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THRESHOLDS IN RIVER REGIMES

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INTRODUCTION

The bed of natural stream channels can be composed of bedrock, fine-grained alluvium, or a coarse-grained gravel pavement. This classification appears to have a "natural" basis, in that most natural channels, or segments thereof, can be assigned unequivocably to one of the three classes. Downstream transitions between any two of the three channel types are abrupt, as are temporal changes resulting from changed hydraulic regime.

Thus natural discontinuities, or *thresholds*, occur between these channel types, and, as will be shown, each type has a distinctive pattern of response to controlling factors. Although thresholds are generally defined as an abrupt *temporal* change in morphology or dynamics of landforms, similar abrupt *spatial* transitions are common (stream banks, excarpment brinks, riffles and pools, stream junctions, etc.), and the transitions between channel types discussed here can be of either type. In evolving landscapes spatial thresholds may be analogous to temporal changes at a point ("substitution of space for time"), such as in the case of an advancing gully network or a retreating escarpment; this is often true for the channel bed transitions discussed here.

Temporal thresholds between channel types are rarely mentioned in geomorphic literature, largely because the generally slow time scales of fluvial response limit the number of directly observable cases. However, a few scattered cases have been mentioned. Howard (in preparation, a) cites examples where badland washes alternate between bedrock and fine-bed alluvial channels as a result of seasonal changes in relative discharge and sediment supply. Bed degradation below dams often converts fine-bed alluvial channels to coarse-bed channels by the armoring process, or in some cases completely scours the alluvium to bedrock (Gessler, 1971; Hammad, 1972; Simons et al., 1975).

The spatial thresholds are better known, although not usually remarked upon as such. Many otherwise alluvial channels have steep rapids or falls where especially resistant rocks crop out. Howard (in preparation, a) discusses abrupt downstream transitions in badland channel networks between bedrock and fine-bed alluvial channels. Steep, shale-floored rills and washes on badland slopes commonly abut fine-bed alluvial pediments (Smith, 1958; Schumm, 1956; Howard, 1970). Yatsu (1955) describes striking downstream transitions from gravel- to sand-bed rivers in Japan, and Shaw and Kellerhals (1977) cite similar transitions in Alberta. The alternating coarse-bed riffles and fine-bed pools common to many rivers may also be examples.

The median grain sizes at 158 stream cross sections throughout the United States sampled by Williams (1978a) are indirect evidence for a threshold between fine- and coarse-bed alluvial beds. The frequency distribution of the medians is bimodal; 65% of the medians lie in the 3ϕ range between 0.125 and 1 mm, 20% in the 3ϕ range from 16 to 128 mm, but only 8% occur in the intermediate range from 2 to 16 mm (Williams, 1978a, Table 11). Other indirect evidence for thresholds between the two types of alluvial channels will be presented in this chapter.

The regime of streams will be considered herein primarily in terms of their long-term adjustment to discharge pattern and to the size, grading, and quantity of sediment supplied by slope erosion. These factors will be termed the *hydraulic regime*, and they will be considered to be the independent (or external) variables, whereas the hydraulic geometry of the channel (in particular, the channel gradient) constitutes the dependent, or internal, variables. This contrasts with the short-term engineering view wherein gradient, bed material size, channel dimensions, and discharge are the independent variables and the bed material load is the dependent variable, and with very long geologic time scales, where tectonism, global climate, and the nature of the bedrock are the independent variables and the entire landscape becomes a dependent set of variables (Schumm and Lichty, 1965.).

Schumm (1974) and Schumm and Beathard (1976) distinguish between *extrinsic* thresholds, in which abrupt changes in morphology or dynamics occur as a result of gradual (progressive) change in the independent variables, and *intrinsic* thresholds, in which the threshold occurs without any necessary change in the external variables. In general, the thresholds discussed here are extrinsic in that gradual downstream or temporal change in hydraulic regime triggers transformations between the channel types. However, erosion of bedrock channels without change in hydraulic regime can reduce the gradient to the point that it is transformed to an alluvial bed, an intrinsic threshold.

Alluvial Channels

Alluvial channels are characterized by one of two types of beds: (1) coarse gravel or cobbles that move only near bankfull discharge, and (2) sand-bed channels in which transport occurs at all but perhaps the lowest flows. The former channels are distinguished morphologically as *coarse-bed* and the latter as *fine-bed*. Hydraulically, these correspond to "threshold" or "stable" channels on the one hand, and to "live-bed" or "regime" channels on the other. The recognition of this dichotomy is not new. For example, Maddock (1969) presents separate regime formulas for gravel- and sand-bed channels, and other authors have recognized that coarse-bed channels are near the threshold of motion for peak discharges (Kellerhals, 1967; Li et al., 1976). Similarly, many studies have been made of sediment transport relationships applicable to natural sand-bed streams. However, it does not seem to be widely recognized that transitional forms between these channel types are rare, and that a hydraulic threshold exists. In fact, these two types of alluvial channels have distinct hydraulic geometries (particularly the *downstream* relationships). Henderson (1966, p. 472) recognized that fine- and coarse-bed channels have different exponents, relating channel gradient to dominant discharge and that these exponents:

> may be used to distinguish between those river channels which have shaped themselves according to live-bed criteria and those others (probably in coarse material) which have shaped themselves according to conditions obtaining at the threshold of motion. But it must be conceded that so far no complete theory has been presented explaining the mechanics of the process.

A major theme in this paper is an attempt toward an explanation of this threshold behavior as well as an examination of the factors determining which of the two channel types will occur in a particular stream.

Bedrock Channels

Bedrock channels are defined here as channel segments whose beds lack a coherent cover of active alluvium, even at low flow. "Bedrock" is employed somewhat loosely, for steep rills on soil slopes and steep channels dissecting cohesive fluvial deposits fall within this definition. The qualification of coherency of the alluvial cover is included to allow for scattered pockets of alluvium within scour holes in otherwise bedrock-floored channels (Einstein, 1964). Short bedrock channel segments are common along many rivers at the outcrop of resistant rock, but large channel systems that are exclusively bedrock are rare. Such channels seem to be common in headwater sections where relief ratios are high, often forming the fingertip tributaries of otherwise alluvial channel systems.

Bedrock channels remain free from alluvial deposition because of their steep gradient; they are always steeper than alluvial channels with the same hydraulic regime imposed from upstream. Whereas the gradient of alluvial streams is determined by its hydraulic regime, the gradient of bedrock channels can be considered to be an independent variable. For example, Howard (in preparation, a) shows that bedrock channels are steeper than alluvial channels in an area of badland topography, and whereas the gradient of the alluvial channels shows a strong correlation with drainage area (due to consistent downstream changes in the controlling hydraulic regime), there is no such correlation for bedrock channels. Rather, the gradient of these channels may have been determined by a number of factors, including the original land slope (for newly dissected topography), the resistance of the bedrock to erosion, and the erosional history of the area.

Bedrock channels have no alluvial deposition at low flows, even though they may transport considerable bed material load and, in the case of ephemeral washes, may have periods of no flow. The reason for the lack of alluvial bed is apparent for bedrock channel sections downstream from alluvial stream sections; the bedrock channel, by virtue of its steeper gradient, has a greater transporting capacity than the upstream alluvial channel for any stage, so that the bedload will be preferentially deposited on the alluvial channel during waning flow. For headwater bedrock channels the upstream sediment supply diminishes more rapidly than the transporting capacity during waning stages because that supply occurs primarily during the runoff stage, whereas transport in the channel continues during base flow recession.

In the following sections the behavior of alluvial and bedrock streams is more closely examined in order to demonstrate the existence of the thresholds and to explore the implications of these thresholds to fluvial sedimentology and to river response to changes in hydraulic regime. This introductory section closes with a discussion of two important concepts, the dominant discharge and the selection of independent and dependent variables of hydraulic geometry.

Dominant Discharge and Status of Variables of Hydraulic Geometry

The role of discharge in sediment transport and bed erosion will be represented by a *dominant* discharge, that is, a steady discharge (not necessarily of continuous duration) that performs the same action as the natural sequence of discharges. For bed material transport this is a constant discharge that would transport the same bedload and suspended load as the long-term average of the actual discharges. This discharge can be determined by summing and weighting the natural discharges according to their transporting capacity, generally using an appropriate transport formula (Shen, 1971b). In some channels the armoring by the coarse fraction at low flows (Emmett, 1976; Bagnold, 1977) and "hiding factors" for small grains resting between larger ones (Einstein, 1950) would have to be taken into account in calculating the dominant discharge for fine-bed material.

The dominant discharge is a function of grain size, with higher values for coarser fractions. Thus, there is actually a suite of dominant discharges for sediment transport that may in turn be considerably different than the dominant or formative discharges determining channel width or meandering characteristics of the stream. However, the mean annual flood or the bankfull discharge is often assumed to approximate the dominant discharge for all these aspects of fluvial behavior.

For reasons that will become apparent, the dominant discharge for bedrock channels is considered to be a constant discharge producing a rate of erosion equivalent to the naturally varying discharges. For some channels where weathering must precede erosion, the concept of a dominant discharge may be inapplicable.

In accord with the time perspective adopted here, the channel gradient for alluvial channels and the rate of bed erosion for bedrock channels are considered to be the primary dependent variables. The hydraulic regime also affects the channel cross-sectional characteristics (in particular, the channel width) and meandering behavior, and these can be considered to be either dependent or independent variables (Maddock, 1969; Schumm, 1969, 1971b). However, for the following reasons these will be considered here to be independent variables which can affect the adjustment of channel gradients or erosion rates:

1. The channel width and degree of meandering are determined by different hydraulic properties than is the channel gradient. In particular, the wash load and the finer suspended load play an integral role in determining channel width and sinuosity (Schumm, 1960, 1971b), whereas the fine sediment has only a secondary effect upon channel gradient. In addition, bank vegetation is a variable factor affecting channel width.

2. Some alluvial channels are constricted by bedrock or talus banks, as the Colorado River in the Grand Canyon. In many other channels the development of free meanders is constrained by the valley walls. These constraints may not directly influence the gradient, but they may do so indirectly through their effects upon the width and sinuosity.

3. Regime formulas for gradient of alluvial channels are presented in their most general form in terms of unit discharges of water and sediment.

4. Channel width can respond rapidly to change in hydraulic regime, in that single large floods can dramatically widen a channel (Schumm and Lichty, 1963; Burkham, 1972), whereas gradient generally responds more slowly. Thus, alluvial channel gradients are adjusted to long-term averages of channel width.

5. Changes of channel width and sinuosity in response to changes in hydraulic regime often have counteracting effects upon stream gradient. For example, channel widening by floods, which should cause steepening of alluvial channel gradients, is often accompanied by decrease in sinuosity, which has an opposite effect. This balance

of effects is cited by Schumm (1969) as the reason for the lack of aggradation or entrenchment of the Murrumbidgee River in Australia following a change in hydraulic regime which drastically changed channel patterns.

REGIME AND HYDRAULIC GEOMETRY OF ALLUVIAL CHANNELS

Natural stream channels are primarily alluvial, that is, they have at low flow stages a bed composed of unconsolidated sediment transported into place by the stream. Except under highly unsteady flow or if bed material is completely scoured during high flows, an alluvial channel carries a capacity load of the size ranges dominating the bed. This assumption is implicit in the use of engineering formulas to calculate the bedload or total sediment load from flow characteristics. Their low-flow alluvial covers occur because transport capacity decreases during falling states; alluvium transported at peak or dominant discharges must be redeposited at low flows. The thickness of the active layer, h, is approximately related to the high-discharge flow depth, D, by (assuming negligible transport at low flows)

$$h = \frac{CD}{S_s(1 - p)} \tag{1}$$

where C is the concentration of the bed-material load, S_s is the specific gravity of the sediment, and p is the bed porosity. The active bed thickness is small, ranging from only a few grains to as much as 2 cm for the Colorado River in the Grand Canyon (maximum flow depths of 10 m and sand-sized total load concentrations of 0.4%). Therefore, the much greater seasonal depth of scour and fill reported in sand-bed streams (up to 4.5 m in the Grand Canyon; Leopold and Maddock, 1953) must be due to other factors, such as (1) migration of bedforms (Foley, 1976), (2) a rapid-weir control mechanism (Silverston and Laursen, 1976), or (3) in small streams at least, to seasonal or flood-by-flood differences in rate of sediment supply from slope erosion.

Alluvial streams with stable hydraulic regimes have gradients that are just sufficient (or graded) to carry the supply of bed material in the size range dominating the bed (after Mackin, 1948). The evidence for this is largely indirect. Under steady flows and a constant sediment supply, flumes, which are presumably models of natural streams, adjust their gradients so that input and output of bed material load are equal. In natural streams the situation is complicated by variable discharges and fluctuating sediment supply, but the same type of adjustment presumably occurs on a statistical basis (Howard, in preparation, a). Alluvial channel gradients respond to changes in hydraulic regime in a manner that is predictable by sediment transport relationships. The gradient of alluvial channels is predictable from bed-material transport formulas using suitably averaged values of discharge and sediment load. Finally, the gradients of alluvial channels decrease downstream in a regular manner as would be predicted by downstream changes in discharge, sediment load, and grain size. The degree of uniqueness of alluvial channel grade is a subject of continuing discussion, largely beyond the scope of this chapter. The nature of alluvial channel regime is given closer attention in Howard (in preparation, a), although much of the data presented in this paper confirms the utility of the concepts of regime and grade.

The occurrence of abrupt spatial (downstream) and temporal transitions between fine- and coarse-bed channels, and the lack of beds with intermediate grain sizes is a consequence of the nonlinear rate laws for sediment transport, and, in particular, of the existence of a critical threshold stress for initiation of motion, which is the greater the coarser the grain size (although additional factors, such as differential abrasion and lack of supply of intermediate grain sizes may also in part account for the paucity of alluvial beds of intermediate grain size). Although the gradient of alluvial channels is roughly balanced at the minimum value sufficient to transport the sediment load supplied from upstream, in some channels the sparse load of coarse cobbles requires the steepest gradient, and in others the abundant fine bedload is the critical factor. The reasons for this dichotomy and the factors influencing which channel type will occur in a given situation will be illustrated by using sediment transport equations to predict the gradient necessary to transport each size fraction of the load supplied from upstream.

Calculation of Required Gradients

The procedure used is a variation of the normal engineering use of transport formulas; rather than calculating the quantity of bed material in transport as a function of velocity (or shear) and grain size of the bed material, the formulas are used to calculate the gradient needed (the *required gradient*) as a function of the size and quantity of bed material supplied from upstream with the dominant discharge. Since the sediment transport functions generally relate the quantity of sediment in transport to either the bed shear or the flow velocity and depth, a flow resistance relationship is needed in addition to relate these parameters to discharge and gradient. For most formulas the Manning equation was assumed:

$$V = \frac{R^{2/3} S^{1/2}}{n} \qquad \text{(mks units)} \tag{2}$$

where V is the average velocity, R the hydraulic radius, and S the gradient. For fine grain sizes the resistance coefficient, n, was given an arbitrary fixed value, for the coarser grain sizes the grain resistance was assumed to dominate and the Manning–Strickler relationship was used:

$$n = 0.04 d^{1/6} \qquad \text{(mks units)} \tag{3}$$

The most critical assumption made in the present calculations is that the spectrum of grain sizes supplied from upstream can be divided into size fractions, each with its characteristic long-term rate of supply, and that the required gradient can be calculated independently for each size fraction. In essence, the assumption is made that each size fraction is transported independently of each other. This requires justification, for there is obviously some interaction between all sizes in transport [e.g., Einstein's (1950) "hiding factor" for small bed-material grain sizes and Maddock's (1973) discussion of the influence of the wash load on bed-material transport rates]. However, in natural streams the grains dominating the bed are a small fraction of the total sediment throughflow. For example, the channel bed of the Colorado River at the Grand Canyon and Lee's Ferry gaging stations is a uniform sand, whereas the river transports a very well-graded mixture to Lake Mead (Fig. 1). The size fractions represented on the bed are presumably

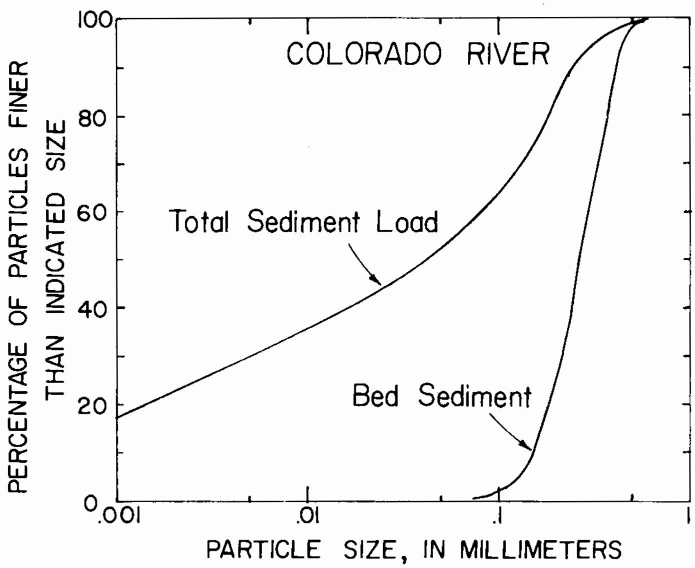

FIGURE 1 *Cumulative histograms of grain size for bed material and total sediment load of the Colorado River in the Grand Canyon. Bed sediment is average of several determinations at Lees Ferry and Grand Canyon gaging station. Total load curve from sedimentation in Lake Mead. (After Smith et al., 1960.)*

those that at that section of the channel are the most difficult to transport, that is, which require the steepest gradient. The proportion of the total sediment load which is finer than the bed material is transported in suspension at below-capacity concentrations (i.e., the wash load). Material coarser than the dominant bed material is carried in such small quantities that it does not accumulate, but remains dispersed on the bed, or is areally sorted (e.g., into riffles or cobble bars). Thus, there appears to be a small range of grain sizes selected from the total range in transport which dominates the bed and which is carried by the stream in capacity quantities. Correspondingly, it seems reasonable to assume that this same grain size range determines the required gradient, and that the critical range of grain sizes (i.e., the resultant dominant bed material grain sizes) from the total range of sizes supplied from upstream can be determined by finding the grain size range that gives the greatest required gradient given the quantity of supplied sediment for that size range.

The assumption of independence between transport of different size ranges certainly is not valid for very high sediment concentrations (i.e., for mudflows). The threshold between fine- and coarse-bed alluvial channels probably ceases to exist for mudflows, for its deposits are essentially unsorted.

The quantity of sediment supplied from upstream as a function of grain size has only rarely been determined for natural channels, the distribution being known most often where all the load cames to rest in a reservoir or natural delta, as in the case of the Colorado River (Fig. 1). For illustration purposes in the following calculations the quantity of supplied sediment was assumed to follow a log-normal frequency distribution with respect to grain size, so that the distribution can be completely characterized by its (logarithmic) mean and variance (Krumbein and Graybill, 1965). The log-normal distribution (or a more complex combination of such distributions) of grain sizes of sediments is commonly assumed to result from the weathering and erosional processes that supply sediment to stream channels as well as by the subsequent processes of abrasion and sorting that act during transport (Spencer, 1963; Visher, 1969; Middleton, 1976). The sediment transported by the Colorado River is clearly not log-normally distributed (Fig. 1), but the basic consequences of the model will hold for a wide range of distributions of

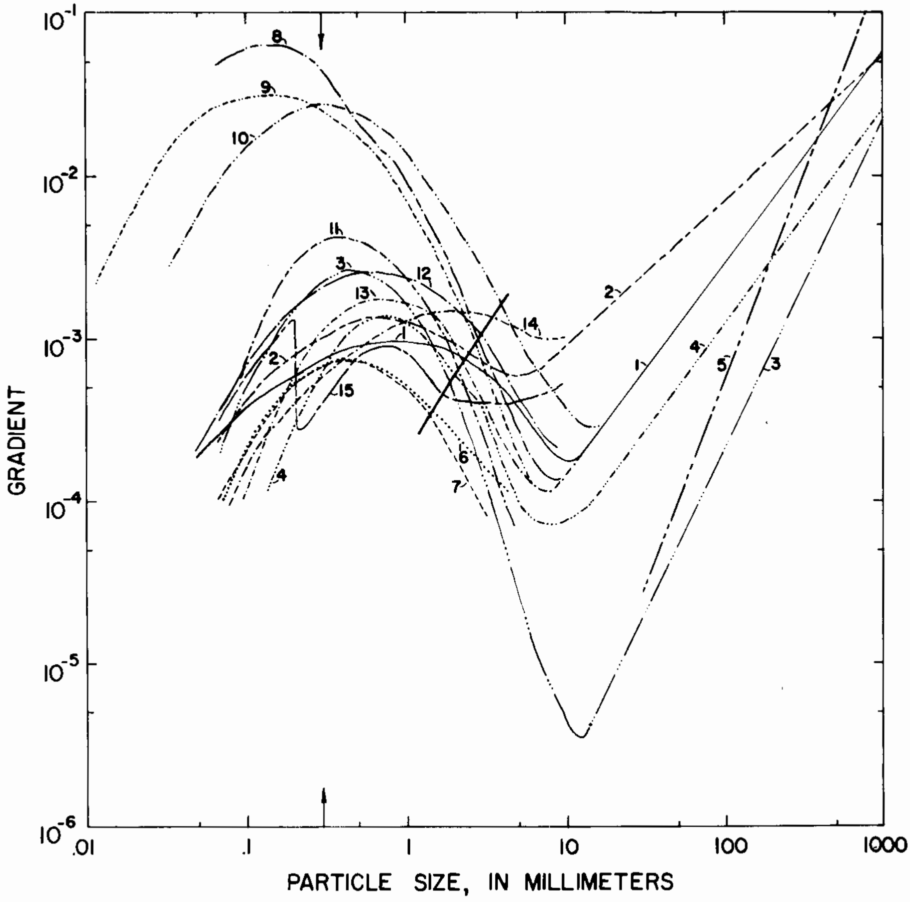

FIGURE 2 *Curves of required gradient versus grain size for several transport theories for nominal values of hydraulic variables. Median grain size is 0.3 mm (arrows). Curves for various transport formulas identified as follows (see Tables 1 and 2 for references): 1. Einstein-Brown with Shields criterion for threshold of motion; 2. Maddock; 3. Yang; 4. Ackers and White; 5. Baker and Ritter (threshold-of-motion criterion); 6. Shen; 7. Engelund-Hansen; 8. Bagnold bedload; 9. Meyer Peter; 10. Yalin; 11. Bagnold total load; 12. Kalinske; 13. Laursen; 14. Bogardi; 15. Peterson. Heavy solid line gives threshold of suspension.*

supplied sediment, so that the particular distribution assumed is not critical. The sediment load is broken into equal logarithmic size ranges, each with a characteristic grain size and quantity of supply, and the gradient required to transport that fraction has been calculated by the use of various bed- and total-load equations for the assumed hydraulic regime.

Only a subset of the numerous transport formulas were used in these calculations (generally the most well known or most recent ones), and a few could not be used because they cannot be easily solved for the required gradient. Some of the formulas are restricted in their range of applicability to fine grain sizes and high transport rates (e.g., the Shen and Bogardi relationships); that is, they are not valid for low transport rates near the threshold of motion. Others, however, like the Einstein, Maddock, and Yang relationships, include the threshold of motion criterion, and a few relationships have been proposed which are concerned solely with initiation of motion (e.g., Baker and Ritter, 1975). Some of the selected formulas were "bedload" and some were "total load" (the latter including the proportion of the bed material transported in suspension as opposed to saltation and rolling).

The variable input parameters to the gradient calculations are the log-mean grain size, d, the variance of the grain size distribution, θ, the water discharge, Q, the concentration of the sediment (sum of all grain sizes), C, the channel width, W, and the roughness, n. In all calculations the sediment specific gravity was assumed to be 2.65, and the water viscosity was held constant. A wide channel was also assumed, so that the hydraulic radius, R, equals the flow depth, D. Variations in the six basic parameters were considered independently with reference to a nominal set of parameter values (d = 0.3 mm; Q = 1000 m^3/s; W = 100 m; θ = 0.87; C = 0.01; n = 0.02). A graph of the required gradient versus grain size is shown in Figure 2 for the various transport formulas using the nominal parameter values. Figures 3 and 4 show, for one of the transport formulas (the Einstein-Brown relationship), the effect of variation of individual parameters with the rest fixed at their nominal values.

Factors Determining Bed Material Size

The curves of required gradient versus grain size for the various transport formulas under nominal conditions show the same basic pattern (Fig. 2): a local maximum lying within the range 0.1 to 2 mm, a local minimum near 5 to 10 mm, and a continuous increase for grain sizes beyond 10 mm, where the threshold of motion is the determining factor. The channel gradient at equilibrium, or in-regime, will presumably be the highest required gradient. But since the required gradient increases without limit for increased grain size in the threshold domain, the question arises why natural channels have finite gradients and, in fact, often have gradients determined by the maximum in the fine-grained range. The limiting factor must be one of supply. The required gradient for threshold conditions in natural channels will be determined by the supply of coarse grain sizes. Above some *critical* grain size the supply of sediment will be insufficient to form a coherent bed. If the required gradient for this size is greater than the fine-grained maximum, the bed will be dominated by this coarse grain size (because the flow will carry a below-capacity load of the size range corresponding to the fine-grained maximum, there will be little tendency toward deposition of the fine grains during waning flows). On the other hand, if the critical grain size is such that the gradient at initiation of motion is below the required gradient of the fine-grained maximum, the low-flow bed will be dominated by grain sizes near the fine-grained maximum (termed for convenience the *capacity* grain size), diluting the representation of the coarse grain sizes. The availability of coarse grain sizes (and hence the critical grain size) depends upon bedrock type, weathering and slope transport processes, upstream sorting and abrasion, and flow

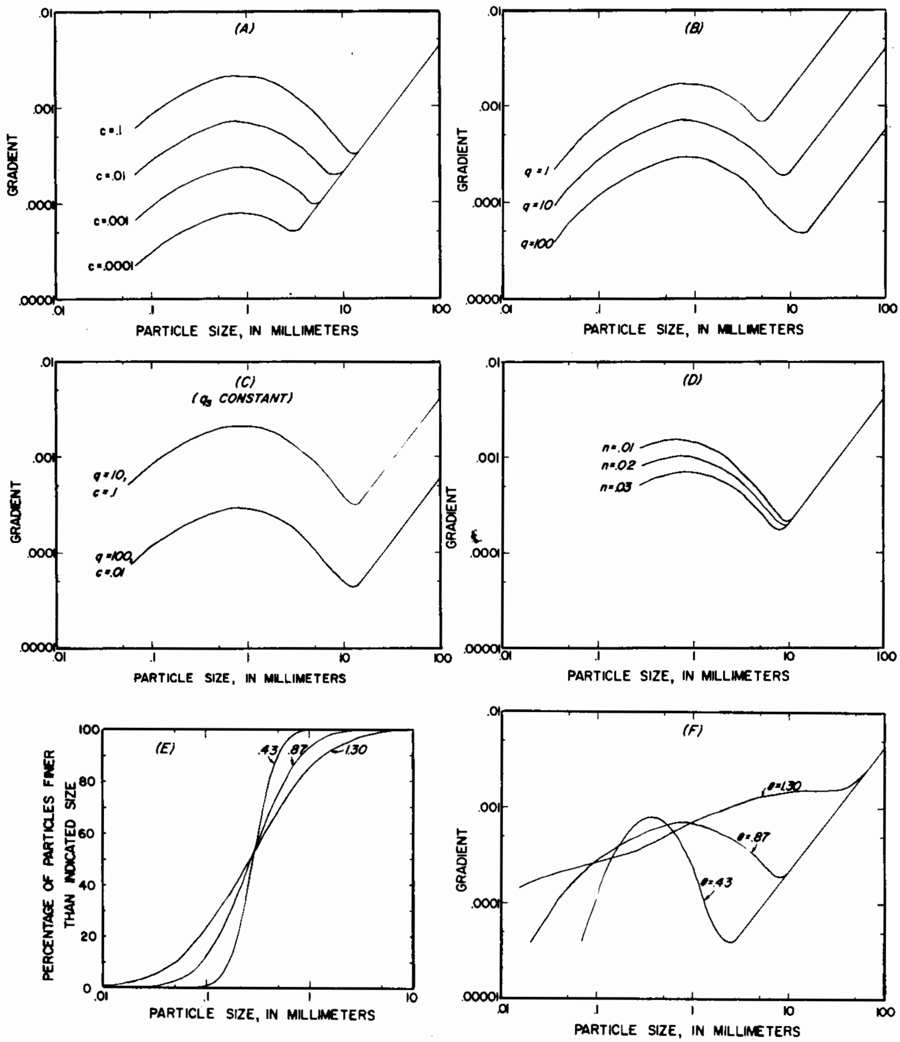

FIGURE 3 *Curves of required gradient versus grain size, showing effect of variation of single variables of the hydraulic regime with other variables constant at nominal values: (A) variation in concentration; (B) variation in specific discharge; (C) variation in specific discharge and concentration with specific sediment discharge fixed; (D) variation in roughness; (E) grain size distributions for different values of variance, θ; (F) required gradient curves for different values of variance.*

frequency characteristics (because the dominant discharge for the coarse grains will be higher than for the finer particles).

The effect of variations of the input parameters and of the derivative parameters $q = Q/W$, $q_s = C \cdot Q/W$, and $Q_s = C \cdot Q$ was investigated for the Einstein-Brown transport function (Figs. 3 and 4). Except as noted below, the behavior for other equations was

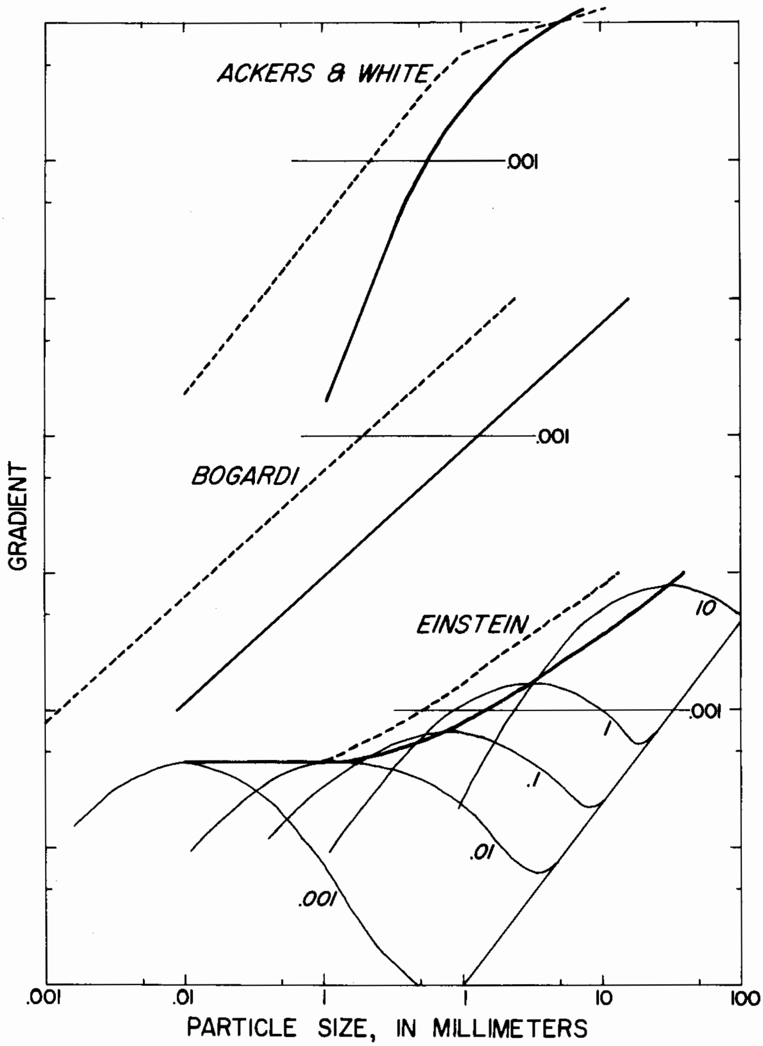

FIGURE 4 *Variation of maximum required gradient for fine-bed alluvial channels with average particle size for three transport relationships. Dashed lines show relationships between average particle size in transport and channel gradient, whereas solid lines show relationship between particle size of bed material and channel gradient. Representative curves of required gradient shown for several average grain sizes for the Einstein-Brown relationship.*

TABLE 1 *Comparison of Downstream Gradient Changes of Fine-Bed Streams with Predictions Using Sediment Transport Formulas*[a]

A. Predicted relationships for $S \propto q_s^{\alpha} d^{\beta} q^{\gamma}$

	α	γ	Value of β for indicated size of bed sediment: 0.1 mm	0.3mm	1 mm	3 mm
Bedload formulas						
1. Bagnold and Yalin	0.95	−0.86	0	0	0	0
2. Meyer-Peter	0.81	−0.86	0	0	0	0
3. Einstein-Brown	0.48	−0.86	0	0.31	0.54	0.64
Total-load formulas						
4. Bagnold	0.84	−1.09	1.48	0.77	0.15	0.16
5. Shen	0.54	−0.95	0.70	0.28	0.26	0.14
6. Laursen	0.60	−0.90	4.54	1.95	0.78	0.45
7. Yang	0.71	−0.99	1.83	1.02	0.41	0.11
8. Maddock	0.47	−0.81	0.58	0.53	0.54	0.49
9. Peterson	0.40	−0.79	—	—	—	—
10. Bogardi	0.35	−0.86	0.91	0.91	0.91	0.91
11. Kalinske	0.57	−0.86	0.57	0.57	0.57	0.57
12. Engelund and Hansen	0.87	−1.48	0.43	0.43	0.43	0.43
13. Ackers and White	0.91	−1.20	2.49	2.31	1.26	0.81

B. Empirical data for fine-bed streams

Geographic Area	Median grain size[b] (m)	$W \propto A^r$ r^c	$Q \propto A^e$ e^c	$Q_s \propto A^p$ p^c	$S = KL^a d^\beta$ (m): K	a	β	R^2[d]
Virginia Badlands[e]	0.003	0.25	0.9	1	0.22	−0.25	—	0.91
Utah Badlands[f]	0.003	0.24	0.9	1	2.15	−0.32	0.15	0.94
Great Plains[g]	0.004	0.13	0.71	0.85	3.31	−0.34	0.16	0.61
Ephemeral, New Mexico[h]	0.02	0.41	0.8	1	0.43	−0.25	—	0.85
Ephemeral, New Mexico[i]	0.008	0.4	0.8	0.9	0.19	−0.19	—	0.41

C. Comparison of predicted versus empirical values for dependence of gradient on equivalent length

$S \propto L^a$

Area	Predicted values of a for indicated transport formula: 1	2	3	4	5	6	7	8	9	10	11	12	13	Av.[j]	Empirical value
Virginia	0.20	0.06	−0.27	−0.11	−0.29	−0.18	−0.15	−0.23	−0.29	−0.40	−0.18	−0.42	−0.13	−0.24	−0.25
Utah	0.20	0.06	−0.27	−0.11	−0.29	−0.18	−0.15	−0.23	−0.29	−0.40	−0.18	−0.42	−0.13	−0.24	−0.32
Great Plains	0.21	0.10	−0.18	−0.03	−0.19	−0.11	−0.07	−0.15	−0.20	−0.29	−0.10	−0.27	−0.05	−0.15	−0.34
New Mexico	0.38	0.24	−0.09	0.12	−0.09	0.01	0.06	−0.06	−0.12	−0.22	0	−0.11	0.12	−0.03	−0.25
New Mexico	0.21	0.10	−0.18	−0.03	−0.19	−0.11	−0.07	−0.15	−0.20	−0.29	−0.10	−0.27	−0.05	−0.15	−0.19

[a] References for sediment transport formulas: 1. Bagnold bedload (Yalin, 1977, p. 117–122); Yalin bedload (Yalin, 1977, p. 143–148); 2. Meyer-Peter bedload (Shen, 1971a, p. 11–22); 3. Einstein-Brown bedload (Henderson, 1966, p. 440–441); 4. Bagnold total load (Yalin, 1977, p. 143–148); 5. Shen total load (Shen, 1971b); 6. Laursen total load (Graf, 1971, p. 213–214); 7. Yang total load (Yang and Stall, 1976); 8. Maddock total load (Maddock, 1976); 9. Peterson (Parker and Anderson, 1977); 10. Bogardi (1974, p. 324); 11. Kalinske (Henderson, 1966, p. 141–142); 12. Engelund and Hansen (ASCE, 1975, p. 208–209); 13. Ackers and White (1973).

[b] Average of individual median values.

[c] Two-decimal values of r, e, and p are estimated from empirical data, while one-digit figures are assumed.

[d] Coefficient of determination.

[e] Unpublished data from Howard (in preparation, a).

[f] Data from Howard (1970).

[g] Data from Schumm (1960, Table 1).

[h] Data from Leopold and Miller (1956, Appendices A and B).

[i] Data from Renard and Laursen (1975).

[j] Omits formulas number 1 and 2 from averages.

similar. However, because of the irregular behavior of the Peterson relationships with grain size, the following generalities do not pertain to these relationships:

1. Although the Einstein-Brown relationship is supposedly a bedload formula, it gives results more similar to the various total load relationships than to the other bedload formulas (Meyer-Peter, Bagnold, and Yalin), both in terms of predicted required gradients and corresponding capacity grain size (Fig. 2) and in the patterns of response to parameter variation (Table 1). The higher required gradients and finer capacity grain sizes predicted by the other bedload formulas apparently occur because they do not take into account suspension of bed material at high flow rates and high sediment concentrations, conditions that occur for the finer grain sizes in the nominal parameter values [the threshold of suspension is indicated on Figure 2 using the criterion of equality of shear velocity and fall velocity (Middleton, 1975)]. The total load equations, apparently including the Einstein-Brown relationship, are more suitable to regime analysis than the bedload equations because appreciable bed material moves in suspension in natural fine-bed streams. This will be apparent in later discussion comparing the downstream hydraulic geometry of fine-grained streams with the predictions based upon the various transport equations (Table 1).

2. Increase in sediment concentration (equivalent also to increase in Q_s since Q and W are fixed) causes a corresponding increase in the required gradient at the capacity grain size, whereas, of course, the initiation of motion gradients are unaffected (Fig. 3A). The capacity grain size is little affected by change in concentration.

3. Increase in the unit discharge, q, with other input parameters constant (equivalent to an increase in Q with W fixed or a decrease in W with Q fixed) reduces the required gradient at the capacity grain size less than the threshold gradient. The capacity grain size remains nearly constant (Fig. 3B).

4. Keeping q_s constant while q increases or C decreases (Fig. 3C) affects both the required gradient at the capacity grain size and the threshold gradients by the same amount, while leaving the capacity grain size unaffected. However, other combinations of threshold criteria and bedload or total load formulas will give slight differences in the rate of change of threshold and fine-grain maximum gradients to variations in q (Tables 1 and 2).

5. Change in roughness, n, through the range of 0.01 to 0.03 (Fig. 3D) reduces the required gradient at the capacity grain size by a factor of 2 or less (although the Yang, and Ackers and White formulas indicate a similar *increase* through this same range). The Maddock formula has no variable resistance parameter. Threshold values of coarse grain sizes are unaffected because of the assumption of the Manning-Strickler relationship.

6. Increase in the variance of grain sizes in transport (with the total concentration constant) broadens the curve of the fine-grained maximum while simultaneously reducing the depth of the valley at intermediate grain sizes (Fig. 3F). Simultaneously, the size of the capacity bed material is increased, whereas the required gradient is relatively unaffected. However, the various formulas differ somewhat in the detail of these variations.

7. The behavior of the predicted values of the required gradient with change of average grain size shows the greatest discrepancy in the various formulas (Fig. 4). In general, however, several conclusions can be made: (1) the capacity grain size is coarser than the average grain size (dashed lines in Fig. 4 show the relationship between the *average* grain size and the required gradient of the fine-grained maximum, whereas the solid lines show

the dependency of the required gradient upon the capacity grain size, the latter presumably being the dominant size of the bed material); (2) the threshold values of the required gradient are unaffected; (3) the width and depth of the "valley" between the fine-grained maximum and the threshold curve decreases as the grain size increases; and (4) the capacity grain size increases as the average grain size increases. It is with respect to this last observation that the various formulas show the greatest differences. Four basic patterns occur: (a) the required gradient is essentially unaffected by the average grain size (the Meyer-Peter, Yalin, and Bagnold bedload formulas, not shown on Fig. 4); (b) the required gradient increases more rapidly for coarser average grain size (the Einstein-Brown relationship); (c) the rate of change of gradient is independent of the average grain size (the Bogardi relationship shown in Fig. 4, and also the Kalinske formula); and (d) the required gradient increases more rapidly for fine than for coarse grain sizes (the Ackers and White relationship on Fig. 4 as well as all remaining total load formulas tested).

These conclusions, based upon the sediment transport formulas listed in Tables 1 and 2, may be partially inaccurate if Bagnold (1977) is correct in suggesting that bedload transport relationships should include a relative depth term, d/R, and that initiation of motion should be specified in terms of a critical power, $(\rho RSV)_c$, rather than a critical shear stress.

Downstream Hydraulic Geometry

The reality of the existence of the division of natural alluvial channels into fine-bed and coarse-bed channels is confirmed by regional empirical data on downstream hydraulic geometry. The bedload transport equations discussed above can be translated into regime equations for channel gradient. For the live-bed regime of fine-bed channels, these equations can be summarized as follows:

$$S = K_1 \frac{q_s^{\alpha} f(d)}{q^{\gamma}} \quad \text{or} \quad S = K_1 \frac{Q_s^{\alpha} f(d) W^{\gamma - \alpha}}{Q^{\gamma}} \tag{4}$$

For a limited range of bed material size $f(d)$ can be replaced by $K_2 d^{\beta}$. For threshold of motion conditions and coarse grain sizes, the channel regime can be represented by

$$S = K_3 \frac{d^{\beta'}}{q^{\gamma'}} \quad \text{or} \quad S = K_3 \frac{d^{\beta'} W^{\gamma'}}{Q^{\gamma'}} \tag{5}$$

In a climatically and physiographically homogeneous area, statistical downstream hydraulic relationships can be used to express the interrelationships among Q, W, Q_s, and the drainage area, A:

$$Q = K_4 A^e \tag{6}$$

$$Q_s = K_5 A^p \tag{7}$$

$$W = K_6 Q^b \tag{8}$$

$$W = K_7 A^r \quad \text{(implying that } r = e \cdot b) \tag{9}$$

TABLE 2 *Comparison of Downstream Gradient Changes of Coarse-Bed Streams with Initiation of Motion Criteria*

A. Predicted relationships for initiation of motion formulas[a,b]

	$S = K_1 d^{\beta''} R^{\epsilon}$			$S = K_2 d^{\beta'} q^{\gamma'}$		
Formula	K_1	β''	ϵ	K_2	β'	γ'
1. Shields	0.078	1.00	−1.00	0.42	1.29	−0.86
2. Yang	0.048	1.33	−1.33	0.46	2.00	−1.33
3. Maddock	0.019	0.50	−0.75	0.22	0.90	−0.60
4. Baker and Ritter	0.18	1.85	−1.00	1.15	2.50	−0.86

B. Regional empirical data for coarse-bed streams using bankfull depth; numbered columns give average ratio of predicted gradient to observed gradient, using formulas listed above

				S_p/S_o[k]					
Geographic area	K_1	β''	ϵ	1	2	3	4	Median grain size[c]	R^2[d]
Virginia and Maryland[e]	0.022	0.48	−0.64	0.82	0.21	0.61	0.15	0.050	0.17
Pennsylvania[f]	0.026	0.56	−1.33	0.71	0.19	0.49	0.13	0.050	0.60
Idaho[g]	0.015	0.15	−1.49	0.19	0.046	0.16	0.023	0.043	0.57
New Mexico[h]	0.104	0.96	−1.19	0.57	0.23	0.24	0.20	0.116	0.47
Various U.S. streams[i]	0.0083	0.30	−0.78	1.03	0.22	0.88	0.17	0.048	0.30

C. Regional empirical data for coarse-bed streams using estimated discharge, with comparison to predictions of initiation of motion formulas and equivalent length regressions

					S_p/S_o				$S = K_3 L^c d^{\beta'}$			
Geographic area	K_2	β'	γ'	R^2	1	2	3	4	K_3	c	β'	R^2
Virginia and Maryland[m]	0.039	0.42	−1.03	0.81	0.83	0.083	1.67	0.059	4255	−0.79	0.50	0.85
Pennsylvania[m]	0.067	0.47	−1.08	0.72	0.62	0.064	1.24	0.056	3376	−0.77	0.45	0.75
Idaho[j,l]	0.031	0.10	−0.89	0.37	0.21	0.021	0.44	0.009	116	−0.53	−0.01	0.38
New Mexico[m]	0.11	0.63	−0.48	0.39	1.13	0.36	1.20	0.22	41	−0.37	0.61	0.48
Various U.S. streams[l]	0.017	0.48	−0.36	0.20	1.53	0.15	3.17	0.10	0.051	0.08	0.63	0.12

[a]Grain size in meters, depth in meters, and q in m^2/sec.
[b]References for initiation of motion formulas: 1. Shields (Henderson, 1966); 2. Yang (Yang and Stall, 1976); 3. Maddock (1976); 4. Baker and Ritter (1975).
[c]Average of individual median values.
[d]Coefficient of determination.
[e]Data from Hack (1957, Table 8).
[f]Data from Brush (1961, Appendix A).
[g]Data from Emmett (1975, Tables 9, 12, 13).
[h]Data from Miller (1958).
[i]Data from Williams (1978b).
[j]Values for K_2, β', and γ' become more similar to other areas listed if d_{90} is used instead of d_{50}. These become 0.29, 0.52, and −1.00, respectively.
[k]Ratios averaged logarithmically.
[l]Uses bankfull discharge.
[m]Uses the mean annual flood.

Defining the ratio of $A/W = L$, the "equivalent length" (Howard, 1970), the regime equations can be expressed in terms of either q or L using the foregoing relationships. For live-bed conditions:

$$S = K_8 L^a f(d) \quad \text{or} \quad S = K_9 q^b f(d) \tag{10}$$

where

$$a = \alpha\left(\frac{p - r}{1 - r}\right) - \gamma\left(\frac{e - r}{1 - r}\right) \tag{11}$$

$$b = \alpha\left(\frac{p - e \cdot b}{e - e \cdot b}\right) - \gamma \tag{12}$$

For threshold of motion conditions:

$$S = K_{10} L^c d^{\beta''} \quad \text{or} \quad S = K_{11} q^{-\gamma'} d^{\beta'} \tag{13}$$

where

$$c = -\gamma'\left(\frac{e - r}{1 - r}\right) \tag{14}$$

The regime of channels at the threshold of motion can also be related to the grain size and hydraulic radius (depth):

$$S = K_{11} d^{\beta'} R^{\epsilon} \tag{15}$$

These relationships are compared in Tables 1 and 2 with empirical hydraulic geometry relationships derived from various sets of areal hydraulic geometry data. Data for coarse-bed alluvial channel networks have been used to estimate by linear regression (using logarithmic transforms of the data) the exponents of equations 13 and 15. These have been compared with the predictions of four formulas for the initiation of motion in Table 2. A similar approach was used for fine-bed channels (median bed material size less than 10 mm), empirically estimating exponents for equation 10 and comparing them with 13 bed material transport formulas (Table 1). Both the basic data and the summary comparisons in Tables 1 and 2 indicate that natural alluvial stream systems fall into two distinct ranges of size of bed material. In addition, the gradients *within* each of the two types of channel systems are similarly related to downstream changes in equivalent length (or unit discharge or channel depth) and grain size, despite widely different locations and climatic regimes, whereas there is a considerable difference for both predicted and empirical gradient relationships *between* the two types of channels. Finally, at least some of the sediment transport formulas give reasonable predictions of the regimes of the two types of streams.

The fine-bed stream systems are quite similar in the equivalent length regression (Table 1), despite widely different environments. By contrast, the predicted values of the equivalent length and grain size exponents (Table 1A, C) are quite varied, both for different transport formulas and because of the different dependencies of W, Q, and Q_S on A in the various areas. The observed dependency of gradient upon grain size is smaller than most of the transport formulas would suggest, but the range of variation of grain size in the empirical data is quite small. The predicted values of the equivalent length exponent are very sensitive to changes in the values of r, e, and p in equation 11. Since the hydraulic geometry relationships of equations 6–9 are estimated with small amounts of data showing a high degree of scatter (or in some cases are merely given assumed values), the high degree of variability in the regional predictions for the equivalent length exponent should be expected. However, the similarity among the regional empirical gradient relationships and the high coefficients of determination (Table 1B) can leave little doubt that the fine-bed streams are regime channels.

Coarse-bed stream systems are also similar to each other in their downstream gradient relationships (Table 2B, C) both in the exponents and in the constants, K_i. The coefficients of determination for the regional data regressions are generally high except that data from streams selected from throughout the United States show less consistent relationships than do the homogeneous areas, as would be expected. The eastern United States data have the greatest degree of regularity. As was the case with fine-bed streams, the fitted hydraulic geometry exponents fall close to at least some of the values predicted by threshold of motion formulas (Table 2A). In addition, since the sediment load does not enter into the formulas, the threshold relationships also predict the magnitude of the multiplicative constants, K_i. The closeness of the predicted threshold gradients, S_p, to the observed values, S_o, can be compared by calculating the average value of S_p/S_o (Tables 2B, C). If the channels are at the threshold of motion at high discharges (specifically, the bankfull discharge or the mean annual flood), these ratios should average unity. They are close to unity for the Shields and Maddock criteria, but are less than unity for the Yang, and the Baker and Ritter formulas. Overall, the values tend to be somewhat less than unity, suggesting that material of median bed size should move before bankfull discharge. However, imbrication and packing affect movement of coarse-bed material, so that the most representative bed material size for threshold conditions is coarser than the median (and is often assumed to be d_{84}), which would bring S_p/S_o values closer to unity. The Idaho data for streams fed by meltwater and draining high mountains may be intermediate in behavior between coarse- and fine-bed streams; these data have the lowest values of S_p/S_o, suggesting that at high flows these streams may be under live-bed rather than threshold conditions.

The upstream (drainage basin) conditions that determine whether a particular alluvial stream will be in "live-bed" or "threshold" regime are illustrated in Figures 2-4. In particular, live-bed streams occur where sediment concentrations are high but the range of grain sizes supplied is narrow. Other factors encouraging live-bed channels are large specific discharge, narrow channels, and a fine average grain size of supplied alluvium. Such conditions are met in most badlands and in the semiarid Great Plains, where erosion of poorly consolidated sedimentary rocks supplies high sediment yields but little coarse detritus. On the other hand, coarse-bed threshold channels occur where sediment loads are relatively low, but where coarse grain sizes are well represented. Thus high mountain areas with resistant bedrock, steep slopes, and a predominant role of physical weathering are likely to form threshold alluvial channels.

DYNAMICS OF BEDROCK CHANNELS

Although the *gradient* of bedrock channels is not directly determined by the hydraulic regime, their *rate of erosion* does depend upon the flow of water and sediment over the bed. Thus bedrock channels have been described by Einstein (1964) as "erosional" and by Gilbert (1877, 1914) as "corrasional." However, strictly descriptive characterization as "bedrock" channels is preferred here, for alluvial channels may also slowly erode their beds (Howard, in preparation, a).

Erosion of bedrock channels requires both weathering and detachment, processes that often go hand in hand. The type of process that is responsible for bed erosion varies with rock type and hydraulic regime. Resistant granites, for example, may be primarily eroded by sediment abrasion. In this case the rate of erosion would vary with the flow velocity, as well as with grain size, density, hardness, and quantity of the sediment supplied from upstream. Soluble rocks such as limestrone may be eroded primarily by chemical attack (particularly in subterranean channels) so that the rate of erosion would be dependent upon flow velocity as well as the chemical composition of the water. A similar situation may occur with some resistant rocks which must be weathered before grain detatchment can occur. In some cases, such as erosion of rills and gullies on soil or in shale badlands, weathering may be of secondary importance to detatchment, so that erosion rates are related either to the abrasion capacity of the bedload or simply to the exceedence of a critical shear stress on the channel bottom.

Under appropriate circumstances the erosion of bedrock channels can be quantitatively modeled. For example, Howard (in preparation, a) finds that erosion rates in badland bedrock channels are functionally related to the channel gradient and to the drainage area in a manner that is consistent with the assumption that erosion rates are proportional to the bed shear stress during high flows. In particular, the average erosion rate, E, increases in proportion to drainage area, A, and gradient, S, as follows:

$$E = KA^{\phi}S^{\sigma} \tag{16}$$

where ϕ and σ are constants and the factor K includes the effects of inherent bed erodibility and of the magnitude and frequency characteristics of the flow. If erosion rates were directly proportional to the bed shear stress, the parameters ϕ and σ would have the values 0.38 and 0.81, respectively, assuming also that bed resistance and channel shape (but not size) remain constant downstream (Howard, 1970; in preparation, a). Empirical measurements in badland topography estimate $\phi = 0.44$ and $\sigma = 0.68$ (Howard, in preparation, a), which is in reasonable agreement with the model.

Bedrock streams have no simple downstream hydraulic geometry, because bed and bank erodibility vary, and the gradient is a semiindependent variable. However, if the erosion rate is nearly uniform throughout a drainage network of bedrock channels on homogeneous rock (such as would occur during long-continued erosion in a high-relief area) and erosion rates follow the rate law above, then a consistent downstream hydraulic geometry results (Howard, 1970). In particular,

$$S \propto A^{-\phi/\sigma} \tag{17}$$

Factors Determining Occurrence of Bedrock Versus Alluvial Channels

The circumstances promoting bedrock instead of one of the two types of alluvial channels are complex, for the gradients of bedrock channels depends upon the past erosional history of the stream system. However, the interactions between alluvial and bedrock channels can be illustrated by numerically simulating possible erosional histories of a stream channel using equation 16, which is a partial differential equation, since the erosion rate, E, is $\partial y/\partial t$ and the gradient, S, is $\partial x/\partial t$, where y is the vertical and x the horizontal directions. The numerical simulations follow the erosion of a representative stream profile, constructed with the assumption that drainage area increases as the square of the distance downstream from the divide. The profiles shown in Figure 5 were evaluated downstream from the point at which the drainage area was unity to the downstream

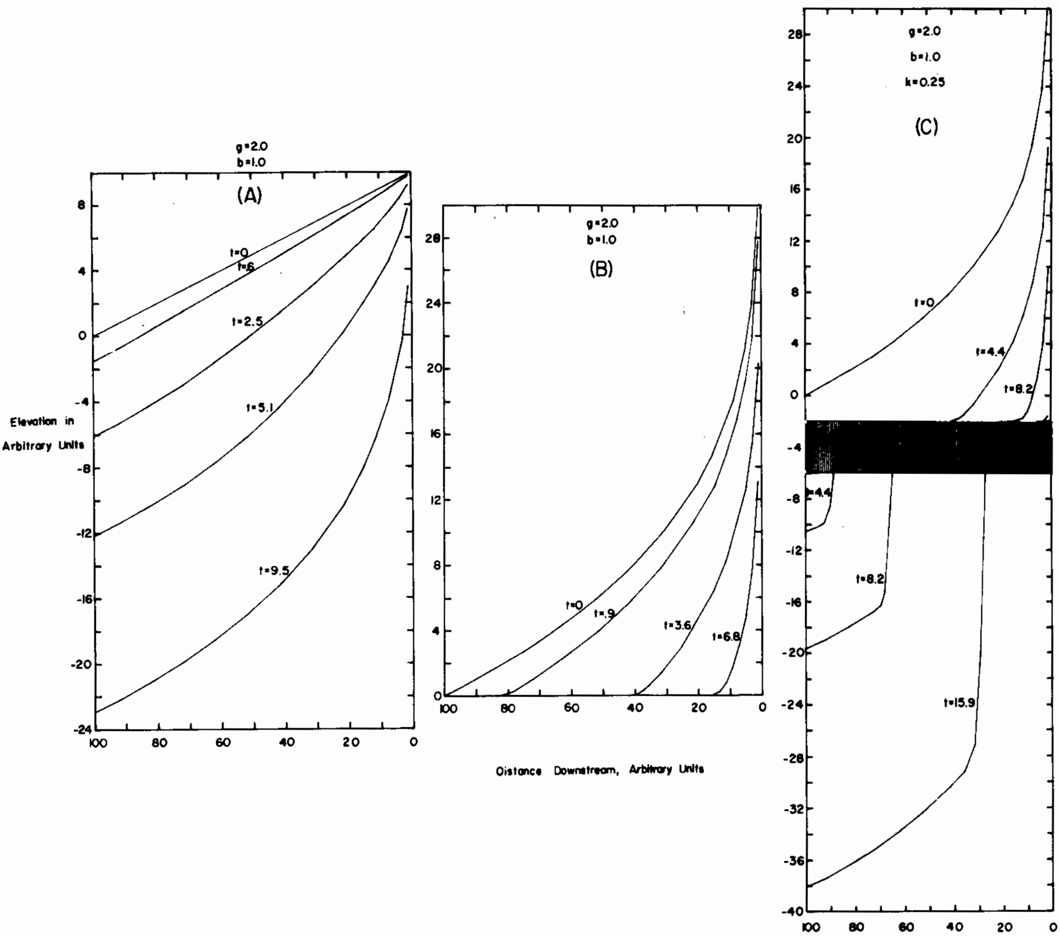

FIGURE 5 *Simulated erosion of bedrock channels under different assumed boundary conditions and bed erodibility. Bed erodibility, k, assumed to be unity except for the dark band in (C), which is four times less erodible. (A) Erosion with constant rate of downcutting of downstream terminus of stream (at position 100, time shown in arbitrary units); (B) erosion to a constant-elevation base level; and (C) erosion of a resistant layer. The value of b is the exponent in the equation $E = k\tau^b$, where τ is the bed shear, E the erosion rate, and k the erodibility. The value of g is the exponent in the downstream hydraulic geometry equation $R = CA_x^g$, where R is the hydraulic radius, A_x the cross-sectional area, and C a constant.*

end at x equal to 100 and a drainage area of 10,000. The boundary conditions are the initial stream profile and the vertical position of its downstream terminus through time. The erodibility (K in eq. 16) can be specified as a function of position (vertically and areally) and time. The numerical methods are outlined in Howard (1970). Several representative simulations were conducted:

Case 1. Uniform rate of lowering of the downstream end of the channel (Fig. 5A). An initially straight profile was assumed and the erodibility was uniform. Near the end of the simulation, a concave profile develops in which the rate of erosion is constant through time and equal throughout the drainage basin, that is, in a steady-state or dynamic equilibrium (Hack, 1960). In steady-state erosion headwater, bedrock channels might change downstream into alluvial streams, and, under sufficiently low overall erosion rates, the channel system might be entirely alluvial. To illustrate these relationships, equations 4–9 and 16 can be used to express the relative gradients required for a bedrock channel, S_b, for the given erosion rate and the gradient required for an alluvial channel, S_a. For this comparison the following additional assumptions are made:

$$K_5 = E \qquad \text{(from eqs. 6 and 16)} \tag{18}$$

$$p = 1 \qquad \text{(eq. 6)} \tag{19}$$

$$d = K_{12}A^u \tag{20}$$

Here E is the steady-state erosion rate. The ratio of gradients, S_b/S_a, for a live-bed alluvial channel is

$$\frac{S_b}{S_a} = K_{13}\frac{E^{1/\sigma - \alpha}}{A^{\alpha(1-r) - \gamma(e-r) + \beta u' + \phi/\sigma}} \tag{21}$$

Downstream grain size changes in fine-bed alluvial channels are generally slight, so βu is essentially zero. For reasonable values of the remaining parameters in the exponents in equation 21, both E and A are raised to positive powers. This suggests that bedrock streams will be favored for high erosion rates ($S_b/S_a > 1$), but that, because of the drainage area term, a downstream transition to a live-bed alluvial channel might occur if $S_b/S_a < 1$ (this assumes the ability of alluvial streams to erode the underlying bedrock up to a finite rate limit while keeping pace with overall basin erosion; such behavior is documented by Howard (in preparation, a) for badland channels in Virginia). Downstream transitions from bedrock rills and gullies to fine-bed alluvial channels are common in badlands (Smith, 1958; Schumm, 1956; howard, 1970; Howard, in preparation, a).

For threshold alluvial channels the equivalent gradient ratio is

$$\frac{S_b}{S_a} = K_{14}\frac{E^{1/\sigma}}{A^{\beta' u + \phi/\sigma - \gamma'(e-r)}} \tag{22}$$

In this case the exponent u is likely to have an appreciable negative value, as a result of downstream sorting and abrasion, so that the area exponent in equation 22 is likely to have a very small positive value, or, more likely, a negative value. Under these circumstances the stream system is likely to be either entirely coarse-bed alluvial or bedrock, depending upon the rate of overall erosion. A complicating factor is that the maximum grain size of bed material for threshold channels may increase with erosion rates, both due to increased rate of supply of all size ranges and due to production of coarser detritus on steeper slopes. This would increase the range of erosion rates over which threshold channels would be expected.

Conditions of essentially steady-state erosion are most closely approximated in areas with high relief or in the headwater areas of large drainage networks where base level influence is minimal (or, more precisely, is felt only over time scales longer than those considered in this paper).

Case 2. Base level constant (Fig. 5B). In this simulation the elevation of the downstream end of the profile was held constant. The initial profile was constructed to be steady-state for an arbitrary constant rate of base level lowering. Erosion to base level occurs in a finite time. More realistically, the lower parts of the stream network would become alluvial as the gradients dropped to the minimum necessary to transport the supplied sediment. The alluvial sections would gradually extend headward at the expense of the bedrock channels. If Schumm's (1963) speculation that orogeny is characterized by short periods of uplift followed by long periods of stability is correct, then during the periods of stability, alluvial channels should gradually extend headward at the expense of bedrock channels in a manner similar to this case.

Case 3. Effect of a resistant layer (Fig. 5C). Like case 1, base level was considered to be lowering at a uniform rate, initially with a steady-state profile in bedrock of uniform erodibility. However, the erodibility (K in eq. 16) was decreased in a horizontal layer, corresponding to a resistant layer. The rate of erosion in a bedrock stream encountering a resistant layer varies from place to place and through time. Through, and immediately below, the resistant layer stream gradients are higher than the steady-state case, whereas just above the resistant layer the gradient and rate of erosion are small. If the gradient becomes sufficiently small, a segment of alluvial stream may form, graded to the outcrop of the resistant layer. Howard (1970; in preparation, a) has outlined several cases in badland topography where alluvial stream segments have formed upstream from a control point in a resistant rock layer.

In summary, the simulations indicate that alluvial channel segments will develop if the base level remains constant or lowers very slowly. Likewise, alluvial channel will form upstream from resistant horizons. Although not illustrated in the simulations, alluvial channels will also occur where the original gradient of the stream channels on a constructional land surface is lower than the value required fo transport of sediment supplied from upstream.

CONCLUSIONS, APPLICATIONS, AND LIMITATIONS

Theoretical and empirical evidence has been presented for the occurrence of three distinct types of channel beds (bedrock, fine-bed alluvial, and coarse-bed alluvial) whose areal and temporal interactions are characterized by abrupt thresholds. In this concluding section some practical consequences of these thresholds to river behavior are detailed. In addition, a few more speculative implications are discussed, and the limitations in the application of the threshold concept are examined.

River Response

The response of rivers to a natural or man-induced change in the hydraulic regime will depend upon the preexisting nature of the bed. Under certain circumstances a small or gradual change in hydraulic regime can trigger a change to a different channel type. A few of many possible examples are discussed below; these examples concern primarily temporal thresholds, although some spatial (downstream) examples are considered.

Although the downstream hydraulic geometry of both alluvial and bedrock channels has been considered, one additional aspect remains for discussion. In going downstream in a drainage basin the unit discharge, q, increases while the sediment concentration remains nearly constant or decreases slightly. In addition, sorting and abrasion decrease coarser grain sizes more rapidly than finer, although Hack (1957) and Brush (1961) show that bed particle size of coarse-bed streams does not necessarily decrease downstream, but may actually increase, depending upon the geologic and physiographic situation. These factors tend to reduce required threshold gradients more rapidly than the live-bed gradients, so that downstream transitions from coarse- to fine-bed streams should be common. This may be illustrated by comparing the required gradient of a coarse-bed stream, S_c, to that of a fine-bed stream, S_f, using equations 4, 5, 21, and 22:

$$\frac{S_c}{S_f} = K_{15}\frac{d^{\beta' - \beta}}{q_s{}^{\alpha} q^{\gamma' - \gamma}} = K_{16}\,E^{-\alpha}A^{-\alpha(1 - r)\; - \;(\gamma' - \gamma)(e - r)\; + \;u\beta'} \tag{23}$$

In the second part of the equation, downstream changes in grain size of the fine material has been discounted. Since $\gamma' - \gamma$ is approximately zero (Tables 1 and 2), substitution of reasonable values of the other parameters suggests that coarse-bed channels may yield downstream to fine-bed channels, as observed by Yatsu (1955) and Shaw and Kellerhals (1977).

Equation 23 suggests that fine-bed channels would be favored by high erosion rates, so that changes in vegetation cover or runoff characteristics leading to higher sediment yields (e.g., by deforestation or construction activities) might convert coarse-bed channels to fine-bed channels by aggradation. However, in other cases where under natural conditions $S_c \gg S_f$, appreciable increases in sediment yield might not raise S_f to the point that aggradation is required. If increase in sediment yield occurs because of increasing relief within a drainage basin, then the indicated effect of erosion rates in equation 23 may be offset by increases in the critical grain size for coarse bed streams, owing both to the increased quantity of coarse load delivered and a probable increase in the average size of coarse detritus due to steeper slope gradients.

Changes in the hydraulic regime can also lead to conversion of an alluvial channel to a bedrock channel, or vice versa. Seasonal alternations between live-bed and bedrock channels in Virginia badlands resulting from higher winter sediment concentrations have been documented by Howard (in preparation, a).

An increase in channel width increases required threshold gradients more than fine-bed gradients. Thus, a local or systematic increase in channel width might trigger conversion to a coarse-bed channel. For example, the Colorado River in the Grand Canyon is a sand-bed channel where the width is constricted by talus or resistent bedrock, but cobble bars are common where less resistant rocks allow channel widening (Howard, in preparation, b). In streams where $S_f \approx S_c$, coarse and fine material may be areally sorted into riffles and pools, respectively. This may in part be due to width effects; Keller and Melhorn (1973) indicate that riffles are associated with divergent flow and pools with convergent flow.

Although much has been written on the response of rivers to changes in hydraulic regime (Schumm, 1969, 1971a; Santos-Cayudo and Simons, 1972; Simons and Senturk, 1976), the effects of channel type upon the pattern and rate of adjustment have been largely ignored. In particular, bedrock, fine-bed, and coarse-bed channels have considerably different characteristic time constants (or "relaxation times"; see Howard, 1965; Chorley and Kennedy, 1971; Allen, 1974) for response to changes in hydraulic regime. Generally, the slowest to respond are the bedrock channels, which in many cases (excepting badlands and soil gullying) can be considered to be noneroding over engineering time scales. Threshold channels may be very slow to respond to changes in regime because of the slow average rate of motion of the coarse bed pavement and the prominant roles of sorting and in situ weathering and abrasion in downstream changes in grain size (Schumm and Stevens, 1973). In fact, it is possible that some of the coarse cobble pavements of high mountain streams originated during periods of accentuated physical weathering and mass movement during the late Pleistocene or to later erosion of this coarse debris (Brush, 1961; Miller, 1958). Live-bed channels respond much more rapidly to change in regime, with response times measured in months, years, or decades, depending upon the size of the drainage network (time scales for aggradation and entrenchment are approximately related to the square of the length of the alluvial channel; Gessler, 1971).

The difference in characteristic time scales between threshold and live-bed channels can result in short-term adjustments to changed hydraulic regime which are different than the ultimate response (i.e., the response can be divided into primary and secondary responses; Howard, 1965). For example, an increase in discharge, a decrease in sediment load, or a man-made increase in gradient due to meander cutoff should require a decrease in gradient of fine-bed alluvial channels by degradation. However, if the channel transports a small percentage of coarse detritus, the scouring of the underlying alluvium may produce a gravel pavement (threshold channel), owing to the slower transport rates of the coarse fraction. Such transitions are frequent below dams, where in addition to vastly reduced sediment supply, the competency of the flow is reduced by clipping of flood peaks. In such circumstances the amount of scouring required to develop a pavement can be calculated from the grain size distribution of the alluvium (Gessler, 1971; Pemberton, 1976; Simons and Senturk, 1976).

The inverse situation may occur if a gradient increase is required through aggradation. If a threshold alluvial channel receives a greater influx of sediment without change in grain size distribution (resulting from, say, deforestation or upstream urbanization), the long-term equilibrium response might be the development of a steeper coarse-bed channel due to increase in the critical grain size. However, if the increased sediment load also increases the required gradient for the fine-bed material above the existing channel gradient, the immediate response may be a temporary conversion of the channel to a fine-bed. An even more dramatic transition will occur if a rise in local base level (such as by construction of a dam) requires upstream alluviation of a threshold channel (assuming the hydraulic regime is unchanged). Because of the slow delivery of the coarse fraction forming the bed, the initial alluviation will occur by deposition of the fine sediment load at a gradient smaller than the ultimate threshold equilibrium. For practical purposes the long-term equilibrium may not be established during the lifetime of the dam.

Deficiencies of Intermediate Bed Particle Sizes

Natural alluvial streams normally carry less than a capacity load of the grain sizes finer than those represented on the bed (i.e., the wash load), although the grain size dividing the suspended and wash load varies with the discharge (Partheniades, 1977). It appears to be universally accepted that the grain sizes represented on the bed are carried at capacity (e.g., Partheniades, 1977; Shen, 1971a; Simons and Senturk, 1976). However, in fine-bed streams grain sizes coarser than the median bed material (the capacity grain size) are also carried in less-than-capacity amounts, although they are represented on the bed because they move by saltation or rolling. This is because the required gradient for transport of this coarse material is less than the actual gradient, which is determined by the capacity grain size (Fig. 2).

This phenomenon may help to explain the commonly observed deficiency of grain sizes in the range of about 1–10 mm in fluvial sediments (Church and Gilbert, 1975; Slatt and Hoskins, 1968; Emmett, 1976; Williams, 1978a; Yatsu, 1955). This grain size range corresponds to the valley in the required gradient curves in Figure 2. Material in this size range, being carried below capacity as bedload, will only be deposited at very low flows; thus, fluvial sediments should be relatively depleted in this size range. By contrast, the still coarser grain sizes near the critical value move much more slowly on the average, since they are moved only during high discharges, so that they are well represented on the bed. Assuming spherical particles, the percentage of the bed area, P, covered by particles of grain size d supplied in quantity q_{sd} and traveling with an average velocity V_p (including periods of no motion) is given by

$$P = 100 \times \frac{6}{4} \frac{q_{sd}}{d V_p} \tag{24}$$

where q_{sd} is measured in units of volume of solids per unit time and channel width. Thus, the strong representation of fine bed material near the capacity grain size is due to high q_{sd} and small d, despite relatively high V_p. On the other hand, the grain sizes near the critical grain size have nearly zero V_p, giving a high P despite large d and small q_{sd} The deficient grain size range has high V_p compared to the supply rate.

The usual alternative explanations for the deficiency of intermediate bed material grain sizes has been either that the original weathering processes produce little detritus in this size range or that abrasion and breaking of grains is rapid for these coarser grains but is less so for sands, which have smaller momentum relative to the fluid drag. Another possible explanation for the deficient grain sizes are the techniques used for size measurement of bed material: seiving for finer grain sizes and transect measurements for pebbles and cobbles. The intermediate grain sizes may possibly be underestimated in abundance, since they fall between techniques. Distinguishing between these possible explanations should be possible by examining the size range of all material deposited by rivers in estuaries, lakes, and oceans. If the bimodal distribution of fluvial sediments is due to the sorting process described above, the missing intermediate grain sizes should be found in the still-water environments in their original proportions, whereas if it is due to source deficiencies or differential abrasion, the paucity should also occur in the downstream sediment sink. In addition, if differential sorting is the mechanism producing the bimodal distribution, the grain size range of the deficient sizes will vary from environment to environment in conjunction with differences in median grain size and variance of the detritus supplied by slope erosion (see Figs. 3 and 4). This may help to account for the apparent lack of bimodal grain size distributions in aggregate averages of grain size distributions from diverse fluvial environments (Shea, 1974).

Miscellaneous Aspects of Alluvial Channel Thresholds

Few "live-bed" alluvial streams (with gradients determined by the capacity grain size) have beds coarser than a few millimeters. The main reason for this is the large quantity of bed material that must be supplied to maintain live-bed conditions for coarse bed sizes. In the Einstein-Brown transport relationship two channels will have equivalent bed dynamics if their values of the parameter Φ are equal:

$$\Phi = \frac{q_s}{wd} \cong \frac{q_s}{Kd^{3/2}} \quad \text{(for coarse-bed material)}$$

Thus, live-bed (large Φ) conditions for coarse-bed sizes requires larger volume of suppled load than for fine-bed sizes, a situation that seems to occur only in proglacial streams (e.g., Church and Gilbert, 1975; Fahnestock, 1963) or in streams draining alpine areas characterized by meltwater flood peaks and abundant coarse debris produced by physical weathering (Emmett, 1976). Such areas are also characterized by a high variance in the distribution of grain sizes in the bed material load. These streams are likely to have a correspondingly large range in the size of bed material, because the required gradient varies very slowly with grain size for large variance (Fig. 3F).

The sharp transitions between bed types may in part be due to the difference in ease of transport of bed material depending upon the type of underlying surface. In eolian transport fine saltating particles rebound much higher when moving over a fixed surface or over coarse grains (Bagnold, 1941; Ellwood et al., 1975). As a result, Bagnold noted that the capacity of the wind to transport saltating grains diminishes where the sand passes onto a loose sand surface (the saltation height is included in theoretical bedload transport relationships of Bagnold, 1973; 1977); as a result, downwind transitions from fixed or coarse-grained beds to fine sand are abrupt. A similar mechanism may help to account for abrupt downstream transitions between bedrock and sand, bedrock and gravel, or gravel and sand beds.

Coarse-bed alluvial channels may occur in two intergrading types. Along downstream portions of channels in areas of homogeneous high relief most of the gravel forming the bed has been transported from headwater areas and has been subject to systematic sorting and abrasion. However, in places (particularly in headwater tributaries), the bed may be steepened by very local additions of coarse debris from adjacent slopes which is not moved downstream except after considerable in situ weathering and abrasion. The smallest tributaries in areas dominated by physical weathering (e.g., in mountains of the Mojave desert) are often floored by coarse detritus only a few degrees less steep than its angle of repose. The distinction between local and basin-wide source for coarse-bed material is very marked along the Colorado River in the Grand Canyon, where riffles composed of rounded gravel in transport along the river contrast sharply with steep rapids in angular boulders added locally at the entrance of small, steep tributaries (Howard, in preparation, b).

Uncertainties and Limitations

Although the present analysis seems to *explain* why thresholds occur between channel types, it offers little aid in making *a priori predictions* of which type of bed would occur in a particular segment of a particular stream except in cases where the stream is clearly not operating close to any of the thresholds between channel types. Prediction of alluvial channel gradients and bed type requires knowledge of the quantity and size distribution of the sediment supplied from slope erosion; since the details of sediment supply cannot be easily predicted from morphological characteristics of the drainage basin, intrastream measurements are required. The capacity grain size and required gradient for fine-bed streams should be calculable to reasonable accuracy from long-term, systematic measurements of bed and suspended loads. However, the critical grain size for coarse-bed streams (and the corresponding required gradient) depends upon the relatively small rate of supply of coarse grain sizes and upon the frequency of flood peaks. Sampling of coarse bedload is notoriously difficult. The problem is further complicated by the variable porosity and fabric of coarse beds, as well as by the action of sorting and abrasion. Finally, because of the long time constant of coarse-bed streams, it is possible that the channel may be disequilibrium with the present hydraulic regime. The occurrence of bedrock channels depends upon past erosional history and resistance of the bedrock to erosion, factors difficult to quantity. However, the measurement of the channel gradient will tell whether it is steep enough that an alluvial bed does not form, except for gradients near the threshold to an alluvial channel.

Maddock (1969) feels that resistance due to bedforms is an independent method by which streams may adjust to changes in hydraulic regime. The resistance relationship used in this paper (the Manning equation) is certainly an oversimplification of the actual dependence of resistance on the hydraulic regime, particularly for fine-bed alluvial streams. The effect of the development of bedforms upon the capacity grain size and the required gradient needs to be further investigated. Actual curves of required gradient may more resemble the complex curve of the Peterson relationship than the simpler peaks in the fine-grain range indicated by the other formulas (Fig. 2), and it is possible that additional thresholds due to change in bed resistance may occur within fine-bed alluval channels. The situation is further complicated by the dependence of bedforms upon river stage and the disequilibrium that may occur between bedforms and the hydraulic regime due to the finite relaxation time of bedforms (Allen, 1974).

The main variable of hydraulic geometry considered in this paper has been the gradient. However, other variables of hydraulic geometry probably also exhibit different behavior, depending upon the type of channel bed. In particular, this should be true of channel width and sinuosity, which up to this point in the paper have been considered to be independent variables. Bedrock channels are commonly narrow and straight, because bedrock also forms the channel banks. Fine-bed channels are quite variable in width and sinuosity, depending upon the quantity of suspended and wash load and the amount of vegetation (Schumm, 1960). The extent of variation of width, sinuosity, and other hydraulic geometry variables (except gradient) with bed type is uncertain, yet it is clear that hydraulic geometry relationships would be more consistent if the data were stratified according to channel type. For example, Li et al. (1976) have derived essentially complete equations of hydraulic geometry for networks of nonbraided coarse-bed channels with banks of the same composition as the bed which appear to fit the Pennsylvania streams measured by Brush (1961).

In conclusion, the occurrence of three distinct types of channel beds (bedrock, as well as fine- and coarse-bed alluvial) is the result of thresholds in sediment transport mechanics. Because temporal or spatial variations in the controlling hydraulic regime often trigger change from one channel type to another, each with a distinct hydraulic geometry and pattern of river response, the threshold behavior should be incorporated into future modeling of fluvial mechanics. Further study of threshold behavior is desirable, including field studies, flume experimentation, and theory.

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