Effect of slope on the threshold of motion and its application to orientation of wind ripples

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ABSTRACT

On sloping sand surfaces, the downwind perpendicular to ripple crests is not, as commonly believed, an unbiased indicator of current direction, for the ripple-forming creep load is deflected downgradient as a function of the surface gradient, the orientation of the surface relative to the wind, and the friction angle of the sand. The force required to initiate motion on sloping surfaces is likewise dependent upon these parameters. The mean direction of sand movement is also deflected downgradient from the applied fluid force, but by a lesser amount than the ripples.

INTRODUCTION

The shear stress necessary to initiate motion of a noncohesive granular bed varies with the inclination of the sediment surface, as particles are easier to transport when the surface slopes downflow. Also, if the surface has a component of slope perpendicular to the direction of fluid flow, the direction of initial motion will be at an angle to the flow. The mathematical formulation of these effects reported below was developed for use in prediction of sediment transport over barchan dunes. However, this phenomenon also explains observed systematic deviations between the direction of the surface wind on such dunes on one hand and the direction of the normal to the strike of the ripples at the same point on the other hand.

INITIATION OF MOTION ON SLOPING SURFACES

The motion of particles on a sand surface begins when the fluid forces, including drag and lift, exert an overturning moment on the grain equal to the cohesion between the particles and the stabilizing moment exerted by gravity. Bagnold (1941, p. 85–86) and White (1940) developed expressions for this balance for the case where cohesion is unimportant (coarse grains), making the assumption that the net fluid forces acts through the center of gravity in the direction of the fluid flow (Fig. 1A). More recent analyses of Chepil and Woodruff (1963, p. 222–229) and by Iverson and others (1976) take into account two fluid forces — surface drag, acting in the direction of the fluid flow, and aerodynamic lift, which acts nearly vertically. In addition, the resultant of the fluid forces acts somewhat above the center of gravity, which slightly reduces the apparent friction angle, α. The effects of lift and cohesion are most important for very fine particles, and their inclusion greatly complicates the threshold equation. For particles of sand size and larger, Bagnold's simpler analysis provides an excellent fit to experimental data (Iversen and others, 1976) and is more tractable to the analysis reported below. The indirect verification of the derived results from wind ripple data suggests that the Bagnold for-

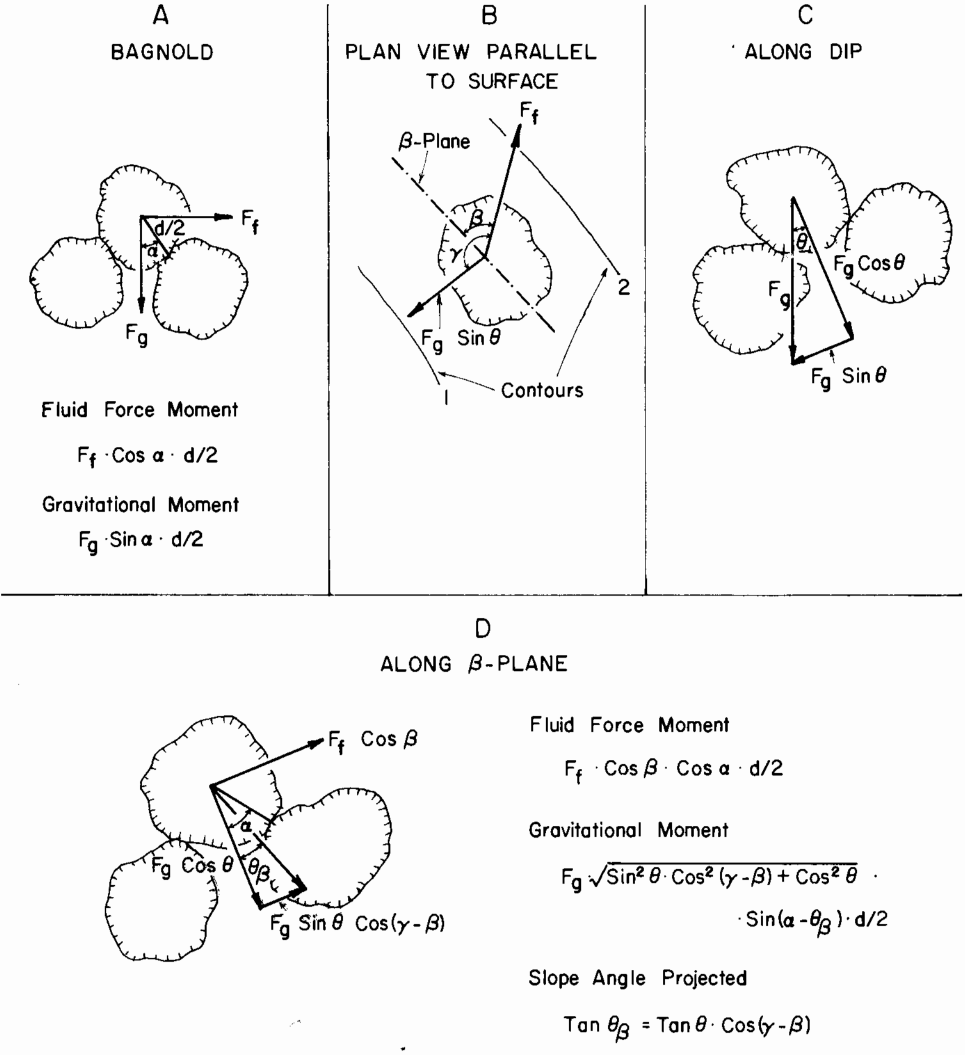

Figure 1. A. Cross section through level sand surface showing force relationships at threshold of motion; B through D illustrate same on a sloping surface.

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mulation of the threshold equation is adequate for sand transport over dunes.

It is evident from Figure 1A that transport on a horizontal sand surface begins when the ratio of the fluid force, F_f, to the gravitational force, F_g, exceeds tan α, where α is an average geometric property of the sand surface, approximately equal to the angle-of-repose of the loose surface sand, and d is the grain diameter:

$$F_f \cos\alpha \frac{d}{2} = F_g \sin\alpha \frac{d}{2}\text{, or } \frac{F_f}{F_g} = \tan\alpha. \quad (1)$$

Bagnold reasons that the fluid force is proportional to the square of the product of the shear velocity and the particle diameter, while the gravitation force is proportional to the density difference between the particle and the fluid, the gravitational acceleration, and the cube of the particle diameter; that is:

$$\frac{F_f}{F_g} = B \frac{V_{*t}^2}{d(\sigma - \rho)g}, \quad (2)$$

where σ is the density of the particle, ρ is the density of the fluid, g is the gravitational acceleration, V_{*t} is the shear velocity at threshold, and B is a constant determined experimentally. The effect of surface slope is to replace tan α in equation 1 with a more complicated function of α, the surface slope, θ, and the angle between the wind and the surface gradient, γ (Fig. 1B). The fluid force is assumed to act parallel to the granular bed in the direction of flow. In general, the particle will feel the greatest overturning moment at some angle β between the direction of the fluid and gravitational stresses, which angle will presumably be the direction of initial motion. The angle β and the ratio of the fluid to gravitational forces in this direction can be determined by projecting the forces onto an arbitrary β-plane and then finding by differentiation that value of β that maximizes the ratio of the projected fluid to gravitational moments. From Figure (B, C, and D), the moment ratio on the β-plane is (α is assumed to be statistically independent of the angle β):

$$\frac{F_f \cos\alpha \cos\beta}{F_g(\sin\alpha\cos\theta - \cos\gamma\cos\beta\sin\theta\cos\beta - \cos\alpha\sin\beta\sin\gamma\sin\theta)}. \quad (3)$$

When the derivative of this function with respect to β is set to zero, the direction of the initiation of motion is found to be given by:

$$\sin\beta = \frac{\tan\theta\sin\gamma}{\tan\alpha}. \quad (4)$$

Upon substitution into equation 3, the ratio of fluid to gravitational forces at the initiation of motion (ratio of moments equaling unity) is found to be:

$$\frac{F_f}{F_g} = \sqrt{\tan^2\sigma\cos^2\theta - \sin^2\gamma\sin^2\theta} - \cos\gamma\sin\theta. \quad (5)$$

This result has been previously derived by Brooks (1963), although he did not calculate the direction of initial motion (equation 4). The force ratio for the special cases where the flow is directly up or down contour ($\gamma = 0^0$) or where the flow parallels the contours ($\gamma = 90^0$) has been derived by several authors, including Carter and others (1953), Sundborg (1956), Henderson (1966, p. 418–419), Smith (1970, p. 41–42), Graf (1971, p. 84–85, 113–116), and Christensen (1972). Stevens and Simons (1971) discuss the more general case with lift and unequal moment arms.

APPLICATION TO WIND RIPPLES

Field observations made by the author on several barchan dunes reveal that the downwind perpendicular to the ripple strike is systematically deflected downslope from the surface wind on sloping surfaces by as much as 35 degrees (Fig. 2). This deviation has apparently not been noted previously, for it is usually assumed that the

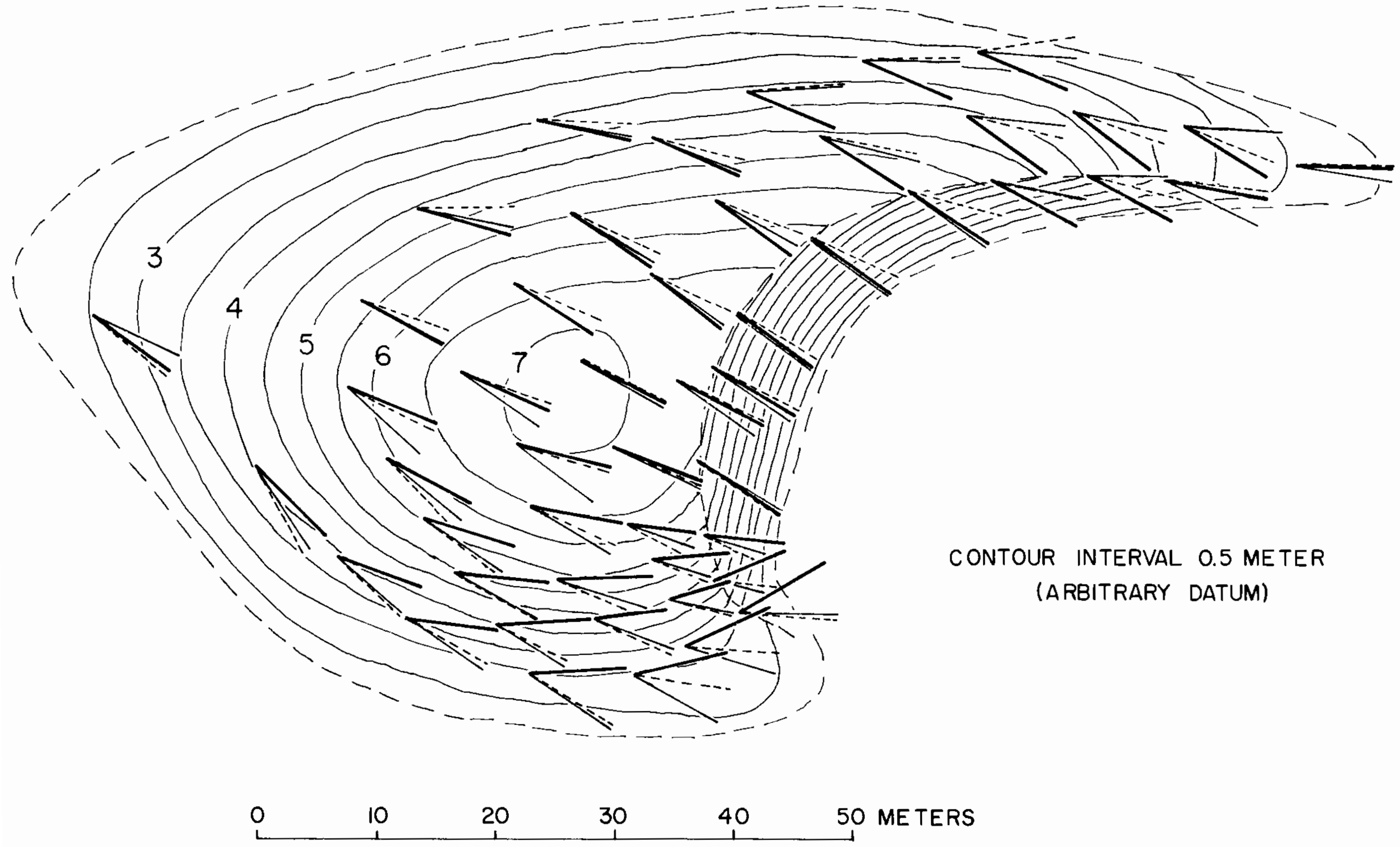

Figure 2. Wind directions at surface (heavy lines) compared with observed (light lines) and predicted (dashed lines) downwind perpendiculars to ripple crests on the stoss slope of a barchan dune near the Salton Sea, California.

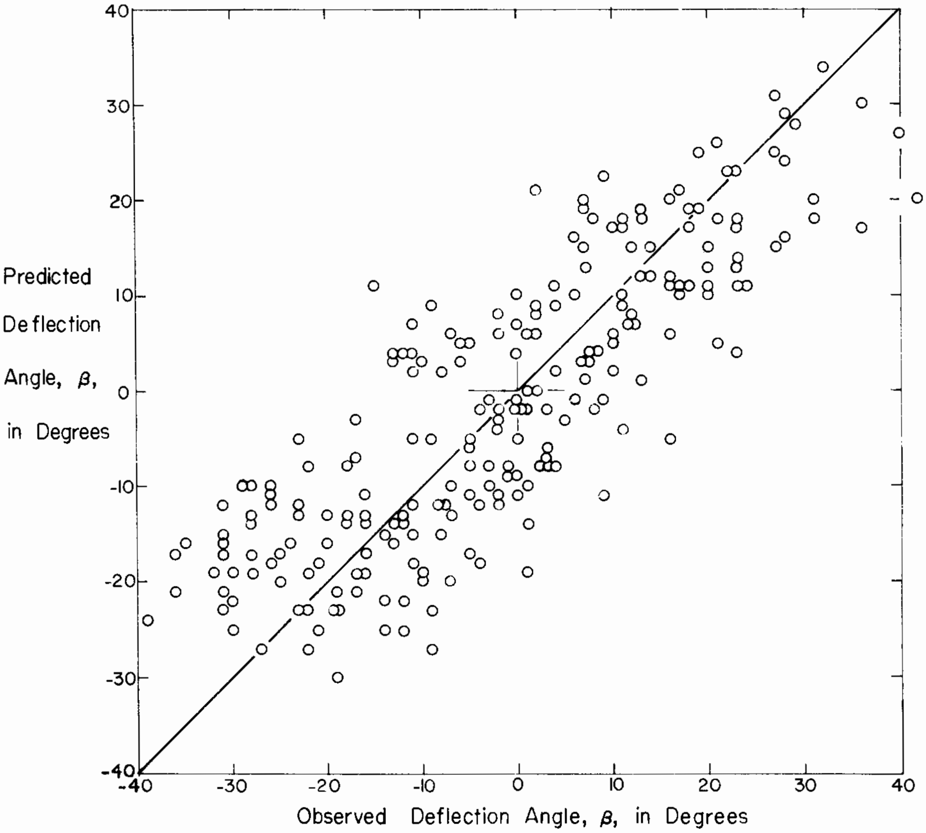

Figure 3. Relationship between measured downslope deflection of ripple perpendiculars and the values predicted by equation 4, showing line of perfect agreement.

ripple marks are oriented perpendicular to the oncoming wind. For example, Allen (1968) makes such an assumption when drawing inferred ripple trains from experiments on model dunes which give the surface flow directions. Conversely, ripple trends on sloping sand accumulations have been used to infer the flow pattern (Stone and Summer, 1972; Verlaque, 1958).

Although several explanations for this phenomenon could be proposed, the two most likely explanations are that the sand flow over the dune has a downslope component of motion and (or) that the increase in sediment-transport rate, upon moving from the base to the summit of the dune, shears the ripples due to their faster movement at higher elevations. The latter explanation seems less likely, for a shearing would tend to break the ripples into short segments (rather than the long crests observed) and would lead to a large variance in ripple orientation as ripples broke up and reformed perpendicular to the wind. The first hypothesis, however, can be tested quantitatively by the criterion for initiation of motion derived above.

Although the mechanics of ripple formation are imperfectly understood, researchers on wind ripples generally agree that the motion of the coarsest grains (that is, the creep load) controls the scale and geometry of the ripples (Bagnold, 1941, p. 145–166; Sharp, 1963; Ellwood and others, 1975). This role of surface creep in ripple formation is indicated by the segregation of the coarsest grains in the ripples (Sharp, 1963, p. 620–623). Because the coarsest grains in motion are close to their threshold condition, equations 4 and 5 might govern their motion. In particular, equation 4 predicts that the movement of the coarsest grains would have a downslope component relative to the direction of the impelling force. The major difference between the conditions described by equations 4 and 5 and the motion of the creep load is that the grains move by a combination of wind drag (and lift) as well as by the impulse of impacting grains. Because the saltating particles closely follow the local wind (see discussion below), both forces together may be identified as the fluid force, F_f, and both determine the value of the constant B in equation 2 (the "impact threshold" of Bagnold, 1941, p. 104).

As a test of this hypothesis, the angle β was equated with the observed angle between the downwind perpendicular to the ripple strike and the surface wind, θ with the slope gradient, and γ with the angle between the dip of the slope and the surface wind. A total of 224 individual measurements of wind and ripple directions were made at 50 to 70 locations on each of three barchan dunes in the Salton Sea dune field (Long and Sharp, 1964; Norris, 1966) during strong winds (one set of measurements on each of three dunes during one period of wind, and one set on one dune on a separate occasion). Surface wind directions were measured from the strike of the scour trough downwind of stakes emplaced on the dunes. The scour troughs were shown to be unbiased indicators of wind direction by comparison with smoke bombs placed nearby on the sand surface. A best-fit regression of equation 4 was made, with the value of the friction angle estimated by the regression slope:

$$\sin\beta = \frac{\tan\theta \sin\gamma}{\tan(30.5^\circ)} . \qquad (6)$$

Seventy percent of the variation in sin β was explained by this relationship (Fig. 3). This high degree of explanation, together with a reasonable value of the friction angle, strongly supports the hypothesis that the ripples are formed by the coarse grains moving at an angle to the surface wind on sloping surfaces. The agreement also indirectly supports the usefulness of equation 4 and 5 in estimating the direction of motion and the fluid force required to initiate motion on a sloping surface, despite the simplified assumptions of the model. The adherence of creep motion to equations 4 through 6 is probably true only on the average. Grain motion on sloping surfaces is irregular; grains appear to closely follow the wind during strong gusts, but the coarse grains tend to roll downhill as the gusts die down, giving a zigzag motion with a resultant apparently obeying equation 6.

The unexplained variance presumably rests with two types of measurement errors. Wind and ripple strikes, measured by Brunton compass, were often difficult to interpret. Wind and ripple orientation could not be measured on the three-dimensional granule ripples found on the upwind toe of the dunes. Slope gradient and slope strike were estimated from plane-table contour maps, leading to errors from inaccuracies in the maps. The pattern of residual errors from measurements made on the three dunes reveals no systematic bias associated with particular relative locations on the dunes.

DISCUSSION

The mean sand transport direction on a sloping surface probably is deflected from the surface wind by an angle proportional to, but less than, the angle β predicted from equation 4. The creep load for naturally sorted desert sand, governed by the above equations, is only about one-fourth of the total wind-driven load, the remainder being carried in saltation (and a small percentage in true suspension). The particles comprising the saltation load are projected an average of 1 to 7 cm into the air flow (depending upon grain size and shear stress; see Zingg, 1953; and Williams, 1964). Although the impact of the saltating grains with the surface and near-surface lift forces impart an initial downslope component of motion, the lateral component of motion of grains rising from the bed is small compared to the velocity reached during the saltation jump as a result of fluid drag. Thus the saltating grains are unlikely to be as strongly deflected downslope as the creep component.

Because the impelling force for the creep component partially derives from saltating grains, the observed ripple deflection, β, may be partially due to downslope deviation of the net force from the direction of the local wind. This would not affect the usefulness of equation 6 in predicting ripple deflection, but the apparent friction angle (30.5°) may be an underestimate of the actual friction angle, α.

In conclusion, the inference of current direction from ripple crest orientation on sloping surfaces can lead to large errors unless the orientation and gradient of the surface is taken into account. As an indication of the magnitude of the slope effect, consider the case where the wind parallels the contours, where the greatest ripple deflection occurs. In this case, the angle β is about 1.7 times the slope angle θ for slopes less than 10°, but the ratio increases to 1.9 for a 20° gradient.

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