating terminology.

equal declivity should never meet except by chance because their predicted angle of junction would be 0°. But the angles of junction of equal order streams (of nearly equal gradient) are not close to zero and are systematically related to the angles of junction of low order streams and high order streams, both within and between drainage basins [*Lubowe,* 1964, pp. 332–337]. Horton's criterion apparently fails in this case because the gradient of the stream below the junction is unequal to the gradient of either incoming stream, and in general the gradient of the three segments may be unequal (Figure 1*b*).

The next section introduces a mechanism that may explain the adjustment of angles of cordant with that of the main stream at a sin- junction to changes in gradients at the junc- stream, the water surface can only be con- tion, and the subsequent section amends Horton's criterion so that it predicts angles of junction more closely.

## ADJUSTMENT OF ANGLES OF JUNCTION

Streams, especially in flood, are nearly always more wide than deep, and the water surface of a stream without rapids is nearly horizontal at right angles to the direction of streamflow. If a low order tributary enters a main gle angle, i.e., that given by equation 1. If the tributary approaches the main stream at an angle less than that predicted by (1), the flow in the tributary will be diverted toward the main stream in the vicinity of the junction. The resulting oversteepened gradient on the upstream side of the junction causes rapid erosion, whereas aggradation (lessened erosion) occurs on the gentler downstream side. As a result the point of junction will migrate upstream, increasing the angle of junction until it matches that predicted by equation 1. In like manner a tributary at an angle greater than that predicted will erode on the downstream side.

Similarly a wide floodplain is analogous to Horton's postulated hillslope, and the tributary tends to follow the line of steepest gradient given by equation 1.

## ANGULAR RELATIONSHIPS AT JUNCTIONS

When two streams join to form their single downstream continuation, the angular relationships at the junction are given by the gradients of the three segments and any two angles between the streams. The angles chosen here are the projections on a horizontal plane of the angles made by the two incoming streams with the upstream extension of the downstream segment (Figure 1*b*), designated $E_1$ and $E_2$.

The Horton criterion is modified below by hypothesizing that each incoming stream obeys equation 1 separately, the main stream in that equation being the segment below the junction (stream 3, Figure 1*b*;Figure 2).

If $E_1$ is the angle of junction for tributary 1 of gradient $S_1$ and the continuing steam has a gradient $S_3$, then

$$\cos E_1 = S_3/S_1 \tag{2}$$

And if $E_2$ is the angle of junction for tributary 2 of gradient $S_2$, then

$$\cos E_2 = S_3/S_2 \tag{3}$$

Therefore the angle between the two tributaries equals $E_1$ plus $E_2$. In particular if two streams of equal declivity join to form a continuing stream of lesser gradient, then the predicted angles of junction are greater than 0°. Should the gradient ratio $S_3/S_1$ or $S_3/S_2$ be greater than unity, the respective predicted angle $E_1$ or $E_2$ would be 0°.

This model must be only a first approximation to the actual processes and geometric relationships at river junctions. Specifically the patterns of sedimentation and erosion by mixing waters are probably complex, and the two streams meet, not as two planes intersecting at a line, but in a triangular zone of complicated geometry. Although these handicaps are serious, the field data introduced below suggest that the model is more accurate in predicting angles of junction of badland rills and washes than the Horton model.

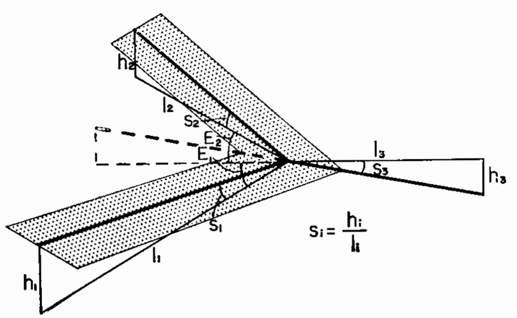


Fig. 2. Junction of two streams that meet concordantly with the segment of the stream continuing downstream. For values of the angles of junction $E_1$ and $E_2$ satisfying the relationships between $S_1$, $S_2$, and $S_3$ given by (2) and (3), the stippled planes horizontal at right angles to the streams that they pass through meet without angular discordance at the segment continuing downstream (stream 3) and at the hypothetical extension of this segment upstream from the junction (indicated by the dashed line).

*Prediction of angles of junction.* Rill and gully gradients and angles of junction were measured at 102 sites on shale badlands developed on Mancos and Morrison shales near the Henry Mountains of southeastern Utah. These sites ranged from small rill networks, perhaps ephemeral, with gradients up to 45° to small, deeply incised gullies with gradients of a few degrees. Gradients were measured with a 12-inch triangular ruler placed in the rill immediately upstream or downstream from the junction. This method of measuring gradients contrasts with that of *Lubowe* [1964, pp. 337–338], who classed junctions by the order of the streams involved and used the average gradient of all *n*th order streams over their entire length in equation 1. The present use of gradients measured close to the junction seems most appropriate if angles of junction are determined by processes acting at the junction. The angles of junction were computed from the azimuth of the stream segments immediately above and below the junction.

The sites selected were not a random sampling of all rill junctions, for the majority of such junctions involve a small tributary entering a main stream so that the gradient of the main stream is little affected by the entrance of the tributary. In such a case one of the gradient ratios of equations 2 and 3 would be nearly unity, and the predicted angle would be very close to that given by equation 1. Rather, rill and gully junctions were selected such that the size of the incoming streams was approximately equal, as judged from rough equality of length, drainage area, and/or width of the incoming channels.

In addition some junctions were eliminated because (1) the rill or gully segments were strongly curved or meandering, (2) the gradients of the rills involved were very irregular, or (3) several rills joined at almost the same location. About 30% of those junctions involving equal size streams were eliminated because of meandering or uneven gradient.

The angles of junction calculated directly from compass readings ($E_1$, $E_2$, or their sum) are termed observed angles, whereas the predicted values are calculated from the gradient ratios by means of equations 1–3. The observed angle subtracted from the predicted angle gives the difference. The bias is the average difference, and the variance and standard deviation of the differences are similarly defined.

The method of application of Horton's criterion (equation 1) to the junction of two streams of nearly equal gradient is uncertain, for neither gradient necessarily remains unchanged through the junction. Two methods of computation were adopted. In the first (Horton 1) $S_m$ is the gradient of the downstream segment and $S_n$ is the gradient of the steepest incoming stream. In the second (Horton 2) $S_m$ is the average of the gradient of the downstream segment and the lower gradient of the incoming tributary and $S_n$ is defined as above. The predicted values of the angle of junction are compared with the total observed angles ($E_1 + E_2$) for the 102 sites. Equations 2 and 3 give two predictions at each site.

The most easily applied test for bias in prediction uses the sign test [*Dixon and Massey*, 1969, pp. 335–340] under the hypothesis that an unbiased estimator should on the average as often overestimate as underestimate the angle of junction. Of the 204 angles predicted by equations 2 and 3, 100 gave positive differences. This proportion is insufficient to reject the null hypothesis at more than a 99% level of significance. With a sample size of 204 the sign test can distinguish between the null hypothesis and an alternative hypothesis that the probability of overpredicting (or underpredicting) is 0.6 or greater at a 95% level of significance. However, both Horton 1 and Horton 2 universally underpredict the 102 total angles of junction and thus fail the sign test at more than a 99% level of significance.

The differences between the observed and predicted angles may be caused by (1) the technique of measuring the channel bottoms of the formerly dry gullies rather than the water surface in times of flow, which introduces measurement variance and perhaps bias; (2) errors in measuring rill gradients due to the crumbly nature of the weathered shale; (3) random factors in hydrology and bedrock resistance that cause variations of the gradients through time (if these gradient changes are more rapid than the adjustment of angles of junction, then random differences would be expected); and (4) errors in measuring the azimuth.

The fourth cause is probably less important than the first three because errors in measuring azimuth (less than 2°) are much smaller than the observed standard deviation of the differences (17.5°). The transformations in equations 2 and 3 complicate the analysis of the effects of causes 1–3. To investigate these effects a statistical model is introduced based on the following assumptions:

1. The observed angle of junction corresponds to the angle predicted by the mean value of the true gradients in equations 2 and 3. Furthermore, errors in measuring the true stream gradients are assumed to be unbiased.
2. Random variations of gradient and errors in its measurement occur independently in each segment entering or leaving the junction and are normally distributed.

No satisfactory analytic expression was found for the distribution of predicted $E_1$ and $E_2$. This distribution was simulated by using a computerized random normal number generator for different mean values and standard deviations of the gradients in equations 2 and 3. Two alternative assumptions were made about variance in the rill gradients:

1. A constant ratio of the population standard deviation $d$ to the mean value of the rill gradient $S$ (constant $d/S$, Figure 3) is assumed. The average predicted angle is in general biased. The amount and sign of the bias, as well as the standard deviation of the differences, vary with the gradient ratio and with $d/S$.
2. A constant standard deviation of the rill gradients (constant $d$, Figure 3) is assumed. The amount of bias and the standard deviation of the differences depend not only on the gradient ratio and on $d$ but also on the magnitude of the gradients.

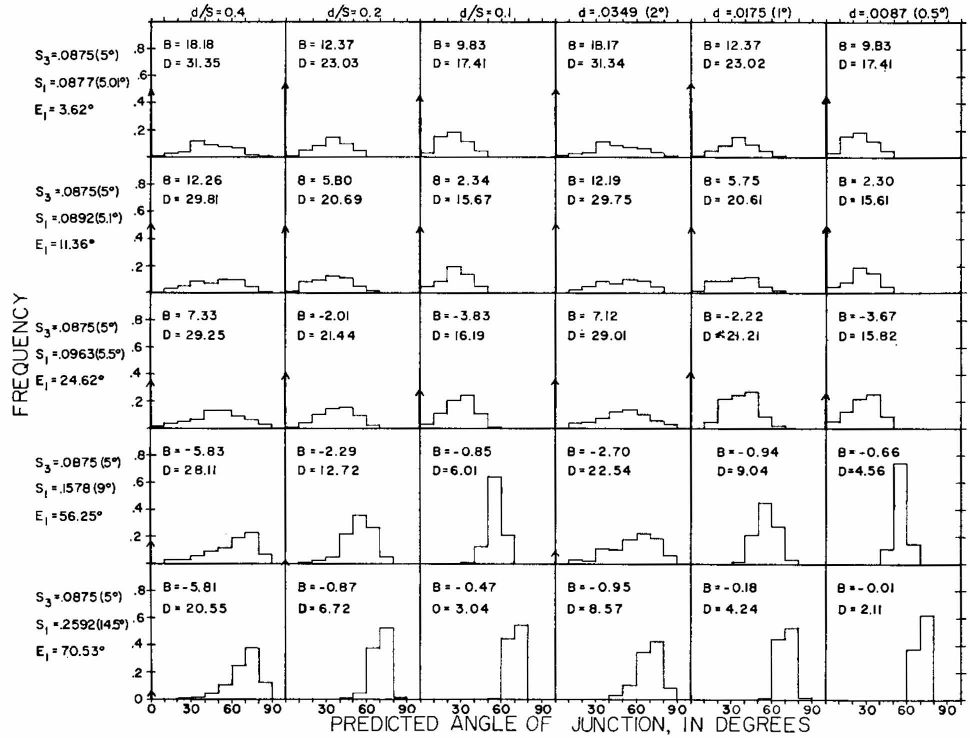


Fig. 3. Frequency distributions of predicted angles of junction for normal, random variations in gradients above and below the junction. The mean values of the stream gradients above and below the junction are given by $S_1$ and $S_3$, respectively. The expected angle of junction $E_1$ is given by substituting $S_1$ and $S_3$ in (2). The standard deviation of the stream gradient is given by $d$, the bias in predicting $E_1$ by $B$, and the standard deviation of the differences by $D$. The vertical arrow on the left-hand side of some frequency distributions indicates the frequency of predicting an angle of junction of 0° ($S_3 \geq S_1$). The first three columns assume a constant value of the ratio of $d$ to the stream gradient; the last three columns assume constant $d$.

The distributions of predicted angles are similar under both hypotheses, because both hypotheses give similar distributions for nearly equal gradients (high gradient ratios). For low ratios the variance of the lesser, downstream gradient predominates.

Figure 3 demonstrates that the average bias and the standard deviation of the differences are not adequate measures of the predictive ability of equations 2 and 3 unless the data are grouped by the value of the observed angle of junction. The distributions of the predicted angles of junction, the bias, and the standard deviation of the differences of the field data (Figure 4) vary with the observed angle in a manner similar to that of the model (Figures 3 and 4). This result suggests that causes 1–3 predominate in producing the differences and that the variations in the gradients are nearly normally distributed. However, the simulation model predicts a small bias for large observed angles of junction (greater than 50°), whereas the field data have a large negative bias. However, because the number of large angles of junction in the field data was small, they may have been influenced by such factors as structural influences or slight meandering.

For constant $d/S$ the standard deviation of the differences should not vary with the gradient, but if $d$ is constant, it decreases strongly with increasing gradient. The field data indicate a slight decrease in the standard deviation of the differences with increasing $S_3$ (Table 1), but the decrease is much smaller than

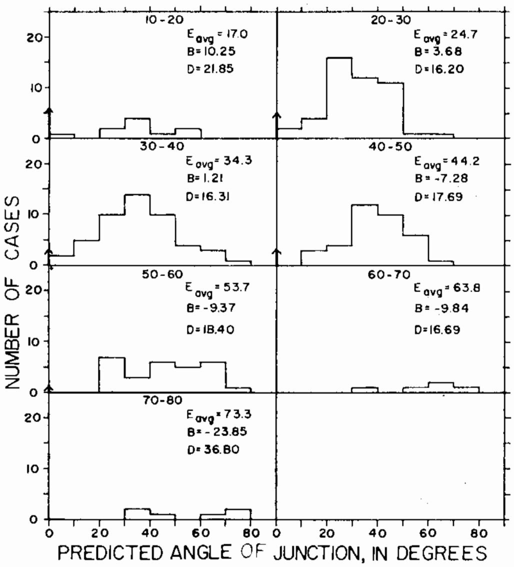


Fig. 4. Frequency distributions of angles of junction predicted from field measurements (using equations 2 and 3) grouped by 10° intervals of observed angle of junction. The average value of observed angle of junction within each range of 10° is given by $E_{avg}$ (cf. Figure 3).

would be expected for constant $d$. Therefore constant $d/S$ is the better model, and comparison of Figures 3 and 4 shows the appropriate value of $d/S$ for the natural junctions to be about 0.3. As would be expected from the model, the bias shows very little systematic variation with $S_3$ (Table 1).

In another test of equations 2 and 3, 81 stream junctions were sampled in the drainage network of Bull Creek on the Hyden West 1:24,000 topographic quadrangle, Kentucky (Figure 5). This area was selected because the gradient decreases rapidly downstream so that the angles of junction are large and can be measured with a small percentage of error. Junctions of streams of approximately equal length and drainage area were selected so that the predictions of the Horton criterion (equation 1) and the present one (equations 2 and 3) could be readily distinguished.

Equations 2 and 3 overestimated the total angle of junction in 63 of 81 cases, whereas the Horton 1 and Horton 2 criteria underestimated it in 72 and 75 of 81 cases, respectively. All these criteria therefore failed the sign test at a high level of significance (more than 99%).

The overprediction by equations 2 and 3 is probably explained by the method of measuring gradients, which were estimated by contour spacing at some distance above and below the junction. Because the gradient decreases downstream, it was probably overestimated for the incoming streams and underestimated for the segment continuing downstream, the result being a large positive bias.

The apparent confirmation of the Horton criterion (equation 1) by the map analysis by *Lubowe* [1964, pp. 337–338] was possibly caused by her use of gradients averaged by order, the result being a smaller gradient ratio than that at the junction; this result causes large predicted angles and compensates for the use of equation 1 instead of equations 2 and 3.

*Angles of junction as related to stream discharge.* Many stream systems in homogeneous lithology have gradients that are closely related to average stream discharge by an equation of the following type:

$$S = KQ^u \qquad (4)$$

where $S$ is the gradient, $Q$ is the discharge, and $K$ and $u$ are parameters. The exponent $u$ generally ranges between −0.2 and −1.0 in natural stream networks for mean annual discharges or mean annual flood [*Leopold,* 1953, p. 619; *Wolman,* 1955, p. 26; *Leopold and Miller,* 1956, pp. 25–26; *Carlston,* 1968]. In drainage basins for which equation 4 is ap-

TABLE 1. Statistics for Predicted Angle of Junction Grouped by Gradient of the Downstream Segment $S_3$

| | $S_3$ = 0° − 5° | $S_3$ = 5° − 10° | $S_3$ = 10° − 15° | $S_3$ = 15° − 20° | $S_3$ = 20° − 25° | $S_3$ = 25° − 30° |
|---|---|---|---|---|---|---|
| Number in sample | 40 | 74 | 44 | 24 | 12 | 8 |
| Average bias | −4.8 | −2.5 | +1.0 | +5.0 | −5.8 | −0.03 |
| Standard deviation of biases | 20.4 | 17.8 | 17.2 | 13.2 | 15.2 | 12.1 |

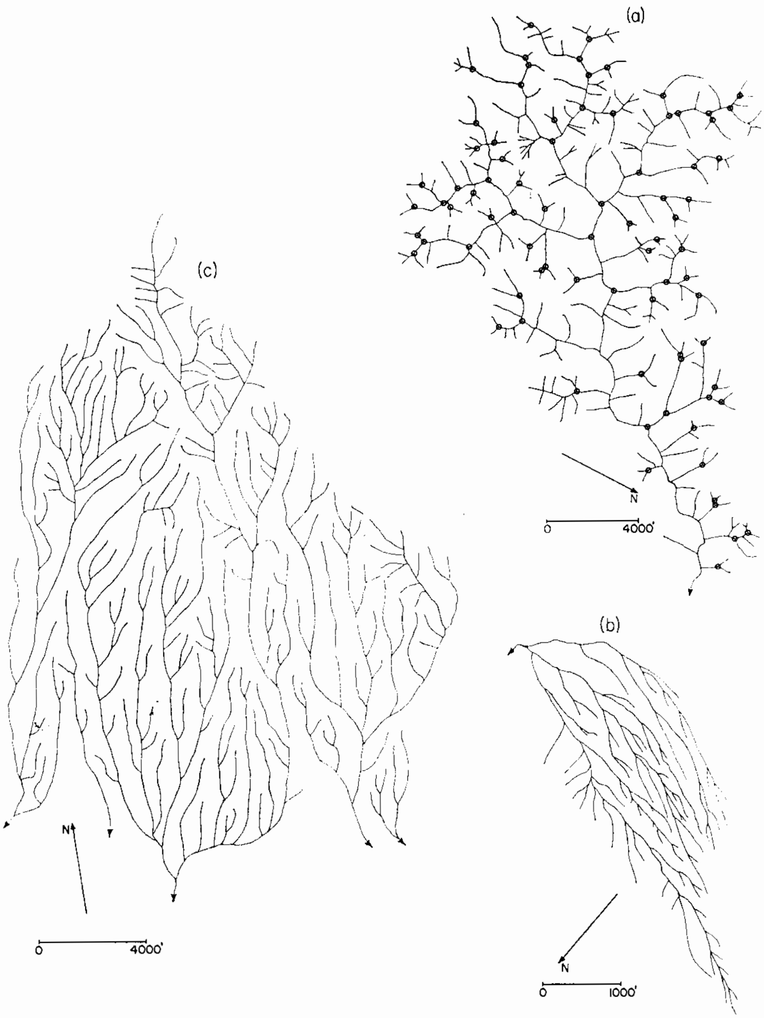


Fig. 5. Natural stream patterns. (*a*) Bull Creek and tributaries, Hyden West 7½ min quadrangle, Kentucky (circles are junctions sampled for measurements). (*b*) Drainage on a very slightly dissected pediment formed during the early Wisconsin (from an air photo of a portion of the Mt. Ellen 2NW 7½ min quadrangle, Utah). (*c*) Portion of the drainage on the Los Olivos 7½ min quadrangle, California (the fact that many of the merging streams curve toward each other above the junction suggests that past decreases in the gradient ratios (equations 2 and 3) resulted in upstream migration of the points of junction).

propriate, its substitution into equations 2 and 3 reduces the number of variables from three to two (for a given value of $u$):

$$\cos E_1 = \left(\frac{Q_1 + Q_2}{Q_1}\right)^u \tag{5}$$

and

$$\cos E_2 = \left(\frac{Q_1 + Q_2}{Q_2}\right)^u \tag{6}$$

where the subscripts refer to the two streams entering the junction.

*Effect of angles of junction on stream patterns.* If streams adjust their angles of junction to their respective gradients, then the areal pattern of the stream network should be influenced by this tendency. However, each stream junction is also influenced by adjacent junctions as well as by at least two other systematic factors:

1. A straight line with varying degrees of superimposed meandering is the most stable connection between two points on a stream lacking intervening junctions. Capture (piracy) tends to straighten indirect courses [*Howard*, 1971].

2. In areas of homogeneous lithology the drainage density tends to be uniform, so that the stream network expands into areas of relatively poor drainage and retreats from areas with exceptionally high drainage.

These tendencies probably cannot be satisfied simultaneously in a stream network; the resulting pattern bears an imprint of each factor in proportion to its importance. The influence of angle of junction adjustments is particularly striking when the gradient decreases rapidly downstream (highly negative $u$), so that angles of junction are large (Figure 5*a*). Such a drainage pattern shows the following regularities:

1. If a major tributary enters a main stream, then the main stream is deflected toward the tributary so that the direction of the main stream changes at the junction.

2. If most tributaries enter the main stream from one side, then the main stream curves away from that side.

At the other extreme, when gradients change very slowly downstream, angles of junction are small and the tributary streams are nearly parallel (Figure 5*b*).

When an angle of junction adjusts to a change in gradient ratio by shifting the point of junction, bends are produced in the segments entering (and possibly leaving) the junction. Because of the greater stability of straight stream courses, the bends in the channels should be lessened through time by lateral shifting of the channels upstream from the junction. However, a small angular change at a junction requires a large lateral shift at points far from the junction; these portions will probably not straighten. In some stream networks systematic gradient changes and angle of junction adjustments may have occurred throughout the stream network, because most junctions show similarly curved incoming tributaries (Figure 5*c*).

In addition to the preceding factors, numerous local influences, such as lithology, structure, microclimate, and past erosional history, affect angles of junction. These local factors, for example, streams imprisoned in deep canyons and trellis drainage patterns, occasionally dominate the development of angles of junction. Stream meandering introduces irregularity to angles of junction in proportion to the degree of sinuosity.

In arid environments tributaries to large channels commonly produce a small alluvial fan in the floodplain near the junction. The volume of deposited sediment may be sufficient to divert the main stream away from the tributary rather than toward it, the result being systematic errors in the predictions of equations 2 and 3.

## STABILITY TO CAPTURE

Under certain conditions a change of angle of junction may occur, not by gradual shifting of the point of junction due to differential sedimentation and erosion, but by discrete shifting, or capture, of one stream segment at the junction by another. For example, for small values of the angle $A$ (Figure 1*d*) the gradient from point 1 to point 2 may be greater than the gradient from point 1 to the junction, if $S_1 > S_2$ is assumed. In such a case advantageous capture [*Howard*, 1971] might occur along the dashed path in Figure 1*d*, the result being changes of angles of junction. It is important, therefore, to ask whether the angles of junction predicted by equations 2 and 3 would be immune to advantageous capture.

If the channel gradients are such that $S_1 \geq S_2 \geq S_3$, then stream 1 (Figure 1*d*) might be subject to advantageous capture by stream 2 or stream 3, and stream 2 by stream 3. In the first case, where $H$ is the vertical drop and $L$ is the horizontal distance between points 1 and 2, then (Figure 1*d*):

$$H = S_1 l_1 - S_2 l_2 \tag{7}$$

and

$$L^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos A \tag{8}$$

For a fixed point 1 on channel 1, the point 2 on channel 2 at which the gradient is a maximum can be found by taking the partial derivative of $H/L$ with respect to $l_2$:

$$l_2 = l_1\left(\frac{S_1 \cos A - S_2}{S_1 - S_2 \cos A}\right) \qquad (9)$$

If $l_2$ is zero or negative (i.e., along the imaginary downstream projection of stream 2 beyond its junction with stream 1), then $S_1$ must be equal to or greater than the gradient to any point upstream from the junction, and advantageous capture is impossible. Because $l_1$ and the denominator on the right-hand side are positive, advantageous capture is not possible if

$$S_1 \cos A - S_2 \leq 0 \qquad (10)$$

Equation 10 is always satisfied for angles of junction obeying equations 2 and 3, so that advantageous capture cannot occur.

Because equations 2 and 3 are derived from the assumption that each incoming stream follows the path of steepest gradient to the junction, advantageous capture of points 1 or 2 by point 3 (Figure 1*d*) cannot occur (this relationship may also be shown by an analysis parallel to that resulting in equations 9 and 10).

### MINIMUM POWER ANGLES OF JUNCTION

Several authors have examined minimum rate of work criteria for stream networks [*Leopold and Langbein*, 1962; *Langbein*, 1964; *Woldenberg*, 1969; *Smart*, 1969, p. 1763]. Angles of stream junction may be examined with regard to the rate of work (power) done by gravity on the water flowing into and out of the junction. If rate of flow varies only slowly with time, then the rate of work per unit time in a segment of stream with no inflow or outflow of discharge is proportional to the product of gradient, discharge, and length of the channel segment (measured on the horizontal).

When two streams (1 and 2) merge to form a third (3) such that they pass through the respective points $(x_1, y_1)$, $(x_2, y_2)$, and $(x_3, y_3)$ (Figure 1*c*), the power may be a minimum if the junction is at some point $(x, y)$. Only the immediate vicinity of the junction will be considered, so that direct addition of discharge to the three segments is negligible. The rate of work in the three segments involved in the junction $W_t$ will therefore be given by:

$$W_t = rg[L_1S_1Q_1 + L_2S_2Q_2 + L_3S_3(Q_1 + Q_2)] \qquad (11)$$

where $r$ is the fluid density, $g$ is the gravitational constant, and $L_i$ are segment lengths. Minimum power angles of junction may be derived from this equation by the following steps:

1. The $L_i$ terms are expressed in terms of the $X$–$Y$ coordinate system (Figure 1*c*).

2. The derivative of $W_t$ is taken with respect to both $x$ and $y$ and the resulting equations are set to zero; the terms of the two equations are appropriately arranged and the equations are squared; the cosines of $E_1$ and $E_2$ are expressed as $X$–$Y$ coordinates; and the four resulting equations are manipulated to eliminate the terms in $(x, y)$ and $(x_i, y_i)$, the result being analytic equations for the angles of junction:

$$\cos E_1 = \frac{S_3^2(Q_1 + Q_2)^2 + S_1^2Q_1^2 - S_2^2Q_2^2}{2S_1Q_1S_3(Q_1 + Q_2)} \qquad (12)$$

and

$$\cos E_2 = \frac{S_3^2(Q_1 + Q_2)^2 + S_2^2Q_2^2 - S_1^2Q_1^2}{2S_2Q_2S_3(Q_1 + Q_2)} \qquad (13)$$

Angles of junction predicted by equations 12 and 13 can be compared with those given by equations 2 and 3 if equation 4 holds within the stream network. In such a case the gradients may be eliminated from equations 12 and 13, the result being two equations in which the discharges and the parameter $u$ are the only variables. The angles predicted by the minimum power criteria show the following correspondences with the predictions of equations 5 and 6 (Table 2):

1. The two criteria are identical for confluent streams of equal discharge (Table 2).

2. For values of $u$ between zero and about $-0.7$, angles predicted by minimum power and by equations 5 and 6 are so nearly identical over a wide range of discharge ratios that it may be impossible to determine which criterion most accurately predicts natural stream networks. Most stream systems apparently fall within this range [*Howard*, 1971].

3. Both criteria give physically impossible answers for $u > 0$. In such a case the predicted angle is 0°, so that the two streams should not meet.

4. For $u$ more negative than $-0.7$, the two

TABLE 2. Predicted Angles of Junction in Degrees for Modified Horton and Minimum Power Criteria

| | Discharge Ratio = 1.0 | | | | Discharge Ratio = 0.1 | | | | Discharge Ratio = 0.01 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $u$ | H1 | P1 | H2 | P2 | H1 | P1 | H2 | P2 | H1 | P1 | H2 | P2 |
| −0.2 | 29.5 | 29.5 | 29.5 | 29.5 | 51.8 | 56.3 | 11.2 | 7.6 | 66.6 | 70.8 | 3.6 | 1.4 |
| −0.4 | 40.7 | 40.7 | 40.7 | 40.7 | 67.5 | 69.7 | 15.7 | 13.6 | 80.9 | 82.8 | 5.1 | 3.6 |
| −0.6 | 48.7 | 48.7 | 48.7 | 48.7 | 76.3 | 73.3 | 19.2 | 22.4 | 86.4 | 84.0 | 6.3 | 9.1 |
| −0.8 | 55.0 | 55.0 | 55.0 | 55.0 | 81.6 | 70.2 | 22.1 | 36.5 | 88.6 | 78.3 | 7.2 | 22.9 |
| −1.0 | 60.0 | 60.0 | 60.0 | 60.0 | 84.8 | 60.0 | 24.6 | 60.0 | 89.4 | 60.0 | 8.1 | 60.0 |
| −1.4 | 67.7 | 67.7 | 67.7 | 67.7 | 88.0 | ... | 29.0 | ... | 89.9 | ... | 9.5 | ... |

The discharge ratio is equal to the discharge of tributary 1 divided by that of tributary 2.

H1 and H2 are the angles of junction of tributary 1 and tributary 2 predicted by equations 5 and 6 (modified Horton criterion), respectively. P1 and P2 are the corresponding angles of junction predicted by equations 12 and 13 (the minimum power equations), respectively. The parameter $u$ is the exponent in equation 4.

sets of predictions diverge. At $u = -1$, the power is independent of the discharge, so that total stream length is minimized, the result being individual angles of junction of 60°. However, equations 5 and 6 in general predict unequal angles of junction. For $u$ much more negative than $-1$, the minimum rate of work predicts physically impossible angles of junction.

The near identity of angles of junction predicted by the two criteria for a range of $u$ typical of natural streams is intriguing, but unexplained. The two criteria are based on different sets of assumptions, and only the angles of junction predicted by equations 5 and 6 appear to be associated with a causal physical mechanism.

## CONCLUSIONS

Angles of junction in stream networks appear to be regulated by erosion and sedimentation at the junction controlled by the gradient relationships at the junction. Thus the junction of channels with erodible banks that have been constructed or altered by man may not be stable for an arbitrary set of angles of junction. In an unstable situation, erosion of one or more channel banks would occur, and a shift of the point of junction would result. In addition, sediment may accumulate on the opposite side of the channel.

Because the junctions of large rivers have rarely been altered by engineering works, little or no literature on junction stability exists. Some erosion problems caused by artificial angles of junction may have been ascribed to meandering tendencies in the involved channels. However, for diverging channels, such as those that occur in cutoff channels or distributary networks in deltas, improper angles of junction may result in change of the angle by erosion or sedimentation or in the silting in of one of the branches [*Axelsson,* 1967, pp. 40–54].

Equations 2 and 3 have been found to be reasonable predictors of angles of junction in networks of ephemeral channels. Additional information, either from field measurements in perennial streams or from flume experiments, should allow more precise prediction of natural angles of junction.

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