QUANTITATIVE GEOMORPHOLOGY: SOME ASPECTS AND APPLICATIONS

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CHAPTER 2

PROBLEMS OF INTERPRETATION OF SIMULATION MODELS

OF GEOLOGIC PROCESSES

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ABSTRACT

Because of the complexity of geologic phenomena, mathematical models of geologic systems commonly can only give numerical predictions by using the technique of simulation. Simulation introduces additional assumptions, whose effects are minimized in the case of solution of differential equations by small spatio-temporal increments and in the case of probabilistic models by sufficient repetitions of the simulation to give accurate values of statistical parameters.

Not all of the assumptions made in complex simulation models are equally important in determining the numerical results. In some cases models with very different assumptions about the nature of the processes involved give similar predictions due to the dominance in determining results by a seemingly less important assumption common to all the models. Relative importance of the various theoretical constructs of the model can generally be determined only by noting effects of modification of the hypotheses upon the outcome.

Nearly identical predictions are sometimes made by competing models which have no apparent common theoretical constructs. Criteria based upon extremal principles such as minimum rate of work often make predictions close to those of mechanistic models, although the former give little insight into natural processes. The reason for these correspondences is at present uncertain. Mechanistic explanations are generally more acceptable to scientists than extremal explanations, so the former can be regarded as explaining the latter where predictive models of both types exist.

Models differ in degree of generality of the various assumptions incorporated in the model, models give more satisfactory explanations of nature to the degree that they incorporate general laws such as those of mechanics instead of experimental, empirical, or ad hoc assumptions.

The presence of inherent randomness in large scale geologic phenomena has been recently proposed. Such assertions encourage a dangerous methodology because they inhibit the search for more precise and more general explanations. Inherent randomness can never be proven or disproven; however, the scientist may show by constructing more accurate deterministic models the maximum limits of any proposed inherent randomness. These limits should contrast with the progress of science.
Figure 2. Simulated erosion of a corrosional stream under a constant rate of lowering of the downstream terminus of the stream. Erodibility is assumed to be uniform, parameters $b$ and $g$ have the values of 1 and 2, respectively, discharge is assumed proportional to the square of the downstream distance, $x$, and hydraulic resistance (Manning's $n$) is assumed constant.
according to a normal distribution with specified mean and variance.

(b) The means of \( S_t \) and \( S_m \) are sufficiently large with respect to their variances that the probability of negative value is negligible.

(c) The mean of \( S_t \) is larger than \( S_m \).

(d) If variations of \( S_t \) and \( S_m \) produce a ratio \( S_m/S_t \) greater than unity, then one of the is assumed to equal zero.

Because of the transformations in equation 1 and the logical operation in assumption 2, an analytic solution to this set of axioms is impractical. Instead, the distribution of the predicted angle of junction can be simulated through the use of a computerized random number generator. Two random numbers representing \( S_t \) and \( S_m \) are generated from distributions with the desired means and variances, and equation 1 is solved for \( a \) (observing also the restrictions b-d above). The process is repeated until an accurate distribution of the predicted angle of junction results (Figure 1). Similar simulations can be made for other assumed values of mean and variance to investigate the effect of their variation upon the distribution of \( a \). The assumptions superimposed by the process of simulation are that the random numbers follow a normal distribution with the desired mean and variance, and that the derived distribution approximates the distribution of \( a \), consistent with the assumptions of the theoretical model. With a sufficient quantity of numbers generated by a suitable procedure the influence of the process of simulation upon the resulting distribution can be made arbitrarily small.

(2) Complicated differential equations can be solved numerically for a given set of initial and boundary conditions. Several such examples in geologic fluid flow, sedimentation, population ecology, and geomorphology are discussed in Barthaugh and Bonham-Carter (1970) and Schieckger (1961, p. 99-117). Howard (1970) has introduced a heuristic differential equation model of erosion in corrasional streams based upon the assumption that the rate of erosion of the bed of the stream is a simple power function of the average shear stress acting upon the bed of the stream. The average shear stress is related to the commonly measured hydraulic parameters by several further assumptions: that the flow is uniform and steady, that the Manning equation adequately describes the resistance to fluid movement, that the hydraulic radius is consistently related to the cross-sectional area of the channel in a downstream direction, and that the gradient is not greater than a few degrees. The equations resulting from these assumptions can be combined to form a differential equation with 6 variables (downstream distance \( x \); the elevation of the stream bed, \( y \); time, \( t \); the erodibility of the stream bed, \( R \) (a function of \( x \) and \( y \)); the discharge, \( Q \) (a function of \( x \)); the hydraulic resistance (Manning's \( n \), assumed to be constant) and two adjustable parameters (the exponent of proportionality between the rate of stream bed erosion and the average shear stress, \( b \), and the exponent of proportionality between the hydraulic radius and the channel cross-sectional area \( g \)):

\[
\frac{dz}{dx} = \frac{Q}{R \left( 3b/(3g+2) \right) \left( b(6g+1)/(6g+4) \right)}
\]

(2)
Figure 3. Simulated erosion of a corrisional stream through a horizontal stratum four times more resistant than the overlying and underlying rocks, under the assumption that the downstream end of the stream lowers at a constant rate. The initial profile of the stream is the steady-state profile developed under a constant rate of erosion. Other assumptions as in Figure 2.
By using the technique of finite differences stream erosion can be simulated for a given set of parameters, initial conditions, and boundary conditions. For example, a uniform rate of downcutting through uniform bedrock (K constant) starting from an initially linear stream profile eventually produces steady-state gradients and a concave profile (Figure 2), and erosion through a resistant layer surrounded by weaker uniform bedrock produces an irregular profile (Figure 3). The superimposed effects of the simulation upon the solution of the differential equation can be made arbitrarily small by making sufficiently small incremental steps in the downstream direction, x, through time, t. However, the size of increments in x and t must be so related that errors introduced by the finite difference approximation do not grow and propagate through the system (Collatz, 1966, p. 305-315).

(3) Simulation models of stream network development usually idealize the drainage network on a square matrix of points (although hexagonal matrices have also been used by Scheldegger (1967a)), in which lines connecting matrix points are taken to represent segments of streams (Figure 4). Most commonly, each stream segment is restricted to flow in any one of the four cardinal directions. Many of these simulation models gradually fill an originally unoccupied matrix of points with stream segments, either by random downstream growth and merging (random walk models investigated by Leopold and Langbein (1962), Schenck (1963), Smart, Burkam and Coombes (1967), and Howard (1971a)) or by headward growth and branching (Howard 1971a, Smart and Moruzzi 1971a, 1971b). Certain capture models involve the modification of an arbitrary initial stream network by piracy, and are based on the assumption that the matrix points are drained by stream segments at all times (Howard, 1971b). The complete filling of matrix points is assumed to correspond to the rather uniform drainage density found in natural drainage basins of homogeneous lithology. This is formalized in the above-mentioned models by assuming that each stream segment had one square unit of area contributing direction to it, and that the basic length unit for measurement of stream length is the distance between adjacent matrix points (in the cardinal directions). These simplifying assumptions obviously restrict the kind of drainage basin features amenable to simulation, although Smart and Moruzzi (1971a, 1971b) have developed somewhat ad hoc techniques for producing realistic simulation with drainage densities less than unity. However, most of the simulation models allow reasonable predictions of the values or topological properties, stream length-stream order ratios, drainage area-stream order ratios, and basin shape parameters that occur in natural drainage networks. All of these simulation models depend on either random decisions during the simulation or random elements in the initial matrix conditions, or both. Therefore, successive simulations with different sequences of random numbers for randomly-varied initial conditions reduce the influence of the simulation process on the statistical properties of the generated networks. When inferences about the properties of large stream networks are desired the effects of the matrix edges (boundary conditions) can be reduced by increasing the size of the matrix.
Figure 4. Simulated stream network developed by headward growth and branching (Howard, 1971a). The network shown is a portion of the drainage developed on a 40 x 40 matrix in which all streams exit to one of the four matrix edges.
PROBLEMS IN INTERPRETATION OF GEOMORPHIC MODELS

One important problem in the interpretation of complex mathematical models is the difficulty in establishing the relative importance of the various theoretical assumptions in producing the numerical predictions. This is particularly true in simulation models where the relationship between the numerical results and the logical structure of the model can be determined only by induction. The effects of adjustable parameters and boundary conditions (including initial conditions) are directly (if often expensively) determined by altering their values in different simulations and observing variations in the results. For example, in the simulation of stream erosion it is verified that the effect of the initial profile gradually disappears as steady-state gradients develop during continued erosion at a constant rate of lowering of the base level (Figure 2). Similarly, the parameter \( b \) is found to affect primarily the transient behavior of the stream during erosion (including the effects of the resistant layer, Figure 3), whereas the parameter \( g \) affects only the degree of concavity of the stream profile. In the simulation of stream networks (Example 3) the statistical properties of the generated networks are generally insensitive to changes in the shape of the spatial boundaries (the matrix edges) (Howard, 1971a). Similarly, the statistical properties of stream networks resulting from long-continued capture are usually, but not in every case, independent of the initial conditions within the network (Howard, 1971a, p. 1367-1370).

However, some of the theoretical constructs in a model may be relatively unimportant in determining the predictions of the model in the sense that the theoretical structure can be extensively modified without affecting the nature of the resultant predictions. A case in point is the remarkable similarity in topological properties, length ratios and drainage area statistics of drainage basins simulated by random walk, headward growth and branching, and random capture models (Howard, 1971a, 1971b), although the shape of the generated basins differs somewhat among the various models.

The near coincidence in the results of simulation by three models with quite different assumptions as to the process of drainage network development raises several questions about the procedures for validating the model hypotheses. If the predictions of a mathematical model are close to the corresponding properties of the natural system, the model-builder is tempted to infer that his model forms a reasonable explanation of natural processes (the discussion here assumes that the goal of the modeling process is not solely the duplication of certain features of the natural system but rather their explanation). Specifically, it might seem reasonable to infer from the successful prediction of stream network properties by a headward growth and branching simulation that such a process is responsible for the development of natural stream networks. But at least two other models are equally successful in their predictions. How can three models widely differing in their logical structure give nearly identical results?

A common element in all three models is random decisions about the order, positions and directions of modifications or additions within the drainage network. The stochastic elements appear to be the most important structural factor in these models, whereas major differences in the mechanistic elements have little effect upon the numerical results. This conclusion is also supported by the close approach

![Diagram of angles Q2, Q1, and Q1+Q2]

<table>
<thead>
<tr>
<th>Q1/Q2 = 1.0</th>
<th>Q1/Q2 = 0.1</th>
<th>Q1/Q2 = 0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>A</td>
<td>A'</td>
</tr>
<tr>
<td>-0.2</td>
<td>150.5</td>
<td>150.5</td>
</tr>
<tr>
<td>-0.4</td>
<td>139.3</td>
<td>139.3</td>
</tr>
<tr>
<td>-0.6</td>
<td>131.3</td>
<td>131.3</td>
</tr>
<tr>
<td>-0.8</td>
<td>125.0</td>
<td>125.0</td>
</tr>
<tr>
<td>-1.0</td>
<td>120.0</td>
<td>120.0</td>
</tr>
<tr>
<td>-1.4</td>
<td>112.3</td>
<td>112.3</td>
</tr>
</tbody>
</table>

A and B are the angles of junction predicted by the erosion-sedimentation model, and A' and B' are those predicted by the minimum power criterion for the given ratios of discharge of tributary 1, Q1, to that of tributary 2, Q2. The parameter Z is the exponent in equation 3 (after Howard, 1971c, Table 2).
of the statistical properties of both simulated and natural stream networks to those of topologically random networks (as defined by Shreve, 1966, 1967). The insensitivity of most of the simulated network properties to the assumed process of development may correspond to a situation in nature in which spatial or temporal variations in the process involved in stream network evolution have little systematic influence on their statistical properties. However, this conclusion must be qualified in several respects. First, slight deviations do occur between the statistical properties generated by different processes and between simulated and natural stream networks; these minor differences, revealed only in large samples, may reflect the influence of the mode of origin of the networks, simulated or real. For example, the capture model of stream network development (Howard, 1971b) appears to provide slightly better predictions of the properties of natural stream networks than do the other models discussed. Second, only a few statistical parameters have been measured on simulated and real drainage networks. Some of these are more sensitive than others to differences in the rules of simulation, and others might be defined which are even more sensitive. Finally, only a few drainage basin properties are simulated in current models; other properties, such as areal variations in drainage density and slope form, may reflect more strongly the mode of origin of drainage basins.

In complex simulation models probably the only way to determine the relative importance of the various hypotheses comprising the model is to modify the theoretical structure of the model experimentally and to examine the consequences. However, most arbitrary modifications of a model destroy the simulation and have little correspondence with assumed processes in nature. Perhaps the best guide is to search for alternative working hypotheses about the nature of the process involved, such as headward growth versus downstream development or capture in the development of stream networks. Even though alternative hypotheses may not be self-evident, a good simulation of a geologic system does not necessarily indicate that all assumptions in the model are reasonable. However, a successful simulation of a complex phenomenon presumably implies that at least some theoretical structures in the model correspond to the natural situation being modeled, but the extent of this correspondence is almost always difficult to determine.

Similar predictions for the same geologic phenomenon are not limited to alternative simulation models with a common theoretical element. At least two distinct criteria appear to give satisfactory prediction of the angles of junction of streams (Howard, 1971c). One of these theoretical models postulates that the angles vary between which the water flow into a stream network. The rate of work is, for steady flow, proportional to the sum of the products of the discharge, gradient, and length of the three stream segments entering and leaving the junction. The second model postulates that differential sedimentation and erosion forces the angles of junction of merging streams to values at which the water surfaces of the streams meet concordantly. Both criteria give nearly identical predictions for stream networks in which the gradients, $S$, is related to the discharge, $Q$, by the following equation (where $K$ and $Z$ are constant parameters):

$$S = K Q^Z$$

(3)

Provided that the values of $Z$ are similar to those characteristic of natural drainage basins, that is, from $-0.2$ to $-0.6$ (Table 1).
Figure 5. The percentage change in rate of work in the drainage matrix produced by the simulated process of capture (Howard, 1971b), acting upon various types of initial network. The abbreviations A-II and A-II-A refer to two types of initial networks generated by headward growth and branching (Howard, 1971a), RW designated initial networks generated by random walk (Smart, Burkan and Considine, 1967; Howard, 1971a), RC refers to networks produced by random capture (Howard, 1971b,p. 1372), and PAR to an initial network composed of parallel, unbranched streams draining to a single matrix edge. The ordinate $z$ is the value of the parameter in equation 3 which prevailed during capture.
A similar parallel is also observed between a minimum rate of work criterion and the process of capture. The total rate of work performed by gravity in an idealized drainage matrix is proportional to the products of discharge and gradient for each individual stream segment (of unit length) summed for all stream segments within the matrix. If the reasonable assumption is made that discharge is proportional to the drainage area, and that, as above, equation 3 holds within the drainage network, then it is observed that the simulated process of capture (Howard, 1971b) acting upon an arbitrary initial network reduces the total rate of work in the drainage matrix so long as capture is assumed to occur only under advantageous conditions (that is, where the gradient between the captor and the captive stream is greater than that of the captive at the point of capture), Figure 5.

The minimum rate of work and mechanicistic explanations of drainage basin phenomena may be related by a common thread or explanation in a manner not understood at present, but they are not logically equivalent; their predictions are similar, but not identical for values of \( n \) in equation 3 within the range for natural drainage basins. Furthermore, for values of \( n \) smaller than those occurring in natural drainage basins, the minimization (or reduction) of work criteria diverge in their predictions from the mechanicistic models. The minimum power criterion fails to give predictions of angles of junction for exponents less than minus one, whereas the mechanicistic model involving erosion and sedimentation continues to do so (Table 1). Similarly, the process of capture under advantageous conditions in the simulation model results in an increase in power expended within the network for a value of \( n \) less than minus one (Figure 5).

A similar dichotomy exists between mechanicistic explanations of hydraulic geometry and explanations based on external criteria such as minimization of rate of work. Considerable progress has been made in the mechanicistic explanation of particle motion in fluid flow and in experimental investigations of sediment transport. Various sets of criteria have been devised under the rubric of "regime theory" to predict aspects of the hydraulic geometry of stream networks. While many or most of these criteria are experimental laws rather than theoretical predictions, most practitioners of the regime approach work from the methodological concept that a mechanicistic or deterministic explanation of hydraulic geometry is possible. This is manifest in the concept of equilibrium in streams, which can be stated as the hypothesis that there exists a unique combination of hydraulic variables in equilibrium with a particular combination of discharge regime and sediment supplied to the channel, and that a disturbance from equilibrium results in the restoration of the equilibrium (Blench, 1966, p. 45-66). A different mechanicistic approach is provided by the differential equation model of erosion in croccional streams discussed above. Various equations of hydraulic geometry can be formulated from the basic assumptions of the model differential equation; the stream profiles resulting from the simulations can be converted to gradients at each point and the other variables of hydraulic geometry, such as velocity, hydraulic radius, and the cross-sectional area of the stream can be predicted (Howard, 1970). By contrast, models of hydraulic geometry offered by Leopold and Langbein (1962), Leopold (1964) and Langbein and Leopold (1966) postulate that hydraulic phenomena so adjust themselves to satisfy a set of extremal relationships such as minimum power, maximum entropy, or the minimum of some objective function such as the sum of the squares of the exponents of certain equations of hydraulic geometry. The predictive power of at least some of these models appears to be the same order of magnitude as the regime approach. Several of these extremal formulations of hydraulic laws are
associated (encumbered may be a better word) with the view that geomorphic phenomena are basically determined. Thus the major difference between the two methods of analysis of hydraulic geometry appears to be this contrast in attitudes about determinism, which is discussed below. Like the regime approach, the extremal formulation of the laws of hydraulic geometry employs a number of empirical laws such as those relating velocity to frictional forces and sediment transport equations (indeed, the similarity of predictions by the regime, differential equation, and extremal models may be due to dominance by the empirical assumptions). Similarly, neither approach is truly mechanistic, for the regime approach covers only equilibrium, and not the dynamics of adjustment. Finally, at least one formulation of regime laws contains statements of optimal behavior quite similar to those of the extremal approach, for example,

"channels with the same water-sediment complex, and the same measure of erosive attack on sides, tend to adjust to the same dissipation of energy per unit mass per unit time" (Biench, 1966, p. 28).

Similar predictions made by different models raise several problems in deciding which of the competing models provides the "best" explanation, especially because of the most common criterion of choice among alternative theories is predictive ability. However, the criterion of predictive ability should include not only the accuracy of the theory in inferring the behavior (or state) of the natural system under a restricted set of conditions, but also the range of phenomena which can be predicted by the theory. Thus an accurate ad hoc explanation of a particular event is generally regarded as being less satisfying than a less accurate explanation by a theory of general application. Unfortunately, most geologic models are used to explain only very limited ranges of phenomena, such as stream junction angles or the numerical properties of stream networks, and the range of prediction may be very similar for the competing models. However, there are a number of indirect ways in which the competing models might differ in their ranges of prediction.

Complex geologic models, especially those incorporating simulation, often have a hierarchical structure. Capping the structure is a general qualitative hypothesis about the nature of a geologic phenomenon, for example, that piracy is an important process in the development of stream networks, that the angles of junction of streams adjust by differential sedimentation and erosion to particular values to equilibrium with the patterns of flow of the merging streams, or that the plan-form of stream networks adjusts to minimize the dissipation of energy in the stream. These general hypotheses must be translated into logical and mathematical statements which can include, not only "implicit definitions" (Hagel, 1961, p. 55-93) of the theoretical terms of the qualitative hypothesis, but also additional assumptions regarding the nature of the process involved (for example, the probability function governing piracy in the capture simulation model, Howard, 1971b, p. 1326). At a still lower level are a host of further assumptions and simplifications necessary to create a predictive model of particular geologic phenomena. Such assumptions in the stream capture model include many of the rules of the simulation, including the use of the matrix to represent the stream network, the initial stream network configuration, and the hypotheses concerning the evolution of the stream gradients during the simulation. It should be emphasized that the hierarchical structure is not necessarily determined by strictly
logical considerations but is in part the result of a value judgement by the model builder concerning which hypotheses he hopes to test by comparing the predictions of the model with data from nature, and in part a judgement concerning the relative generality of the hypotheses. Thus, the scientist is usually most interested in validating his general hypothesis. However, because of the many interposed additional assumptions or varying generality, at least two problematic situations may occur:

(1) The general hypothesis may be valid, but because of inappropriate subordinate assumptions the predictions may be inaccurate. This result would be disappointing, but it should not be dangerous.

(2) The subordinate assumptions may dominate the logical structure of the model, producing results which are essentially independent of the main hypothesis.

As discussed previously, situations of the second type occur in some geologic models so that models with quite different main hypotheses but similar subordinate hypotheses provide similar predictions. One possible means for distinguishing between the predictive ability of the alternative models is to search for properties of the natural system which the various main hypotheses predict somewhat differently. Another means of distinction may be to find ranges of initial or boundary conditions under which the predictions of alternative theories diverge (as discussed for the two stream junction angle models). However, in this case, appropriate natural data may not exist.

A generally more promising approach is to search for deductive consequences of the main hypothesis which result from its conjunction with a different set of subordinate hypotheses and which thus form an independent means of confirmation of the main hypothesis. For example, field studies or model experiments may give evidence regarding the importance and manner of occurrence of stream capture, headward growth, and junction angle modification. The recent study of stream network evolution on an experimental miniature landscape subjected to artificial precipitation is illustrative of this type of approach (Parker and Schumm, 1975). In this experimental approach the subordinate hypotheses are assumptions regarding the nature of natural geologic processes forming drainage basins and their relationship to the processes active on the model as well as hypotheses concerning the scaling of experiments on miniature landscapes to equivalent forms in nature.

The range of prediction of a geologic model is indirectly extended to the extent that the individual assumptions can be demonstrated to be specific consequences of more general physical laws. Least translatable into the framework of more general laws are ad hoc assumptions specifically introduced to increase the predictive power of the model, for example, the use of an adjustable parameter whose adopted value has no theoretical or experimental justification other than its salutary effect upon the outcome of the model. The automaton ecology and sedimentation model introduced by Barbour (1966) contains one adjustable parameter which are "tuned" to give the most realistic simulation. Similar tuning occurs in a simulation model of stream network development introduced by Smart and Moruzzi (1971b). The inclusion of such elements in a theoretical framework seriously limits the range of applicability of the model and introduces a strong possibility that a reasonable simulation implies little about the more fundamental assumptions in the model.

Experimental laws of varying generality are often included in geologic models.
At one end of the spectrum are relationships specific to the system under investigation, such as equation 3. Such empirical generalizations are often included in geologic models because of incomplete knowledge of the structure and dynamics of the natural system. Several dangers arise from their inclusion. The empirical law may dominate the deductive consequences of the model. Examples of this have been discussed. In addition, the empirical equation may be used unwittingly in an inappropriate manner by over-extending its range of predictive ability or by failure to recognize that the empirical law may be a consequence of, or at least may interact with, the general law in the model. For example, the relationship between gradient and discharge in a stream system may be directly affected by the process of capture within the stream network rather than being independent of it as assumed in the capture simulation model (Howard, 1971b, p. 1358-1359). At the other end of the spectrum are experimental laws of a wide range of applicability, such as resistance equations in turbulent flow which, if used correctly, are not likely to promote misleading conclusions.

Finally there are well-established theoretical laws, such as equations of continuity and the laws of mechanics which have an essentially unlimited range of applicability.

The "best" geologic models are those whose assumptions contain the fewest a hoc of experimental laws. However, treatment of many geologic phenomena is intractable without their use.

THE PROBLEMS OF EXTERNAL LAWS

The above criteria primarily concern geologic models employing mechanistic explanations. The evaluation of models employing external principles requires further discussion. Such hypotheses have the appeal of very general formulation; however, like the laws of thermodynamics, models employing external hypotheses do not provide a mechanistic explanation, although they may employ mechanical criteria such as the rate of work. For example, the minimum power criterion for stream junction angles gives no explanation of the processes by which junction angles are presumed to adjust. This lack of assumptions concerning processes probably accounts for the reluctance of geomorphologists to accept such models. A similar dissatisfaction existed with regard to classical thermodynamics because of a lack of integration with mechanical laws, despite its predictive ability, until the mechanistic hypotheses of statistical mechanics were shown to imply the laws of thermodynamics (at least under the restricted conditions suitable to mathematical manipulation). A similar "reduction" (Wagel, 1961, p. 336-337) may be possible with respect to external laws in geology. For example, as noted above, a model envisioning differential erosion and sedimentation at stream junction gives predictions similar to the minimum power criterion. Similarly, the occurrence of capture under advantageous conditions in a simulation model of stream network evolution tends to reduce the total rate of work in the stream network. In any case, external formulations of geomorphic laws should not be dismissed out-of-hand, for they may imply the presence of general physical laws in geomorphic systems. At a minimum they should be considered as empirical laws (if satisfactorily confirmed by observations) which are awaiting a mechanistic explanation.
However, a number of cautionary remarks should be raised about a proposed extension of the extremal formulations of geomorphic laws to build thermodynamic analogues of landform features and evolution as proposed by Leopold and Langbein (1962) and Scheldegger (1967a). For example, Scheldegger (1967b) hypothesizes that thermodynamic temperature is analogous to elevation of the land surface above sea level, and that the differential of heat associated with changes of temperature is equivalent to the differential of mass involved with changes of land elevation. Similarly, formulations in the landscape are made analogous to pressure, volume, work, and entropy. One problem with this general analogy is that the types of variables equated may not be equivalent; Chorley (1967, p. 72-73) points out that absolute temperature is measured on a ratio scale whereas elevation is measured on an interval scale. The use of a thermodynamic analogy may also introduce more confusion than clarification into geomorphic thinking when such terms as temperature and heat are used both in their strict thermodynamic sense and also in their analogous sense. Finally, the use of the thermodynamic analogy may suggest (although it does not imply) that analogies should also exist to further concepts such as those of ideal gasses and chemical potentials, and that a "statistical mechanics" can be formulated for geomorphology involving ensembles of some discrete physical entity behaving in some mechanistically regular way. However, the prospects are not bright that very much of the complexity in natural landforms of the multitude of processes involved in landform evolution can be translated into thermodynamic analogies.

THE PROBLEM OF RANDOMNESS

The widespread introduction of statistical generalizations and probabilistic explanations in geology has promoted speculations that the large-scale phenomena treated by geology may be based on the random behavior of micro-phenomena, and thus would be incapable of deterministic explanation and thus also inherently unpredictable, even if the state of the system under investigation were completely characterized and all the forces which affect the system were known. (For a general discussion of this viewpoint see Mann, 1970). In geomorphology the most explicit assertion of inherent randomness, or indeterminism, has been made by Leopold and Langbein (1965). The position taken by Leopold and Langbein has been vigorously criticized by Watson (1962, 1963), Simpson (1970), and Kennedy (1971), but the continued speculation about irreducible uncertainty in geologic phenomena (Mann, 1970; Smalley, 1970) suggest that further discussion of the danger of the determinist position is needed.

It is certainly true that there is not, nor can there be, a logical proof that nature is or is not completely deterministic (Bagel, 1961, p. 33). However, nature is not completely random, as our ability to predict future events demonstrates. Many phenomena cannot at present be completely described within a completely mechanistic framework. For example, turbulence and vorticity in fluid flow is imperfectly understood and is generally treated by probabilistic laws. It is uncertain whether a mechanistic description of turbulence is possible, although some progress may have been made in this direction (Sani, 1971).

However, the most important point in the context of this discussion is not to question whether some particular geologic phenomena such as the hydraulic regime of streams shows a certain level of irreducible uncertainty (only
further research can show the maximum limits of inherent randomness, but rather it is the fact that a very dubious methodology is likely to result from assertion of inherent randomness in geologic phenomena.

The universal paradigm of science (the "scientific method") is based on the assumption that natural events are determinate, and that the role of the scientist is to increase the accuracy and generality of his predictions through the construction of empirical and theoretical laws. A corollary to this assumption is the view that a lack of accuracy in a prediction is due either to a shortcoming of the theory, to the influence of uncontrolled factors, or to incomplete specification of initial conditions. The determinist methodology, properly formulated, is a working hypothesis rather than a dogma, and is subject to exception without loss of its general value as in the case of quantum mechanics. To posit an alternative indeterministic methodology based on the assumption of inherent randomness in nature is in effect to abandon the search for complete explanations. It would be to assert the impossibility of discovering explanations of increasingly greater generality, accuracy, and simplicity, and would abandon the scientific quest as we know it. Thus, while it may be attractive to attribute any failure or prediction to "inherent randomness" in nature, such an approach entirely undermines the method by which theories have traditionally been tested. It would leave research with no guide nor goal.

Many of the quantitative variables that are used in geomorphology incompletely specify the state of the system, and indeed are statistical parameters. In hydraulic geometry discharge, velocity, stream depth, and similar parameters are all statistical descriptors which together form an incomplete characterization of the state of the stream; it is therefore unreasonable to expect that empirical laws relating these will show no scatter or that theoretical predictors will be accurate in specific cases although they may give accurate (determinate) predictions about ensembles of streams.

Potentially more troublesome is the interpretation of geologic theories which have probabilistic premises. Particularly critical is the question of whether a premise incorporating uncertainty makes an assertion about inherent randomness in nature. An instructive case is the hypothesis of topological randomness in stream network patterns advanced by Shreve (1966). Shreve's single hypothesis is that in an ensemble of stream networks with the same number of first-order streams all distinct topological arrangements of the first order streams in a dendritic pattern are equally likely to occur. From this premise Shreve and others (Shreve, 1966, 1967, 1969; Smart, 1969) deduce numerous corollaries about the expected numerical properties of stream networks which are closely approached by samples of natural networks (Smart, 1969; Kruecken and Shreve, 1970). But again, the probabilistic conclusions are strictly deterministic from the probabilistic premises.

The accurate prediction of the statistical properties of stream networks by a probabilistic theory could be interpreted as a manifestation of inherent randomness in the formation of stream network topology. However, in light of the above comments, the most conservative interpretation is that factors such as lithology, structure, microclimate, vegetation, and erosional history determine network topology but vary so complexly both in space and through time that small scale stream network topology shows no large scale overall regularity and so can be treated as a random phenomenon when predicting the statistical properties of large stream networks or ensembles of small networks (see similar expositions by Watson (1966, p. 491-494), Howard (1971a, p.43) and Smart (1971)). This does not mean that there is no small scale regularity.
nor does it rule out the logical possibility that sufficient detailed examination would reveal it. An alternative phrasing of the same interpretation would be that there is no geologic process affecting stream network topology on a large scale which acts with sufficient spatial and temporal consistency to produce a regular small scale network topology. However, a prediction concerning the topologic evolution of any particular network could, in principle, be constructed from an understanding of the way in which geologic processes influence network topology, the distribution of these processes in time and space, and the initial conditions in the drainage basin.

There is some vindication of the deterministic methodology in recent investigations which show some deviations in natural stream networks from perfect compliance with Shreve's theory of topologic randomness (Smart, 1969; James and Krubeln, 1969; Krubeln and Shreve, 1970; Howard, 1971b). These discrepancies are presumably due to one or more processes influencing network topology which act with some consistency. The search for these regularities and the attempts at identification of the processes responsible can be based only on the working hypothesis of determinism.

Some confusion is likely to result from the recent usage in geomorphology of the terms "non-deterministic" and "indeterminate" in a manner that is not synonymous with "accusal" or "inherently random". Some geomorphic systems, especially stream channels, are often assumed to show a unique equilibrium to a specific and inclusive set of imposed external forcing factors, that is, the equilibrium state is assumed to be independent of initial conditions in the system. For example, the initial gradients of the stream in the differential equation model discussed above gradually assume steady-state values in response to a constant rate of base-level lowering (Figure 2). The assumption in regime theory of a unique equilibrium to imposed hydrologic variables has been mentioned. By contrast, Muddock (1969) gives evidence that velocity, width, depth and gradient of a straight alluvial channel with fixed values of discharge and sediment load may take various equilibrium values depending upon the initial type of bedform in the channel. Similarly, Schumm (1965, p. 35) suggests that the gradient of a channel may respond to a change in regimen not only by valley degradation or alluviation, but also by adjustments of sinuosity; therefore, the initial valley profile must be specified in addition to discharge and sediment load to determine equilibrium. Similar dependence on initial conditions is shown by many physical systems describable by non-linear differential equations, for example, the long-term climatic equilibrium shown by numerical solution through time of the differential equations for the state of the atmosphere (Lorenz, 1965). The description of such systems as "non-deterministic" does not imply unusual behavior, but merely that initial conditions be supplied to obtain an unique equilibrium. Less confusion will result if such behavior is described as "intransitive" (Lorenz, 1965, p. 2) and the equilibrium characterized as "non-unique".

In short, depending upon the nature of the investigation, one may choose among various kinds of models - mechanistic, probabilistic, empirical or extremal. In any of these models the failures in prediction are as important as the successes; science seems to be best served when these failures are ascribed to shortcomings or incompleteness of the model rather than inherent randomness in nature. Authors proposing "random" or indeterminate events in nature should make clear the philosophical sense in which these terms are used.
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